

RESEARCH ARTICLE

On Application of Dynamic Program Fixed Point Iterative Method of Optimization in the Determination of the Shortest Route (Path) Between Government House and Amuzukwu Primary School, All in Umuahia, Abia State

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ABSTRACT

In this research, dynamic programming seeks to address the problem of determining the shortest path between a source and a sink by the method of a fixed point iteration well defined in the metric space (X, d) , d the distance on $X = U$ the connected series of edges that suitably work with the formula

$$\begin{aligned}x_{n+1} &= f(x_n) = x^* = F(X) \\ &= \text{dist}(S_0, S_k) = \min[U_j, i] \\ &= \sum_{i,j}^n \min[u_j, i] = \min \sum_{i,j}^n [u_i + d_{ij}, i], \\ U_{ij} &\geq 0, S_0 = \text{source}, S_k = \text{sink}\end{aligned}$$

Such that

$$d_{ij} + d_{kj} \leq d_{ik}, i \neq k, j \neq k, i \neq j$$

with the pivot row and pivot column being row k .

Then, evaluation of the shortest route between Government House and Amuzukwu Primary School all in Umuahia and Abuja by the above method revealed it to be 720 m by taking the route SACDFG.

It was remarked that the longest route which is the route from government House to Ibiama road, to Aba road, to Warri road, to Club road, to Uwalaka road, and finally to Amuzukwu Road which now terminates at our Destination, Amuzukwu Primary School with Road distance of 2590 m does not possess other advantages while it should be made use of. The shortest routes were necessarily recommended to road users as the best route to use because its route SACDJFGT is the shortest route with the distance of 1790 m.

Key words: Complete metric space, dynamic programming, Dijkstra's algorithm, Greedy and Prim's algorithm, pseudo contractive fixed point method, source and node

2010 Mathematics Subject Classification: 46B25, 65K10

INTRODUCTION

There are few basics of dynamic programming problems which must be discussed before the details. Those basics are discussed below. A route is defined as a course of travel, especially between two distant points/locations while shortest is sound

to be a relatively smallest length, range, scope, etc., than others of its kind, type, etc. Therefore, we say that the shortest route is the relatively smallest of its kind, especially between two distant points/locations. The route must be accessible/useable by a motor vehicle, the route may be single or double lane. The routes may possess bus stops, junctions, interconnected streets, or venues for join. The routes may be traced or not but should be wide enough to be used by a motor vehicle. The routes may short or interconnect with another route that

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started from Umuahia to end at Abuja that is the route should have a source and a sink.^[1-3]

Therefore, “the application of shortest route/path in Dynamic programming” can be seen as the practical use of the shortest course of travel by road users among other routes of its kind, especially between two points/locations.

It is, however, disturbing to note that much of the available routes from Umuahia to Abuja have one traveling challenge or the other such as long distance, police menace, traffic jams, and bad road networks.^[4] Perhaps, it is necessary to answer some questions to really appreciate the issue on ground. These questionnaires are how do road users view the various existing routes from Umuahia to Abuja? Which of the routes are preserved by road users? and What route should one undertake to minimize time and distance of travel/from the researcher’s observation, it was clear that the route quality such as shortest distance freedom from police menace and traffic jam as well as good road networks, all contributed to affect road users decision of route to make use of when traveling from Umuahia to Abuja in Nigeria. Definitely, it is important to note that:

- a. This work is limited to time and distance of travel by motor vehicle on road excluding the effects of traffic jams, police menace, bad road network, etc., and number of routes from Umuahia to Abuja.
- b. This study will be of major significance to travelers and transporters (who are major beneficiaries).
- c. The study will help us appreciate the importance and practical use of dynamic programming in determining the shortest route of travel when traveling from one location to the other.
- d. To carry out the study, the following hypothesis was formulated for investigation.
 - i. Any part of the shortest route from Umuahia to Abuja is itself a shortest path [Table 1].
 - ii. Any part of an optimal path is itself optimal. The above two hypotheses are also known as “the principle of optimality [Table 2].”
 - iii. **Walk:** A walk is simply a route, in the graph along a connected series of edges. *BCAD* is a walk from *B* to *D* through *C* and *ABDE* is a walk from *A* to *E* in a walk edges and vertices may be repeated [Table 3].
 - iv. **Trail:** When all the edges of a walk are different, the walk is called a trail. *BCD* is

a trail from *B* to *D*. A closed trail is one in which the start and finish vertices are the same. *ADECDBA* is a closed trail [Table 4].

- v. **Path:** This is a special kind of trail if all the vertices of a trail are distinct then the trail is a path *ABCE* is a path, all edges and all vertices are distinct in a path [Table 5].
- vi. **Cycle:** A cycle ends where it starts and all the edges and vertices in between are distinct *ABDA* and *ABCEDA* are as vertices have been repeated [Table 6].
- vii. **Tree:** This is a connected graph which contains no cycles.
Note that, a tree with n vertices has $n - 1$ edges.
- viii. **Vertex Degree:** The degree of a vertex is the number of edges touching the vertex
- ix. **Directed Graph or Diagraph:** It is a graph in which each of a diagraph is called an arc
- x. **Weight:** The edges of a graph are often given a number which can represent some physical property, for example, length, cost time, and profit. The general term for this number is weight.
- xi. **Network:** A graph whose edges have all been weighted is called a network.
- xii. **Stage and State:** The stage tells us how “Far” the vertex in question is from the destination vertex while the states refer directly to the vertices.
- xiii. **Action:** This refers to possible choices at each vertex.
- xiv. **Value:** The numbers calculated for each state at each stage are referred to as values.
- xv. **The Optimal Value:** The optimal value is the label which is assigned to the vertex. The value is also known as the Bellman function.

Major introduction [methods of determining the shortest route/path]

There abound several methods of determining the shortest route/path from one location/point to the other in this section we shall do well to review some of the existing methods of finding the shortest route.

The dynamic programming technique

The network below [Robert and Lynda (1999)] can help us explain the dynamic programming technique.

To find the shortest (or longest path from S to T in the above network), we begin at the destination vertex T . The vertices next to T best route from these are examined. These are Stage 1 vertex the best route from these to T is noted. We now move to the next set of vertices, moving away from T toward S , i.e., the Stage 2 vertices. The optimal route from these vertices to T is found using the already calculated optimal route from the Stage 1 vertices. Then, this process is repeated until the start vertex, S , is reached. The optimal route from S to T can then be found that the principle of optimality is used at each stage, the current optimal path is developed from the previously found optimal path.

Since the method involves starting with the destination vertex and working back to start vertex, it is often called backward dynamic programming.

Dijkstra's algorithm

Dijkstra's algorithm is a method of determining the shortest path between two vertices. The shortest path is found stage by stage. In finding the shortest route to a vertex, we assign to the vertex various numbers. These numbers are simply the length of various paths to that vertex. As there may be many possible paths to a vertex then several different numbers may be assigned to it. Of all possible numbers assigned to a vertex, the smallest one is important. We call this smallest number a label.^[5] The label gives the length of the shortest path to the vertex, suppose we wish to find the shortest path from S to T in a network, the algorithm can be presented in three steps. Since the algorithm can be applied to both graphs and diagraphs, the word "arc" can be replaced "edge" in the following steps [Taha (2002)] $\min [U_j, i] = \min [U_j + d_{ij}, i]$; $d_{ij} \geq 0$, outlined in the following details below.

Step 1: Assign a label O to S

Step 2: This is the general step. Look at a vertex which has just been assigned to Label, say the vertex is A via a single are, say that this vertex is B to B assign the number given by (label of $A + weight(AB)$). If a vertex is reachable by more than one route assign to it the minimum possible such number. Repeat this process with all vertices that have just been assigned a label and all vertices that are reachable from them. When all reachable vertices have been assigned a number the minimum number

is converted into a label. Repeat step 2 until the final vertex T is assigned a label.

Step 3: Steps 1 and 2 have simply found the length of the shortest route this step finds the actual shortest route, we begin at the destination vertex T an arc AB is included whenever the condition label B of $A = weight$ of AB holds true. This route may not be unique.

Greedy and Prim's algorithm

These algorithms are used mainly by television and telephone companies in competing the cities by a cable so that their Carle television and telephone facilities are made available to them,^[6] that is, these algorithms help to solve problems known as minimum connector problem, which means connecting cities with minimum amount of cable [Oyeka (1996)]

$$d_{ij} + d_{jk} < d_{ik}$$

In graph theory terms, the cities are vertices and the cable is edge. If the vertices are connected in such a way that a cycle exists, then at least one edge could be removed leaving the vertices still connected. Recalling that a connected graph which contains no cycles called a tree, it is clear that the best way of connecting all the vertices would be to find a tree which passes through very vertex. The networks below illustrate this.

A tree which passes through all the vertices of a network is called a spanning tree. Spanning tree which has the shortest total length is a minimum spanning tree. There may be more than one minimum spanning tree. The problem faced by the television or telephone companies is to find a minimum spanning tree of the network.

There are two [Oputa (2005)] algorithms which may be used to find a minimum spanning tree

- (i) The Greedy algorithm
- (ii) Prim's algorithm

They are essentially the same algorithm and really only differ in the way they are set on.

The Greedy algorithm builds up the tree adding one vertex and one edge with each application. Any vertex can be used as a starting the vertex added at each stage unused vertex nearest to any vertex which is already a part of the tree and that the edge added is the shortest available edge. The Greedy algorithm may be summarizing as follows:

Step 1: Choose any vertex as a starting vertex.

Step 2: Connect the starting vertex to the nearest vertex.

Step 3: Connect the nearest unused vertex to the tree.

Step 4: Repeat step 3 until all vertices have been included.

Prim's algorithms uses a tabular format making it more suitable for computing purposes since as mentioned earlier greedy and Prim's algorithms are basically the same, it will be enough to illustrate how the Greedy and Prim's algorithms are used by working through a specific example in chapter three.

BASIC RESULTS

Preliminaries

Let X be a non-empty set and d or ρ a function defined on $X \times X$ into the set of real numbers R such that [Stafford (1969)]

$$d(\dots): X \times X \rightarrow R$$

satisfying the following conditions

- (i) $d(x,y) = 0$ if and only if $x = y$
- (ii) $d(x,y) = d(y,x)$ for all $x,y \in X$
- (iii) $d(x,y) \leq d(x,z) + d(z,y)$ for all $x, y, z \in X$

The number $d(x,y)$ is called the distance between x and y , d is called the metric and the pair (X,d) is called the metric space.

Definition 2.1 [Danbury (1992)]: A subset A of a metric space is said to be bounded if there is a positive constant M such that $d(x,y) \leq M$ for all $x,y \in A$.

Definition 2.2 [Danbury (1992)]: A subset A of a metric space is called a closed set if every convergent sequence in A is its limit in A .^[7]

Definition 2.3 [Chika (2000)]: A subset of a metric space is called compact if every bounded sequence has a convergent subsequence.^[8]

Definition 2.4 [Chika (2000)]: A mapping from one metric space into another metric space is called continuous if $x_n \rightarrow x$ implies that $T(x_n) \rightarrow Tx$ that is $\lim d(x_n, x) = 0 \Rightarrow \lim d(T(x_n), Tx) = 0$.

Theorem 2.1 [Robert and Lynda (1999)]: Every bounded and closed subset of R^n is compact.

Definition 2.5 [Stafford (1969)]: A sequence in a metric space $X = (X,d)$ is said to converge or to be convergent if there is an $x \in X$ such that

$$\lim d(x_n, x) = 0$$

x is called the limit of $\{x_n\}$ and we write

$$\lim_{n \rightarrow \infty} x_n = x$$

or simply $x_n \rightarrow x$.

If $\{x_n\}$ is not convergent, it is said to be divergent.

Lemma 2.2 [Chika (2000)]: Let $X = (X,d)$ be a metric space, then

(a) A convergent sequence in X is bounded and its limit is unique

(b) If $x_n \rightarrow x$ and $y_n \rightarrow y$ in X , then $d(x_n, y_n) \rightarrow d(x, y)$.

Definition 2.6 [Danbury (1992)]: A sequence $\{x_n\}$ in a metric space $X = (X,d)$ is said to be Cauchy if for every $\varepsilon > 0$, there is an $N = N(\varepsilon)$ such that

$$d(x_m, x_n) < \varepsilon \quad \text{for every } m, n > N$$

The space X is said to be complete if every Cauchy sequence in X converges.

Theorem 2.2 [Chidume (1998)]: The Euclidean space, R^n is a complete metric space.

Definition 2.7 [Chidume (1998)]: A metric can be induced by a norm if a norm on X defines the metric d on X as $d(x,y) = \|x - y\|$ and the normed space so defined is denoted by $(X, \|\cdot\|)$ or simply X .

Definition 2.8 [Chika (2000)]: Let (X, d) be a continuous complete metric space with the metric $d(X_1, X_2)$ induced by the norm $\|x_1 - x_2\|$. If $T: X \rightarrow X$ is a map such that

$$Tx = d(x_1, x_2) = \|x_1 - x_2\| = \|x\| \quad \forall x_1, x_2 \in X$$

Then, x is a fixed point of the set X .

Definition 2.9 [Chika (2000)]: If x be a norm induced by the metric d such that the operator $T: X \rightarrow X$ is such that $\|Tx_1 - Tx_2\| \leq K\|x_1 - x_2\|$ $\forall x_1, x_2 \in X$ and $K > 1$, then such a Lipschitzian map is called a contractive map and non-expansive or a pseudocontractive map if, on the other hand, $K = 1$, but if $K > 1$, the map becomes a strong pseudocontraction.

Main Result

The above-mentioned definitions and results served as a guide in developing the facts below which form the basis of our main result used in determining the shortest route problem solutions.

Facts

- i) The domain of existence of the shortest route path dynamic programming problem is the complete metric space with the set $X = R$, a closed and bounded set.
- ii) The fixed point iterative operator is continuous in the domain of the closed set R and converges at a unique sink (x_{n+1}) where the initial iterate x_0 is the source [Figures 1-5].

- iii) The distance function sometimes is linear and sometimes nonlinear, hence, the reason for the use of the metric induced by the norm $d(x_1, x_2) = x_1 - x_2$ [Figure 6].
- iv) That the shortest route problem of the dynamic programming problem satisfies the strong pseudocontractive condition of the fixed point iterative method [Figure 7].
- v) That the shortest route method of the dynamic programming problem is a reformulation of the modified Krasnoselskii's method of the fixed point iterative method for strongly pseudocontractive maps.

Theorem 2.3

Let (X, d) be a complete metric space and T a strongly pseudo contractive iterative map of the shortest route problem in (X, d) induced by the norm $x_1 - x_2$ well posed in the Banach space such that the solution method

$$\begin{aligned} Tx &= \min [U_j, i] = \sum_{ij}^n \min [U_j, i] \\ &= \sum_{ij}^n [U_i + d_{ij}, i], d_{ij} \geq 0 \end{aligned}$$

has the unique fixed point

$$d_{ij} + d_{kj} < d_{ik}$$

With $i \rightarrow k$ becoming $i \rightarrow j \rightarrow k$ and $i \neq k, j \neq k, i = j$; the pivot row with pivot column being row k and the triple operation, $i \rightarrow j \rightarrow k$ holding in each element d_{ij} in $D_{k-1} \forall i, j$ such that when $d_{jk} + d_{kj} \leq d_{ij}$ ($i \neq k, j \neq k, i \neq j$) is satisfied, then we

- I. Create D_k by replacing d_{ij} in D_{k-1} with $d_{ik} + d_{kj}$
- II. Create S_k by replacing S_{ij} in S_{k-1} with k and setting k in $k + 1$ and repeating step k .

Proof

Let (X, d) be a complete metric space, the closed and bounded distance function space of the dynamic programming containing all the various paths linking the various nodes beginning from the source to the sink [Figure 8-15]. We aim to establish that the dynamic programming method of the shortest route is a strongly pseudocontractive iterative method of the modified Mann. That is, if

$$\begin{aligned} \|x_1 - x_2\| &\leq \|(1-r)(x_1 - x_2) - rt(T(x_1) - T(x_2))\| \\ &= \|(1+r)I - rtT\| \|x_1 - x_2\| \geq \|x_1 - x_2\| \end{aligned}$$

so that

$$\|(1+r)I - rtT\|$$

and then

$$\begin{aligned} Tx &= \min [U_j, i] = \sum_{ij}^n [U_j, i] \\ &= \sum_{ij}^n [U_i + d_{ij}, i]; d_{ij} \geq 0 \Rightarrow \sum Kd_{ij} \geq 0 \end{aligned}$$

Provided $K \geq 0$ where K is the contraction factor. If $K \geq 0$, then the iterative method is strongly pseudocontractive and so the modified Mann's iterative method in this case the Dijkstra or the Greedy and the Prim's method becomes the suitable iterative method.

$$x^* = Tx = d_{ij} + d_{kj} < d_{ik}$$

which converges to the unique fixed point whenever $i \Rightarrow k$ is $i \Rightarrow j \Rightarrow k$ and $i \neq k, j \neq k, i = j$; the pivot column becomes row k provided the operation $i \Rightarrow j \Rightarrow k$ holds in each element d_{ij} in D_{k-1} for each i, j such that $d_{jk} + d_{kj} \leq d_{ij}, i \neq k, j \neq k, i = j$ when is satisfied and

- i. D_k is created by replacing d_{ij} in D_{k-1} with $d_{ik} + d_{kj}$
- ii. S_k is created by replacing S_{ij} in S_{k-1} with k and setting k in $k + 1$ and repeating step k .

Applications

In this section, we should only apply this work to three out of the six reviewed algorithm or methods, i.e.,

- (i) Dynamic programming technique
- (ii) Dijkstra's algorithm
- (iii) Greedy and Prim's algorithm

Figure 3 gives the route of study. However, I is important to note that the Government House to Amuzukwu road is a closed and bounded distance network which is continuous in the metric $d(s_0, s_k)$ induced by the norm $\|x - y\|$ such that $x, y \in d(s_0, s_k)$ where s_0 is the source. Government house and s_k is the sink, Amuzukwu Primary School, Amuzukwu all in Umuahia. The computation is done using the iterative method of theorem (2.1) above and the sequence of results is displayed in Table 1 and consequently other associated tables that follow [Figure 16-19].

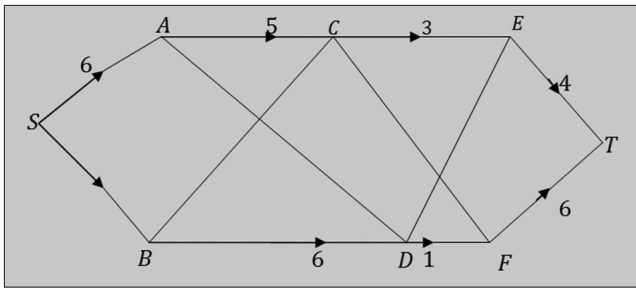


Figure 1

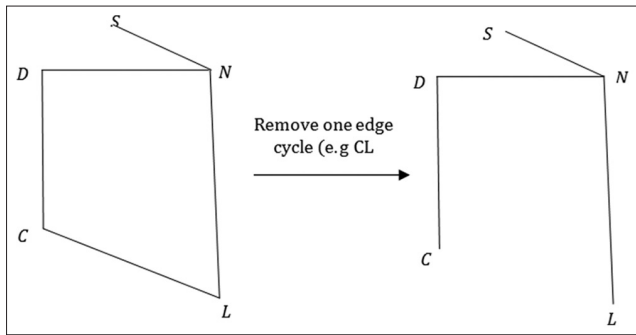


Figure 2

Backward dynamic programming

For ease of reference, we repeat the network drawn in Figure 3 as in Figure 4 be Where the Dijkstra’s algorithm began at *S*, the dynamic programming technique work backward from *T* to *S*. We begin by considering the vertex joined directly to *T*, namely *G*, this is the Stage 1 vertex. The best route from this to *T* is noted. We now move to the next set of vertices that are joined directly to *G*, namely *F* and *H* – these are Stage 2 vertices. The best route from these to *G* is found using the optimal routes from the Stage 1 vertices. This process is repeated once again, until *S* is reached. The principle of optimality is used at each stage and the current optimal path is obtained using the previously obtained optimal paths [Figure 20-25].

Stage 1

From *G*, there is only one choice and the distance *GT* is 720 m. We, therefore, label *G* with 720 m as this is the length of the shortest route to *T*, also *GT* is optimal, we indicate it with

From *C* to *G*, there are four possible routes
 CDEFG, CDJFG, CDEFHG, and CDJHG
 Length CDEFG length of CDEFHG+ label *G*
 = CD+DE+EF+FG+FG+label *G*
 =180+540+150+210+720=1800m
 Length CDJFG = length of CDJFG+ label *G*
 = CD+DE+EF+FG+FG+label *G*
 =180+70+90+210+720=1270m
 Length CDEFJHG = length CDEFJHG = length CDEFJHG + label *G*

=CD+DE+EF+FJ+JH+FG+label *G* =
 180+540+150+90++50+600+720 = 2330m
 Length CDJHG = length CDJHG + label *G*
 =CD+DJ+JH+FG+label*G*=180+70+50+600+720
 = 1620m
 Furthermore, from *I* to *G*, we have one route IHG
 Length IHG = length IHG+ label *G* = IH +HG
 +label *G* = 300+190+1270 = 1760m
 Length AC = length AC +label *C* = AC +label *C* =
 350+1270 = 1620 m

Since we are looking the shortest route, *A* is assigned the label = min (1760, 1620)

We then have
 The stars indicate the optimal routes

Stage 3

From *S*, there are two choices. We may choose a route through *A* or *I*

- i) If we choose, *A*, the shortest route has length:= length SA +label *A* = 170+1620 = 1790 m
- ii) If we choose *I*, the shortest route through has length = length SA +label *A* = 350+1820 = 2170

The shortest route then passes through *A* and is of length 1790 = min (1790, 2170)

We then have.
 The shortest route is obtained by starting at *S* and using SA, AC, CD, DJ, JF, FG, GT,

$$S \rightarrow A \rightarrow C \rightarrow D \rightarrow J \rightarrow F \rightarrow G \rightarrow T$$

Application using Dijkstra’s algorithm

Furthermore, for reference, we repeat the network drawn in Figure 3 and from their proceed as below.

Step 1

S assigned a label 0, i.e., the shortest distance of *S* to *S* is 0 (label is denoted by a number in a box)
A: label *S* +weight of SA+ O+170 m
I: Label *S* + weight of SI + O + 350 m
 The minimum connector is 170 m; (this is connected into a label) (Numbers are given in brackets).

Step 2

A has just been assigned a label. The vertices reachable from *A* are *B* and *C* and their numbers are calculated.
B: label *A* +weight of AB = 170+300 = 470 m
C: label *A* +weight AC = 170+350 = 520 m
 We make a label for *B*

Step 3

B has just been assigned, a label *C* is reachable from *B*.
C: label *B* +weight of BC = 470+ 190 = 660 m
 We have also seen

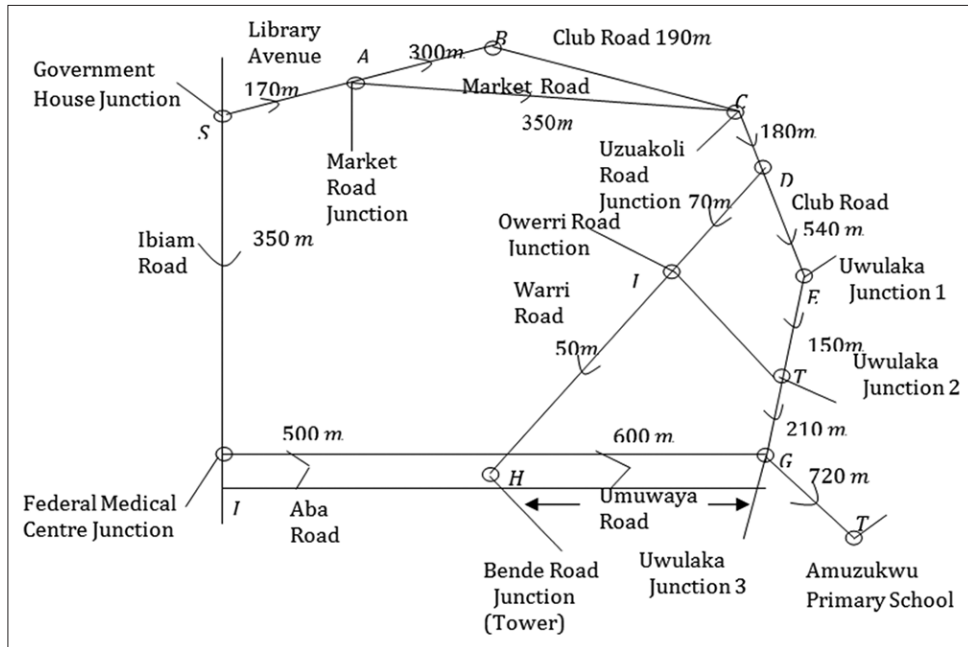


Figure 3

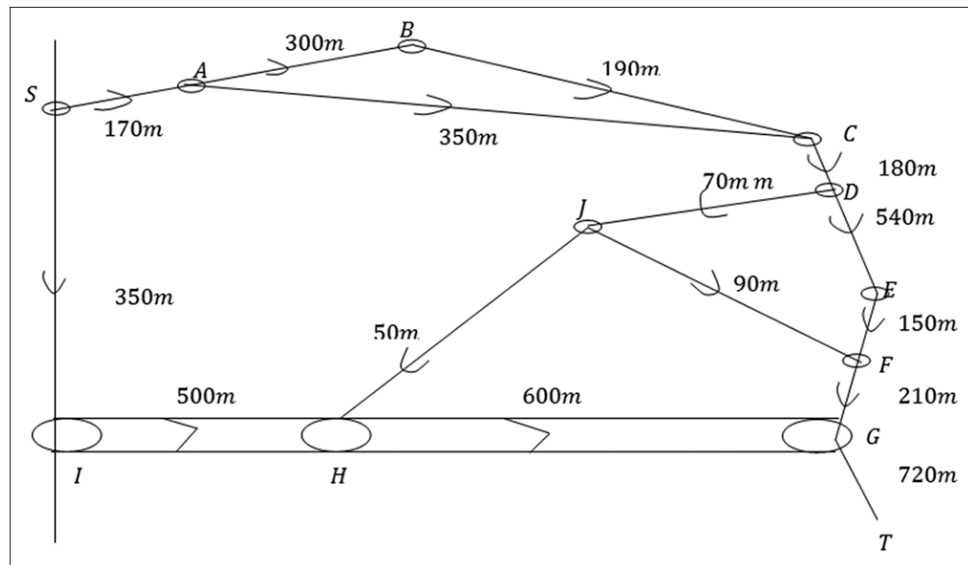


Figure 4

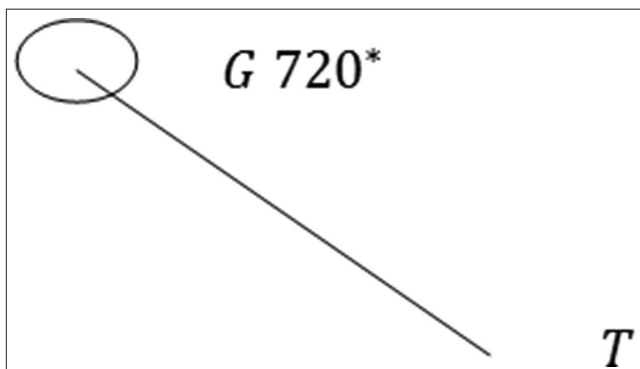


Figure 5

C: label A+ weight of AC = $170 + 350 = 520$ m
 The minimum number is 520 m and C is assigned a label of 520.
 D label of C+ weight of CD = $520 + 180 = 700$ m: So, D is assigned the label of 700 m.

Step 4

F: label of D+ weight of DE + weight of EF = $700 + 540 + 150 = 1390$ m

F: label of D+ weight of J+ weight of F = $700 + 70 + 90 = 860$ m

The minimum number is 860 m, so we can give F label 860

Step 5

J: label of D+ weight of DJ = $700 + 70 = 770$ m: J: label of F+ weight of DJ = $860 + 90 = 950$ m

The minimum number is 770 m: So, we have J labeled 770 m

Step 6

I: label of S+ weight of SI = $0 + 350 = 350$ m: We have I labeled 350

H label of I + weight of H = $770 + 50 = 820$ m: We label H 820 as the minimum number

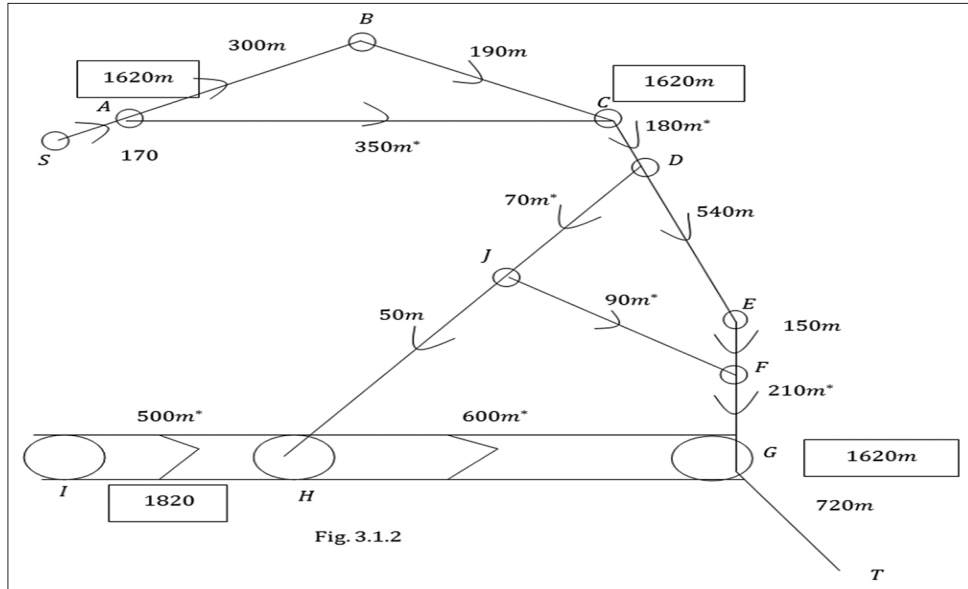


Fig. 3.1.2

Figure 6

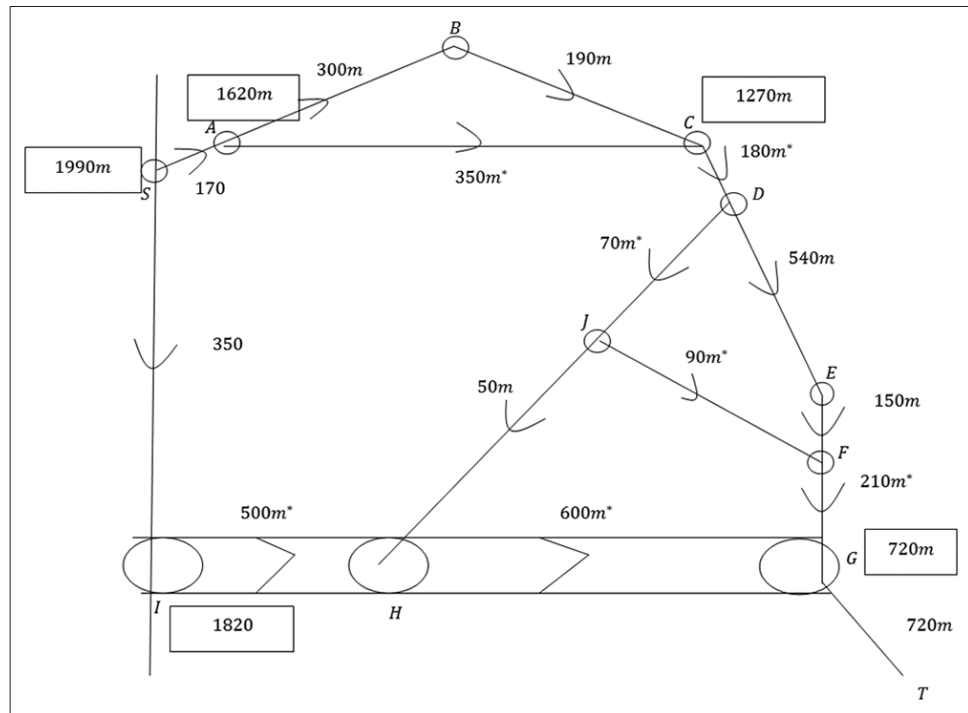


Figure 7

E and F have been given a label. We now find the number of G: $F \text{ label } G \text{ weight } G = 860 + 210 = 1070$

$G \text{ label } I + \text{weight } IH + \text{weight } HG = 770 + 50 + 600 = 1420$: $G \text{ label } S + \text{weight } SI + \text{weight } IH = D + 350 + 500 + 600 = 1450$.: The minimum number is 1070, so, we label C.

Step 7

G has just given a label. T is only reachable from G and the numbers for T found next. $G \text{ label of } F + \text{weight } GT$: The minimum number is 1790 M; so, G is given a label 1790 M

At this stage, we know that the shortest distance from S to T is 1790 m. However, we do not yet

know the path which achieves his shortest length. Step of algorithm finds that path.

Step 8

We start at the destination vertex, T. We include an edge when the weight of the edge is given by the difference of the label of the vertices t the end of the edge.

Stage 9

Label of G – label of T = $1790 - 1070$ include GT = 720M.

Stage 10

Label of G – Label of F = $1070 - 860 = 210$ m: Weight of FG = 210 include FG
Label of G – Label of I = $1070 - 35 = 72$: Weight

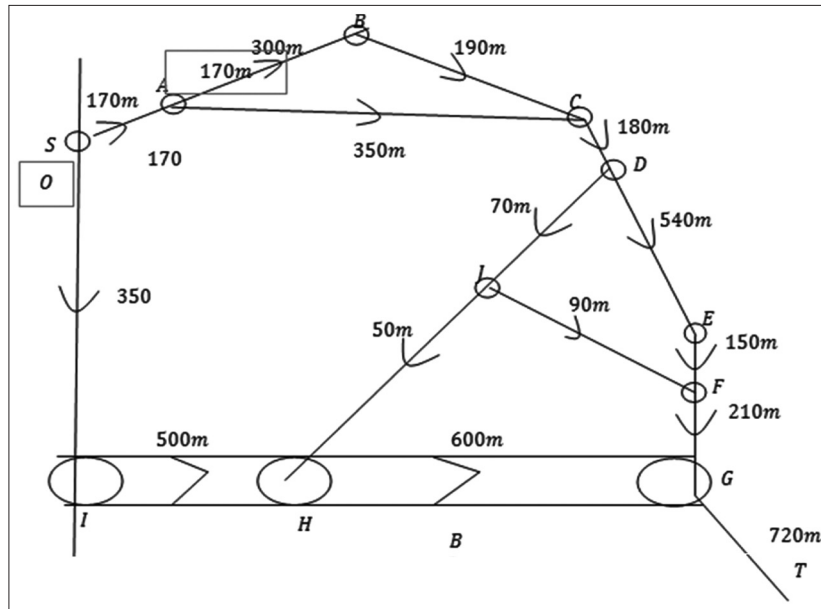


Figure 8

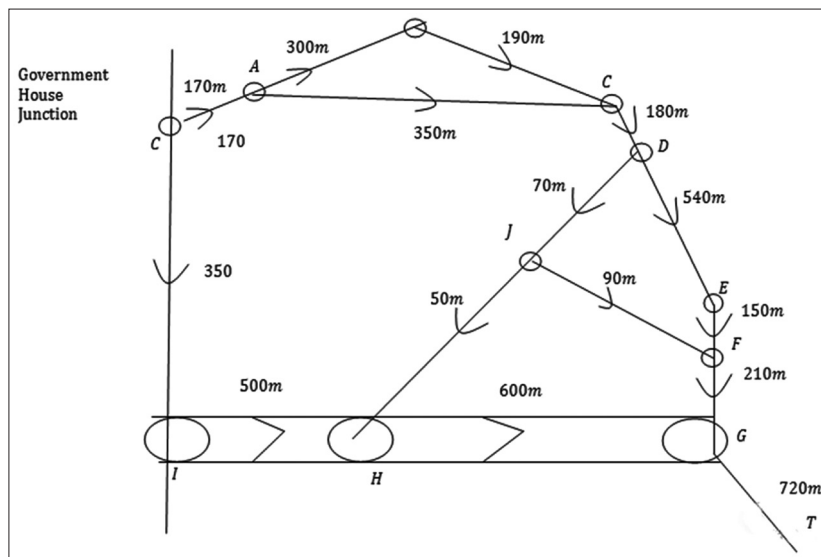


Figure 9

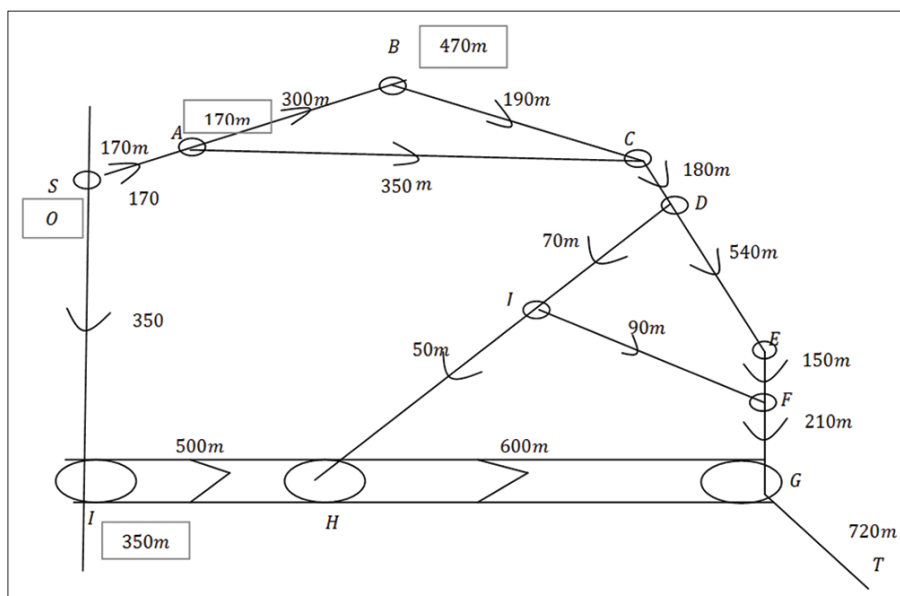


Figure 10

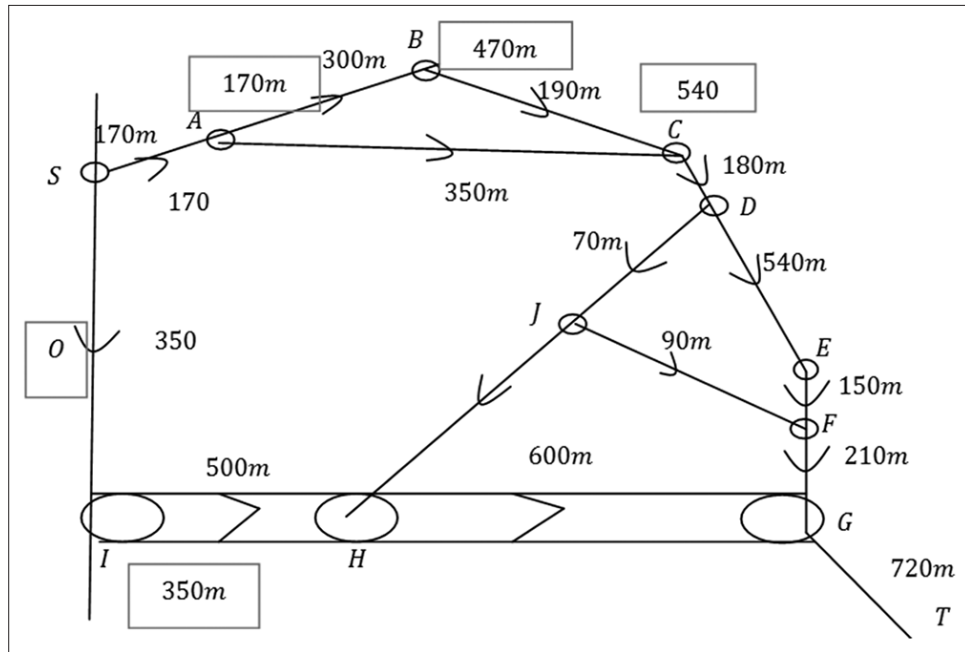


Figure 11

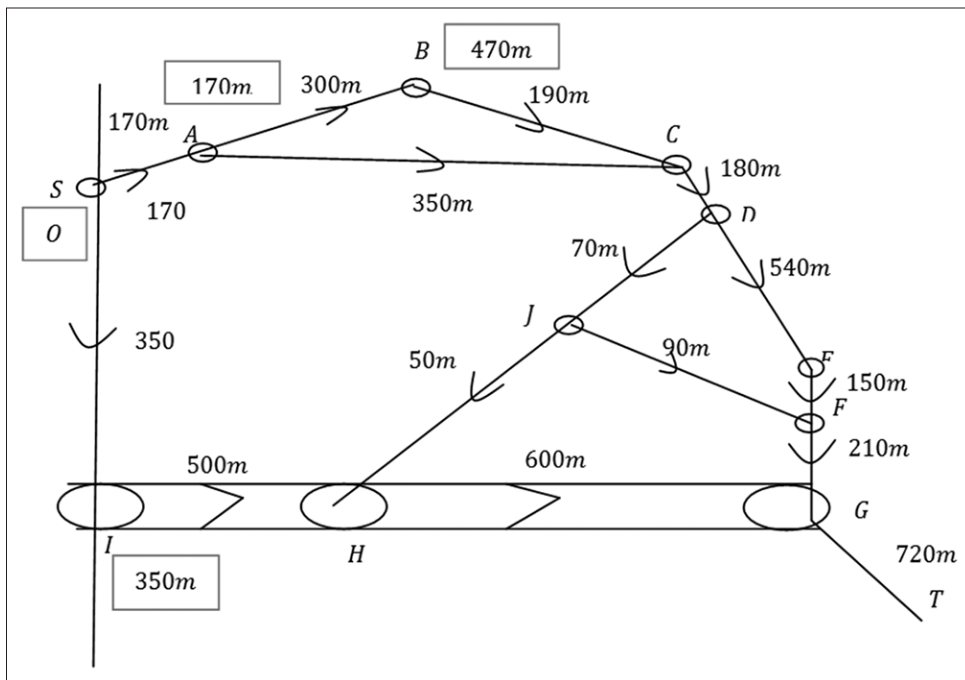


Figure 12

of $IG = 1100$) do not include IG
 Label of $G - \text{Label } H + \text{weight of } HG = 1070 - (820+600) = 1070 - 1420 = -350$ do not include JG

Stage 11

Label of $D - \text{Label of } C = 860 - 770 = 90$
 (Weight of $JF = 90$) include JF
 Label of $G - \text{Label of } H = 860 - 124$
 Weight of $HG = 150$) do not include IJ

Stage 12

Label $J - \text{Label of } D = 770 - 700 = 70$
 (Weight of $DJ = 70$) include DJ

Stage 13

Label $D - \text{Label of } C = 700 - 520 = 180$
 Weight $CD = 180$) include CD

Stage 14

Label $C - \text{Label } B + \text{weight of } BC = 520 - (470 + 190) = 529 - 660 = -140$
 Weight $CA = 490$) Do not include CA
 Label $C - \text{label } A = 520 - 170 = 350$: (Weight of $CA = 350$) include CA .

Stage 15

Label $A - \text{label } S = 170 - 0 = 170$
 (Weight of $SA = 170$) include SA (Label $IS = 350$)

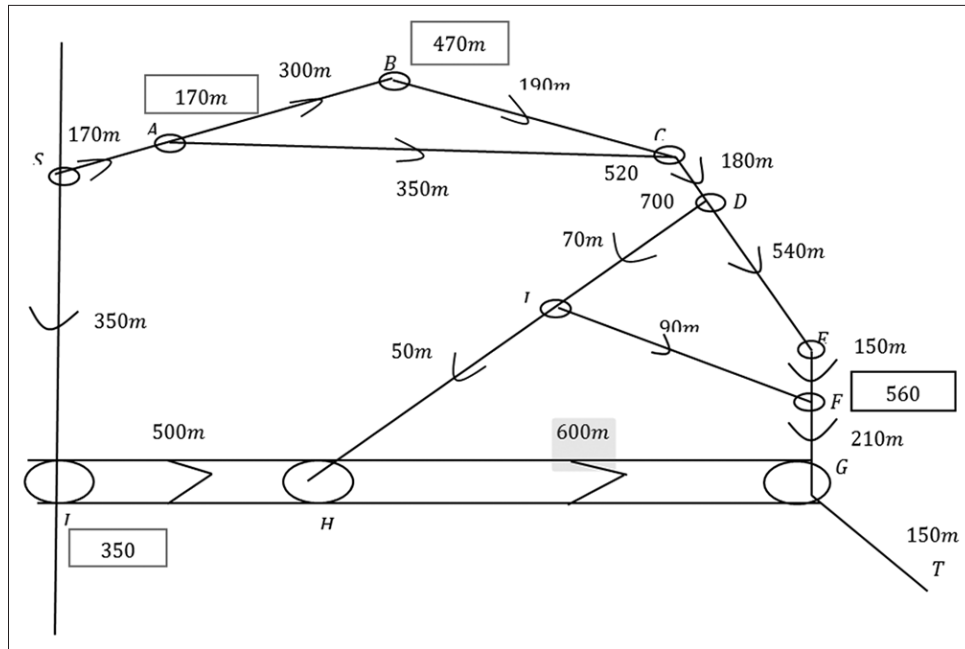


Figure 13

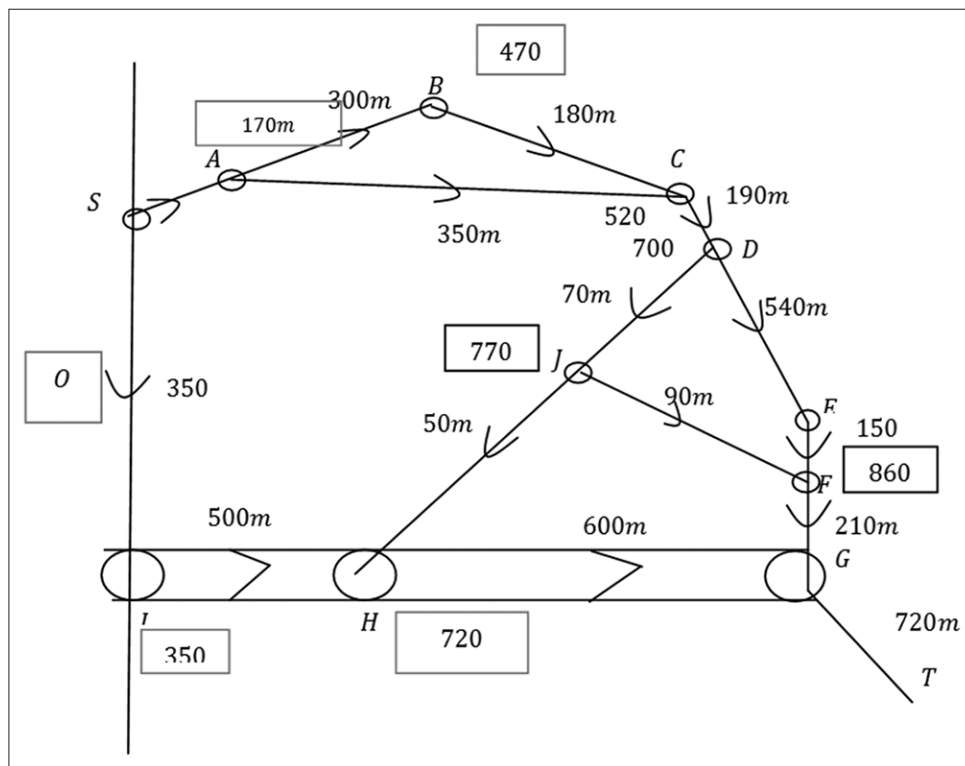


Figure 14

do not include IS. Hence, the shortest route from S to T is SACDJFGT.

Application using Greedy and Prim’s algorithm

Application by Greedy’s algorithm

Applying Greedy’s algorithm in the figure, i.e., Figure 3, we have been considering. We use the following procedure.

Choosing any vertex as a starting vertex, say S
 The nearest vertex to S is A
 The nearest vertex to A is C
 Also the nearest vertex to A is C
 The nearest vertex to D is J
 The nearest vertex to J is F
 The nearest vertex to F is G
 The nearest and only vertex to G is T
 The total length of the figure is 4470 m, in this example, the minimum spanning tree is not unique

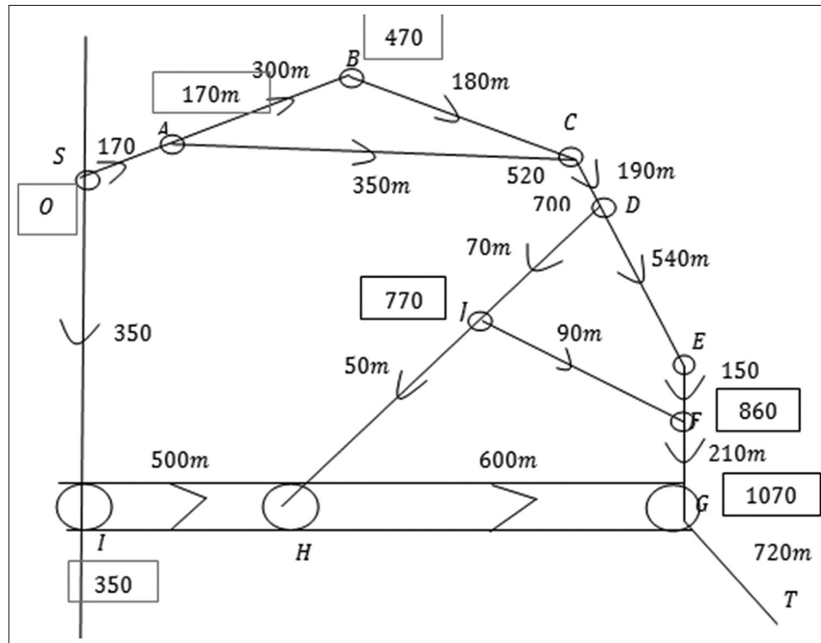


Figure 15

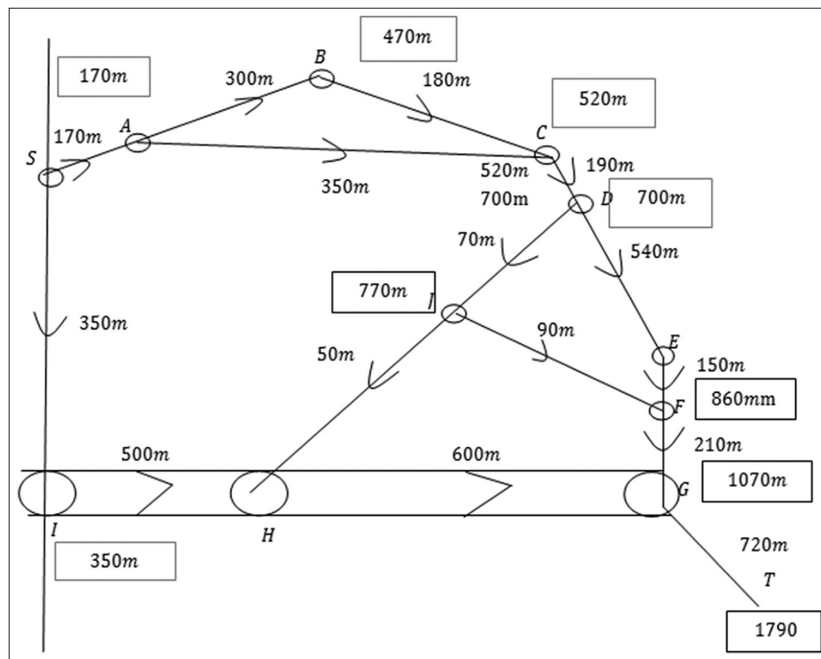


Figure 16

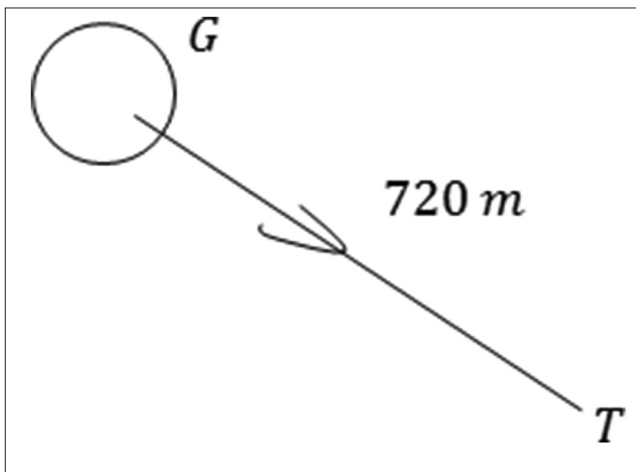


Figure 17

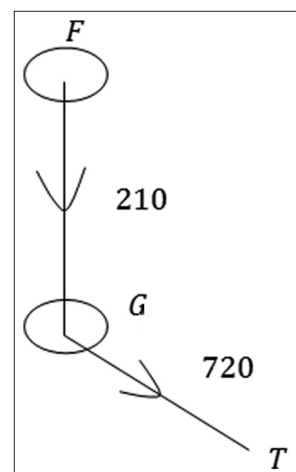


Figure 18

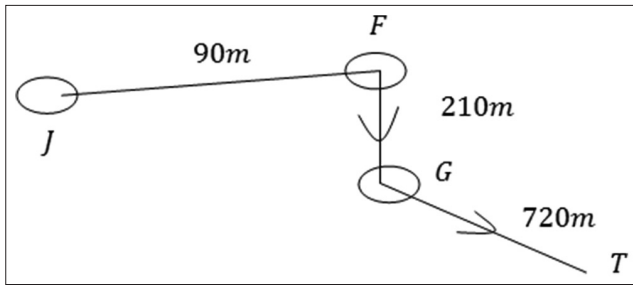


Figure 19

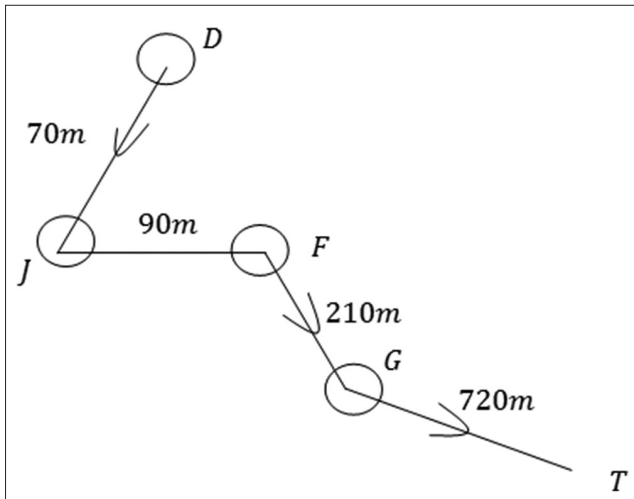


Figure 20

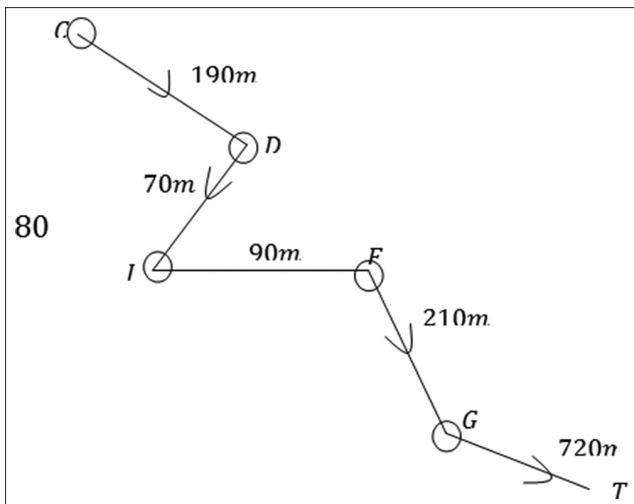


Figure 21

since at each step, in algorithm, we have an alternative in deciding the next vertex and edge. The shortest route is the path SACDJFGT with 1790 metres distance.

Application using Prim's algorithm

Prim's Algorithm uses the table format below to find the shortest route (Table 1).

Reorganize the table to take vertical shape and herisenta. From a close study of the table above, we come up with the following resolutions,

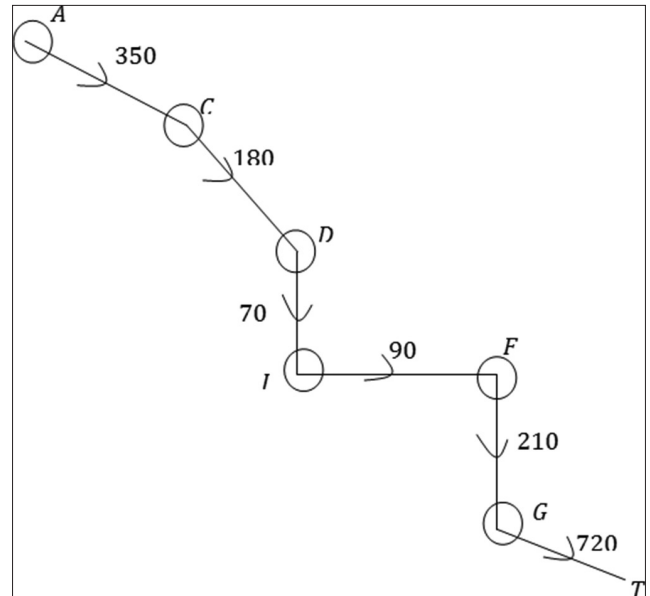


Figure 22

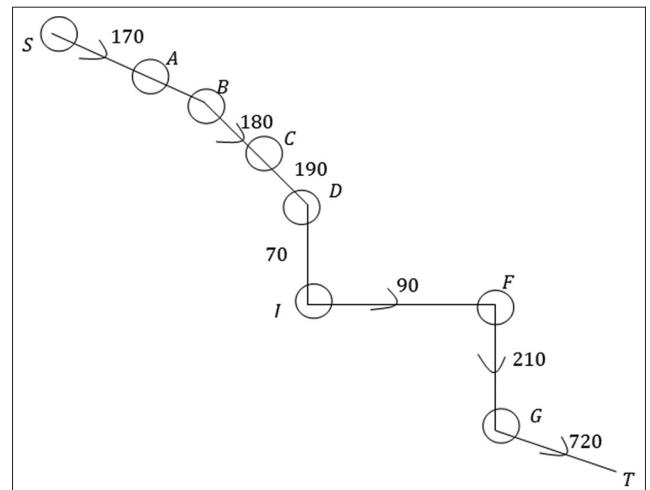


Figure 23

- i. There is a zero distance from S to S; therefore, we eliminate row and column S
- ii. The nearest distance from A to A so we eliminate row and column A
- iii. We resolve to make use of points ACDEFGT, ACDJEGT, and ACDJHGT, hereby over looking point S in our further steps for ease of possible manipulation of the data and table to give accurate result.
- iv. We represent the shortest route from one point to another with the least number derivable in that route.

We now illustrate, the shortest route chosen as follows:

A critical look into the table above we'll help us resolve as follows:

- i. That AC and CD remains constant with value 350 and 180, then we eliminate A and C
- ii. The shortest route diagram includes SACD

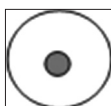


Figure 24

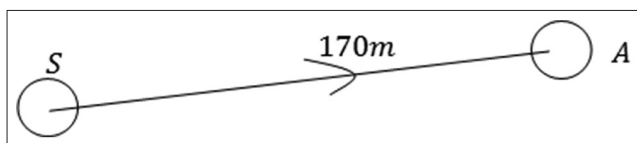


Figure 25

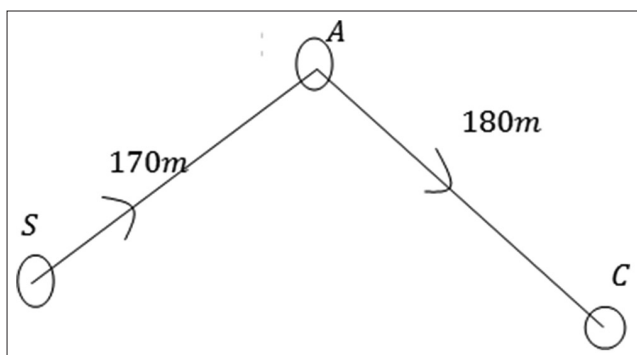


Figure 26

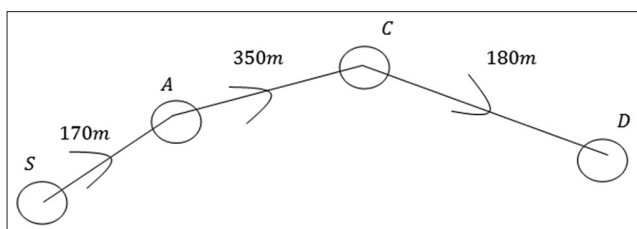


Figure 27

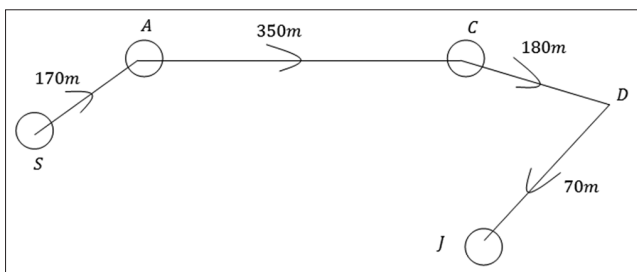


Figure 28

iii. Furthermore, from the table, I to F have shortest route to E and H; therefore, we eliminate E and H.

The smallest number in the 1st row is 70, so we eliminate J and the shortest route diagram includes *SACDJF*.

The smallest number in the J row is 90, so we include in the shortest route diagram as thus *SACDJF*.

The smallest number in the F and G row is 210, respectively. Hence, we include G in the shortest route diagram as thus *SACDJFG* we eliminate F. Then, table becomes or reduces to the smallest and only number in the G row is 720, representing T, so, we include T in the shortest route diagram and elimination. The shortest route diagram now becomes *SACDJFG* with shortest route distance of 1790 m. The shortest route diagram is now represented by

Remark

With all what have done in the three algorithms or methods of shortest route, we took time to apply, we can see clearly that three methods got a particularly shortest route proving the accuracy of the work and showing the methods were rightly applied [Figure 26-18]. Hence, we want to say here that irrespectively of the algorithm or method you may want to use, the shortest should remain the same except for methods like the Eulerian and non-Eulerian graph, min-max and max-min route were some additions would be made on the shortest route, but irrespectively of the additions, the fundamental shortest route will remain the same [Figure 29-31].

Note: By inspection, the longest route is the route from government house through Ibiam road to Aba road then to Warri road also through Club road to Uwalaka road and finally to Amuzukwu road which terminate at destination, Amuzukwu Primary School Umuahia. The longest route covered a total distance of 2590 metres which passed through the path *SIHJDEFT*, i.e., [Figure 32-35]

$$S \rightarrow I \rightarrow H \rightarrow J \rightarrow D \rightarrow E \rightarrow F \rightarrow G \rightarrow T$$

DISCUSSION, CONCLUSION, AND SUGGESTIONS

The minimum length of travel on any route goes a long way in determining the route of travel of road user where the case of alternative routes of travel exists. It is true that there exists other factors competing with minimum length in winning the choice of the route to use such as security, good road network, absence of police menace, and traffic jams, still in a city such as Umuahia where every thing is done to save time and for the purpose of this study where other factors were put on constraints, we have no other alternative but to appreciate

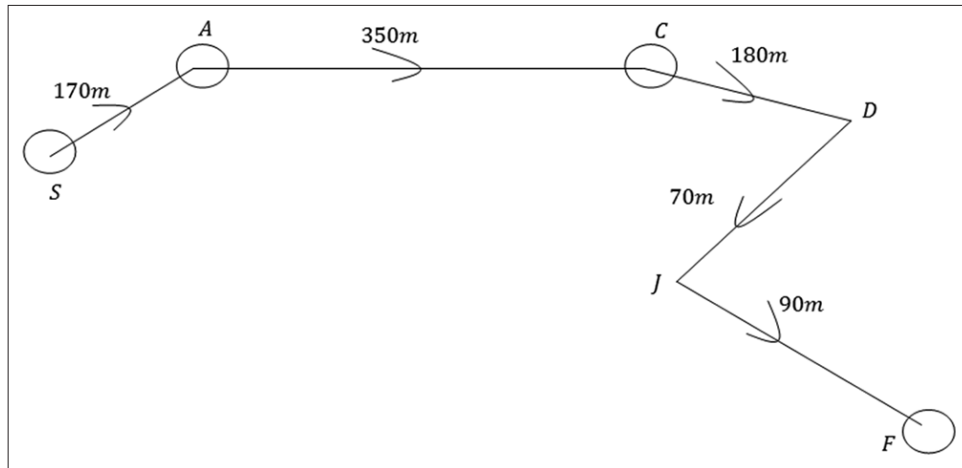


Figure 29

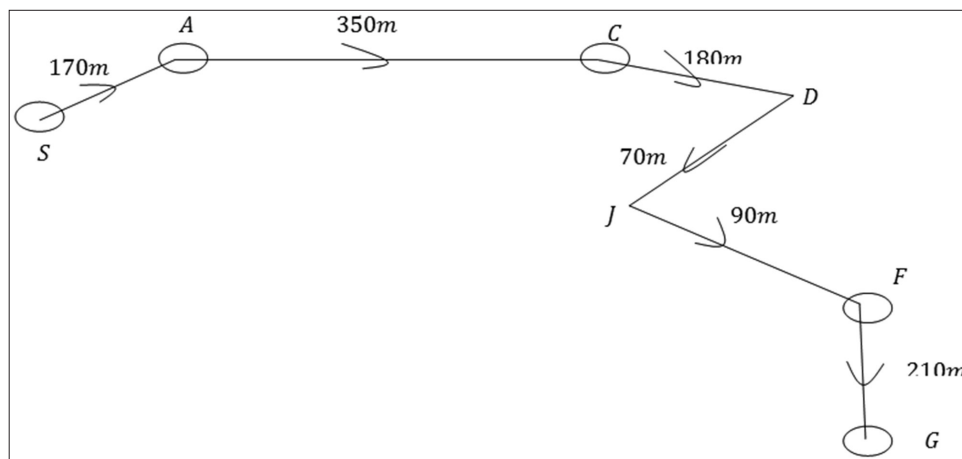


Figure 30

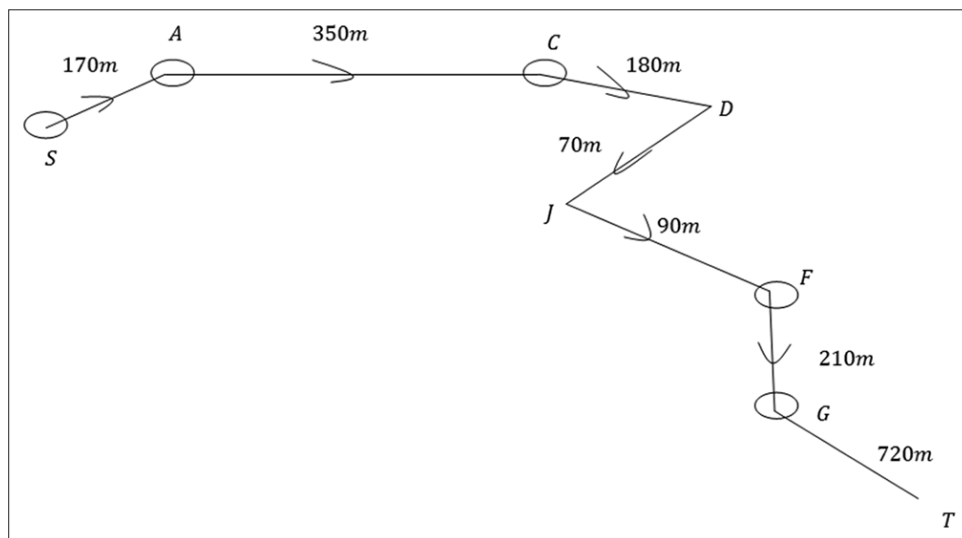


Figure 31

the good gesture done to us by the application of shortest path to dynamic programming.

The application of shortest route in dynamic programming has been attributed to some factors which we have listed in the course of this work. This chapter, therefore, discussed and summaries

the findings of the study and makes conclusion based on empirical findings of how shortest route is applied in dynamic programming. Hypothesis was put forward and analyzed using mathematical tool of application which explained the data collected in the course of the study.

Table 1

	S	A	B	C	D	E	J	F	H	G	I	T
S	-	170	470	520, 660	700, 840	1240 1380	770, 910	860 960 640 850	1190 960 640 850	200, 1600 2130, 1420, 1070, 1450 1740 2270 1560, 1230	350	2170, 920, 2460, 2320, 2990, 2850, 2280, 2140, 1790, 1950
A	170	-	300	490, 350	670, 530	1210, 1070	740, 610	1360 1240, 830, 700	790, 640 660, 690	1570, 1430, 1840, 600, 1250, 1390, 1280.	520	1240, 2870, 2250 2690, 3010, 1730,
B	470	300	-	190	370	910	440	1060, 530	1200, 490	740 1800, 1090, 1270,	820	1090, 740, 1800, 1270,
C	520, 840	490, 350	190	-	180	720	70	870, 340	300	1080, 900, 550	820	1270, 1800, 1620
D	1240, 1380	670, 540	370	180	-	540	70	160, 690	830, 120	370, 900, 1430, 720	1000	1090, 1620, 2150, 1440
E	770, 910	1210, 1080	910	720	540	-	610, 240	150, 700	290, 2040	360, 910, 1260	1440	1080
J	860, 080, 1390, 1530	740, 610	440	70	70	610, 240	-	90	50	650 300	550	1020, 1460
F	1190, 960 640	1360, 1240, 700, 830	1060, 530	870, 340	160, 690	150 700	90	-	140	740, 210	640	930, 1460
H	850	790, 660	1200,	300	830,	2040,	50	140	-	600	500	1320
G	1170, 1740, 1560 1420, 870, 1450, 1600	1570, 1430, 840, 600 1250, 1396, 1280	740, 1800, 1090, 1270	1080, 550, 900	370, 900, 1430, 720	360, 910, 1260	300, 650	740, 210	600	-	1100	720
I	350	520	820	870	1000	1440	550		500	1100	-	1820
T	2320, 1730, 1590, 2170, 2460, 2140, 2280	2140, 2280, 2870, 2690, 3010, 1730	1270, 1090, 740, 1800	1620, 1800, 1270	1090 1620, 2150, 1440	1080	1020, 1460	930, 460	1320	720	1820	-

Discussion of finding

Our finding on choice of routes, road user within Umuahia metropolis makes use of showed that most Umuahia road users have one problem or the other traveling on road. The problems range from

potholes, traffic jams, and long distance route. Existing route is even been covered by market and traders, thereby increasing traffic jam. Our finding also discovered that road users and motor vehicles are increasing at geometric progression which the route/networks are increasing

Table 2

	A	C	D	E/J	F/H	G	T
A	-	350	530	1070/660	1120/690	1330/1290	2050/2010
C	350	-	180	720/180	870/300	550/180	1270,1800,1620
D	530	180	-	540/70	160/720	370/720	1090/1440
E/J	1070/600	720/180	540/70	-	150/90	360/300	1080/1020
F/H	1120/690	870/300	160/120	150, 90,	-	210,600	930,1320
G	1330/1290	550,	370/720	360/300	210,600	-	720
T	2050/2010	1270,1800	1090/1440	1080,1020	930,1320	720	-

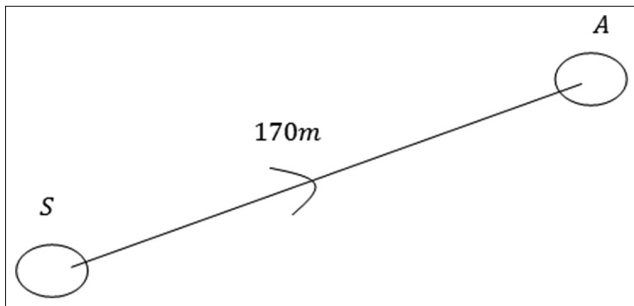


Figure 32

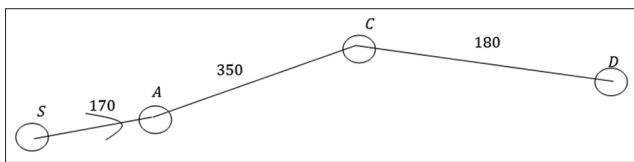


Figure 33

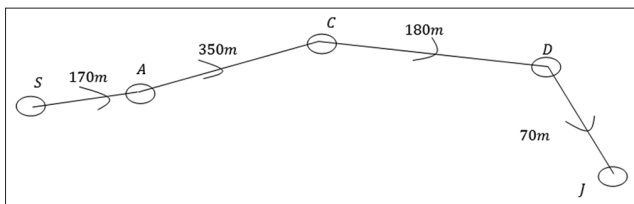


Figure 34

at arithmetic progression or even estimated not increasing. Flood, during rainy season, contributed its quota to hinder minimum length and time travel. Thus, putting other factors affecting movement from one point to the other in constraint and focusing on distance, we will certainly agree that shortest route makes travel interesting. These follow the hypothesis that any path of shortest route is itself a shortest path and we say that any part of an optimal route is itself optimal.

Suggestion

Based on our findings in the course of this study, the researcher suggests as follows:

1. That route is created from one geographical location to another by any responsible authority, especially government.

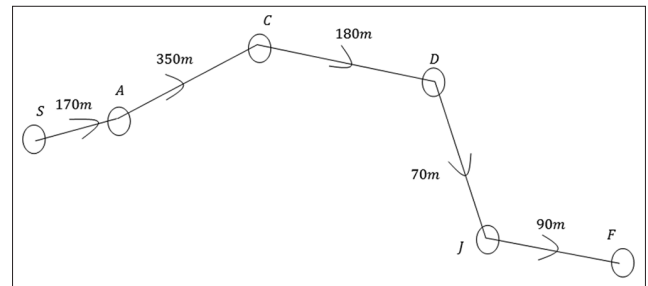


Figure 35

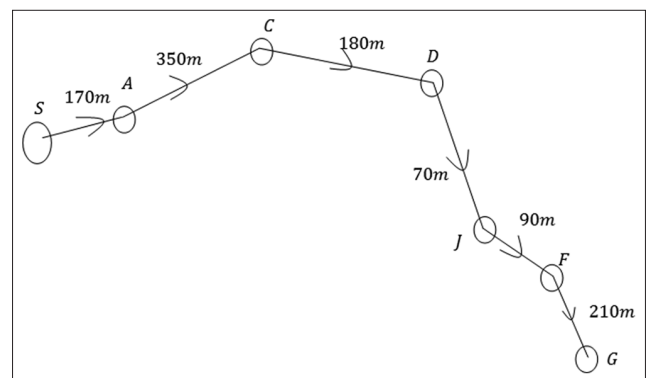


Figure 36

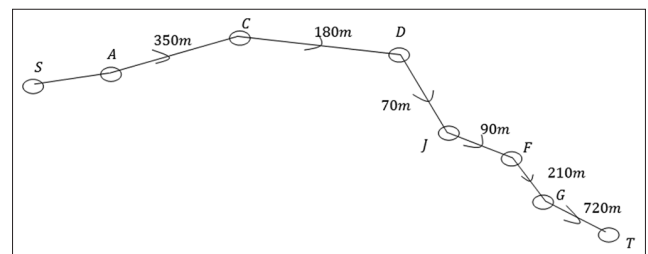


Figure 37

2. Maintenance activity/works should be done on a regular basis on the existing routes.
3. Road directions and warnings should be positioned at strategic junctions to enable travelers locate their destination from their source and have enough information to prevent accidents.
4. Branched network should be attached to reduce the rate of traffic jams on our route.
5. Police menace on our road (routes) of travel should be discouraged

Table 3

	D	J	F	G	T
D	-	70	370	370	1090
J	70	-	300	300	1020
F	160	90	-	210	930
G	370	300	210	-	720
T	1090	1020	930	720	-

Table 4

	J	F	G	T
J	-	90	300	1020
F	90	-	210	930
G	300	210	-	720
T	120	930	720	-

Table 5

	F	G	T
F	-	210	930
G	210	-	720
T	930	720	-

Table 6

	G	T
G	-	720
T	720	-

6. Road users should be cautions as they the road.
7. Safety providing agencies should make themselves available in every route of travel within Umuahia.
8. Road users should make the shortest path their route of travel to minimize length and time of travel.

CONCLUSION

In the transportation world today, the routes are regarded as king in the sense that they provide channels/links between two geographical points/location. The routes do not just come into existence. They are created or built by men to facilitate movement from one point to the other. Though these routes cannot catapult any one

geographical location to the other on their own by when they exist, and good once, even without locomotion machines like motor vehicles one can still make a journey by foot

Government on their own should make building and maintenance of roads and networks paramount projects. It is expected that Wise Travelers having known that there exists short and long route may decide to choose traveling through the shortest route.

Suggestions for further research

This work has examined the application of shortest route in dynamic programming considering the factors of minimum distance. Further studies could still be carried out to understand more factors which could likely determine the minimum or maximum distance between two locations and other applications excluding the one used in this work to determine the shortest path/route between one location/point to the other.

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