

## RESEARCH ARTICLE

## An Approach to Analyze Non-linear Dynamics of Mass Transport during Manufacturing of a Hybrid Comparator Circuit: On Increasing of Integration Rate of Elements of This Circuit

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**ABSTRACT**

In this paper, we introduce an approach to increase integration rate of elements of a hybrid comparator with the first dynamic amplifying stage and the second quasi-dynamic latching stage. Framework the approach, we consider a heterostructure with special configuration. Several specific areas of the heterostructure should be doped by diffusion or ion implantation. Annealing of dopant and/or radiation defects should be optimized.

**Key words:** Hybrid comparator, increasing integration rate of field-effect transistors, optimization of manufacturing

**INTRODUCTION**

An actual and intensively solving problem of solid-state electronics is increasing of integration rate of the elements of integrated circuits ( $p$ - $n$ -junctions, their systems).<sup>[1-8]</sup> Increasing of the integration rate leads to the necessity to decrease their dimensions are using several approaches. They are widely using laser and microwave types of annealing of infused dopants. These types of annealing are also widely using for annealing of radiation defects, generated during ion implantation.<sup>[9-17]</sup> Using the approaches give a possibility to increase integration rate of the elements of integrated circuits through inhomogeneity of technological parameters due to generating inhomogeneous distribution of temperature. In this situation, one can obtain decreasing dimensions of elements of integrated circuits<sup>[18,19]</sup> with account Arrhenius law.<sup>[1,3]</sup> Another approach to manufacture elements of integrated circuits with smaller dimensions is doping of heterostructure by diffusion or ion implantation.<sup>[1-3]</sup> However, in this case, optimization of dopant and/or radiation defects is required.<sup>[18]</sup>

In this paper, we consider a heterostructure. The heterostructure consists of a substrate and several epitaxial layers. Some sections have been manufactured in the epitaxial layers. Further, we consider doping of these sections by diffusion or ion implantation. The doping gives a possibility to manufacture field-effect transistors framework a hybrid comparator circuit with the first dynamic amplifying stage and the second quasi-dynamic latching stage so as it is shown in Figure 1. The manufacturing gives a possibility to increase density of elements of the operational amplifier circuit.<sup>[4]</sup> After the considered doping, dopant and/or radiation defects should be annealed. Framework the paper, we analyzed dynamics of redistribution of dopant and/or radiation defects during their annealing. We introduce an approach to decrease the dimensions of the element. However, it is necessary to complicate technological process.

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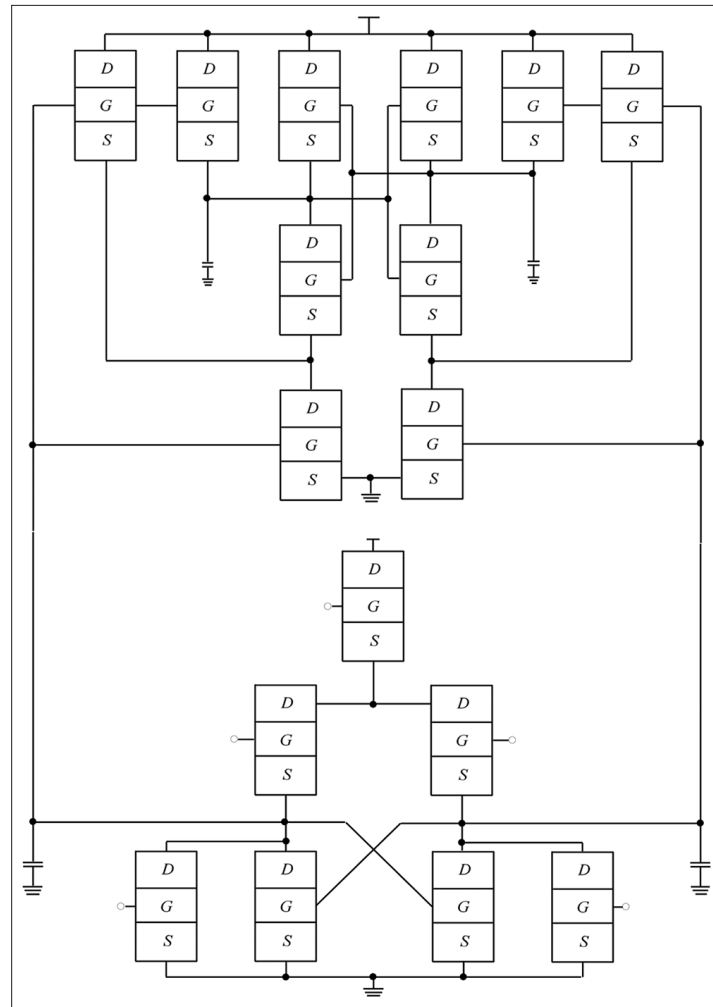


Figure 1: The considered hybrid comparator<sup>[4]</sup>

### METHOD OF SOLUTION

In this section, we determine spatiotemporal distributions of the concentrations of infused and implanted dopants. To determine these distributions, we calculate appropriate solutions of the second Fick's law.<sup>[1,3,18,19]</sup>

$$\frac{\partial C(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_c \frac{\partial C(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_c \frac{\partial C(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ D_c \frac{\partial C(x, y, z, t)}{\partial z} \right] \quad (1)$$

Boundary and initial conditions for the equations are

$$\left. \frac{\partial C(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \left. \frac{\partial C(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \left. \frac{\partial C(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \left. \frac{\partial C(x, y, z, t)}{\partial y} \right|_{x=L_y} = 0, \left. \frac{\partial C(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \left. \frac{\partial C(x, y, z, t)}{\partial z} \right|_{x=L_z} = 0, C(x, y, z, 0) = C(x, y, z) \quad (2)$$

The function  $C(x, y, z, t)$  describes the spatiotemporal distribution of the concentration of dopant;  $T$  is the temperature of annealing;  $D_c$  is the dopant diffusion coefficient. Value of dopant diffusion coefficient could be changed with changing materials of heterostructure, with changing temperature of materials (including annealing), with changing concentrations of dopant and radiation defects. We approximate dependences of dopant diffusion coefficient on parameters by the following relation with account results in Kozlivsky,<sup>[20]</sup> Gotra,<sup>[21]</sup> Vinetskiy and Kholodar,<sup>[22]</sup> Fahey *et al.*<sup>[23]</sup>

$$D_C = D_L(x, y, z, T) \left[ 1 + \xi \frac{C^{\gamma}(x, y, z, t)}{P^{\gamma}(x, y, z, T)} \right] \left[ 1 + \zeta_1 \frac{V(x, y, z, t)}{V^*} + \zeta_2 \frac{V^2(x, y, z, t)}{(V^*)^2} \right] \quad (3)$$

Here, the function  $D_L(x, y, z, T)$  describes the spatial (in heterostructure) and temperature (due to Arrhenius law) dependences of diffusion coefficient of dopant. The function  $P(x, y, z, T)$  describes the limit of solubility of dopant. Parameter  $\gamma \in [1, 3]$  describes average quantity of charged defects interacted with atom of dopant.<sup>[20]</sup> The function  $V(x, y, z, t)$  describes the spatiotemporal distribution of the concentration of radiation vacancies. Parameter  $V^*$  describes the equilibrium distribution of concentration of vacancies. The considered concentration dependence of dopant diffusion coefficient has been described in details in Kozlivsky.<sup>[20]</sup> It should be noted that using diffusion type of doping did not generation radiation defects. In this situation,  $\zeta_1 = \zeta_2 = 0$ . We determine spatiotemporal distributions of the concentrations of radiation defects by solving the following system of equations.<sup>[21,22]</sup>

$$\begin{aligned} \frac{\partial I(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial y} \right] \\ &+ \frac{\partial}{\partial z} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial z} \right] - k_{I,V}(x, y, z, T) I(x, y, z, t) V(x, y, z, t) \\ &- k_{I,I}(x, y, z, T) I^2(x, y, z, t) \end{aligned} \quad (4)$$

$$\begin{aligned} \frac{\partial V(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[ D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial y} \right] \\ &+ \frac{\partial}{\partial z} \left[ D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial z} \right] - k_{I,V}(x, y, z, T) I(x, y, z, t) V(x, y, z, t) \\ &+ k_{V,V}(x, y, z, T) V^2(x, y, z, t) \end{aligned}$$

Boundary and initial conditions for these equations are

$$\begin{aligned} \left. \frac{\partial \rho(x, y, z, t)}{\partial x} \right|_{x=0} &= 0, \left. \frac{\partial \rho(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \left. \frac{\partial \rho(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \left. \frac{\partial \rho(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \\ \left. \frac{\partial \rho(x, y, z, t)}{\partial z} \right|_{z=0} &= 0, \left. \frac{\partial \rho(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \rho(x, y, z, 0) = f_{\rho}(x, y, z) \end{aligned} \quad (5)$$

Here,  $\rho = I, V$ . The function  $I(x, y, z, t)$  describes the spatiotemporal distribution of the concentration of radiation interstitials;  $D_{\rho}(x, y, z, T)$  is the diffusion coefficients of point radiation defects; terms  $V^2(x, y, z, t)$  and  $I^2(x, y, z, t)$  correspond to generation divacancies and di-interstitials;  $k_{I,V}(x, y, z, T)$  is the parameter of recombination of point radiation defects;  $k_{I,I}(x, y, z, T)$  and  $k_{V,V}(x, y, z, T)$  are the parameters of the generation of simplest complexes of point radiation defects.

Further, we determine distributions in space and time of concentrations of divacancies  $\Phi_V(x, y, z, t)$  and di-interstitials  $\Phi_I(x, y, z, t)$  by solving the following system of equations.<sup>[21,22]</sup>

$$\frac{\partial \Phi_I(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial y} \right]$$

$$+ \frac{\partial}{\partial z} \left[ D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial z} \right] + k_{I,I}(x, y, z, T) I^2(x, y, z, t) - k_I(x, y, z, T) I(x, y, z, t) \quad (6)$$

$$\begin{aligned} \frac{\partial \Phi_V(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[ D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial y} \right] \\ &+ \frac{\partial}{\partial z} \left[ D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial z} \right] + k_{V,V}(x, y, z, T) V^2(x, y, z, t) - k_V(x, y, z, T) V(x, y, z, t) \end{aligned}$$

Boundary and initial conditions for these equations are

$$\left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0,$$

$$\left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0,$$

$$\Phi_I(x, y, z, 0) = f_{\Phi_I}(x, y, z), \quad \Phi_V(x, y, z, 0) = f_{\Phi_V}(x, y, z) \quad (7)$$

Here,  $D_{\Phi_\rho}(x, y, z, T)$  is the diffusion coefficients of the above complexes of radiation defects;  $k_I(x, y, z, T)$  and  $k_V(x, y, z, T)$  are the parameters of decay of these complexes.

We calculate distributions of the concentrations of point radiation defects in space and time by recently elaborated approach.<sup>[18]</sup> The approach based on transformation of approximations of diffusion coefficients in the following form:  $D_\rho(x, y, z, T) = D_{0\rho} [1 + \varepsilon_\rho g_\rho(x, y, z, T)]$ , where  $D_{0\rho}$  is the average values of diffusion coefficients,  $0 \leq \varepsilon_\rho < 1$ ,  $|g_\rho(x, y, z, T)| \leq 1$ ,  $\rho = I, V$ . We also used analogous transformation of approximations of parameters of recombination of point defects and parameters of generation of their complexes:  $k_{I,V}(x, y, z, T) = k_{0I,V} [1 + \varepsilon_{I,V} g_{I,V}(x, y, z, T)]$ ,  $k_{I,I}(x, y, z, T) = k_{0I,I} [1 + \varepsilon_{I,I} g_{I,I}(x, y, z, T)]$  and  $k_{V,V}(x, y, z, T) = k_{0V,V} [1 + \varepsilon_{V,V} g_{V,V}(x, y, z, T)]$ , where  $k_{0\rho 1, \rho 2}$  is the their average values,  $0 \leq \varepsilon_{I,V} < 1$ ,  $0 \leq \varepsilon_{I,I} < 1$ ,  $0 \leq \varepsilon_{V,V} < 1$ ,  $|g_{I,V}(x, y, z, T)| \leq 1$ ,  $|g_{I,I}(x, y, z, T)| \leq 1$ ,  $|g_{V,V}(x, y, z, T)| \leq 1$ . Let us introduce the following dimensionless variables:  $\tilde{I}(x, y, z, t) = I(x, y, z, t) / I^*$ ,  $\tilde{V}(x, y, z, t) = V(x, y, z, t) / V^*$ ,  $\omega = L^2 k_{0I,V} / \sqrt{D_{0I} D_{0V}}$ ,  $\Omega_\rho = L^2 k_{0\rho, \rho} / \sqrt{D_{0I} D_{0V}}$ ,  $\vartheta = \sqrt{D_{0I} D_{0V}} t / L^2$ ,  $\chi = x / L_x$ ,  $\eta = y / L_y$ ,  $\phi = z / L_z$ . The introduction leads to transformation of Equations (4) and conditions (5) to the following form.

$$\begin{aligned} \frac{\partial \tilde{I}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \frac{D_{0I}}{\sqrt{D_{0I} D_{0V}}} \frac{\partial}{\partial \chi} \left\{ [1 + \varepsilon_I g_I(\chi, \eta, \phi, T)] \frac{\partial \tilde{I}(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right\} + \frac{\partial}{\partial \eta} \left\{ [1 + \varepsilon_I g_I(\chi, \eta, \phi, T)] \right. \\ &\times \left. \frac{\partial \tilde{I}(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right\} \frac{D_{0I}}{\sqrt{D_{0I} D_{0V}}} + \frac{D_{0I}}{\sqrt{D_{0I} D_{0V}}} \frac{\partial}{\partial \phi} \left\{ [1 + \varepsilon_I g_I(\chi, \eta, \phi, T)] \frac{\partial \tilde{I}(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right\} - \tilde{I}(\chi, \eta, \phi, \vartheta) \\ &\times \omega [1 + \varepsilon_{I,V} g_{I,V}(\chi, \eta, \phi, T)] \tilde{V}(\chi, \eta, \phi, \vartheta) - \Omega_I [1 + \varepsilon_{I,I} g_{I,I}(\chi, \eta, \phi, T)] \tilde{I}^2(\chi, \eta, \phi, \vartheta) \end{aligned} \quad (8)$$

$$\begin{aligned} \frac{\partial \tilde{V}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \frac{D_{0V}}{\sqrt{D_{0I} D_{0V}}} \frac{\partial}{\partial \chi} \left\{ [1 + \varepsilon_V g_V(\chi, \eta, \phi, T)] \frac{\partial \tilde{V}(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right\} + \frac{\partial}{\partial \eta} \left\{ [1 + \varepsilon_V g_V(\chi, \eta, \phi, T)] \right. \\ &\times \left. \frac{\partial \tilde{V}(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right\} \frac{D_{0V}}{\sqrt{D_{0I} D_{0V}}} + \frac{D_{0V}}{\sqrt{D_{0I} D_{0V}}} \frac{\partial}{\partial \phi} \left\{ [1 + \varepsilon_V g_V(\chi, \eta, \phi, T)] \frac{\partial \tilde{V}(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right\} - \tilde{I}(\chi, \eta, \phi, \vartheta) \end{aligned}$$

$$\begin{aligned} & \times \omega \left[ 1 + \varepsilon_{I,V} g_{I,V}(\chi, \eta, \varphi, T) \right] \tilde{V}(\chi, \eta, \varphi, \vartheta) - \Omega_V \left[ 1 + \varepsilon_{V,V} g_{V,V}(\chi, \eta, \varphi, T) \right] \tilde{V}^2(\chi, \eta, \varphi, \vartheta) \\ & \left. \frac{\partial \tilde{\rho}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \right|_{\chi=0} = 0, \quad \left. \frac{\partial \tilde{\rho}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \right|_{\chi=1} = 0, \quad \left. \frac{\partial \tilde{\rho}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \right|_{\eta=0} = 0, \quad \left. \frac{\partial \tilde{\rho}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \right|_{\eta=1} = 0, \\ & \left. \frac{\partial \tilde{\rho}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \right|_{\varphi=0} = 0, \quad \left. \frac{\partial \tilde{\rho}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \right|_{\varphi=1} = 0, \quad \tilde{\rho}(\chi, \eta, \varphi, \vartheta) = \frac{f_\rho(\chi, \eta, \varphi, \vartheta)}{\rho^*} \end{aligned} \quad (9)$$

We determine solutions of Equations (8) with conditions (9) framework recently introduced approach,<sup>[18]</sup> i.e. as the power series.

$$\tilde{\rho}(\chi, \eta, \varphi, \vartheta) = \sum_{i=0}^{\infty} \varepsilon_\rho^i \sum_{j=0}^{\infty} \omega^j \sum_{k=0}^{\infty} \Omega_\rho^k \tilde{\rho}_{ijk}(\chi, \eta, \varphi, \vartheta) \quad (10)$$

Substitution of the series (10) into Equations (8) and condition (9) gives us possibility to obtain equations for initial order approximations of the concentration of point defects  $\tilde{I}_{000}(\chi, \eta, \varphi, \vartheta)$  and  $\tilde{V}_{000}(\chi, \eta, \varphi, \vartheta)$  and corrections for them  $\tilde{I}_{ijk}(\chi, \eta, \varphi, \vartheta)$  and  $\tilde{V}_{ijk}(\chi, \eta, \varphi, \vartheta)$ ,  $i \geq 1, j \geq 1, k \geq 1$ . The equations are presented in the Appendix. Solutions of the equations could be obtained by standard Fourier approach.<sup>[24,25]</sup> The solutions are presented in the Appendix.

Now, we calculate distributions of the concentrations of simplest complexes of point radiation defects in space and time. To determine the distributions, we transform approximations of diffusion coefficients in the following form:  $D_{\Phi_\rho}(x, y, z, T) = D_{0\Phi_\rho} [1 + \varepsilon_{\Phi_\rho} g_{\Phi_\rho}(x, y, z, T)]$ , where  $D_{0\Phi_\rho}$  is the average values of diffusion coefficients. In this situation, the Equation (6) could be written as

$$\begin{aligned} & \frac{\partial \Phi_I(x, y, z, t)}{\partial t} = D_{0\Phi_I} \frac{\partial}{\partial x} \left\{ [1 + \varepsilon_{\Phi_I} g_{\Phi_I}(x, y, z, T)] \frac{\partial \Phi_I(x, y, z, t)}{\partial x} \right\} + k_{I,I}(x, y, z, T) I^2(x, y, z, t) \\ & + D_{0\Phi_I} \frac{\partial}{\partial y} \left\{ [1 + \varepsilon_{\Phi_I} g_{\Phi_I}(x, y, z, T)] \frac{\partial \Phi_I(x, y, z, t)}{\partial y} \right\} + D_{0\Phi_I} \frac{\partial}{\partial z} \left\{ [1 + \varepsilon_{\Phi_I} g_{\Phi_I}(x, y, z, T)] \frac{\partial \Phi_I(x, y, z, t)}{\partial z} \right\} \\ & - k_I(x, y, z, T) I(x, y, z, t) \\ & \frac{\partial \Phi_V(x, y, z, t)}{\partial t} = D_{0\Phi_V} \frac{\partial}{\partial x} \left\{ [1 + \varepsilon_{\Phi_V} g_{\Phi_V}(x, y, z, T)] \frac{\partial \Phi_V(x, y, z, t)}{\partial x} \right\} + k_{I,I}(x, y, z, T) I^2(x, y, z, t) \\ & + D_{0\Phi_V} \frac{\partial}{\partial y} \left\{ [1 + \varepsilon_{\Phi_V} g_{\Phi_V}(x, y, z, T)] \frac{\partial \Phi_V(x, y, z, t)}{\partial y} \right\} + D_{0\Phi_V} \frac{\partial}{\partial z} \left\{ [1 + \varepsilon_{\Phi_V} g_{\Phi_V}(x, y, z, T)] \frac{\partial \Phi_V(x, y, z, t)}{\partial z} \right\} \\ & - k_I(x, y, z, T) I(x, y, z, t) \end{aligned}$$

Farther, we determine solutions of above equations as the following power series.

$$\Phi_\rho(x, y, z, t) = \sum_{i=0}^{\infty} \varepsilon_{\Phi_\rho}^i \Phi_{\rho i}(x, y, z, t) \quad (11)$$

Now, we used the series (11) into Equation (6) and appropriate boundary and initial conditions. The using gives the possibility to obtain equations for initial order approximations of concentrations of complexes of defects  $\Phi_{\rho 0}(x, y, z, t)$ , corrections for them  $\Phi_{\rho i}(x, y, z, t)$  (for them  $i \geq 1$ ), and boundary and initial conditions for them. We remove equations and conditions to the Appendix. Solutions of the equations have been calculated by standard approaches<sup>[24,25]</sup> and presented in the Appendix.

Now, we calculate distribution of the concentration of dopant in space and time using the approach, which was used for analysis of radiation defects. To use the approach, we consider the following transformation of approximation of dopant diffusion coefficient:  $D_L(x,y,z,T)=D_{0L}[1+\varepsilon_L g_L(x,y,z,T)]$ , where  $D_{0L}$  is the average value of dopant diffusion coefficient,  $0 \leq \varepsilon_L < 1$ ,  $|g_L(x,y,z,T)| \leq 1$ . Farther, we consider solution of Equation (1) as the following series:

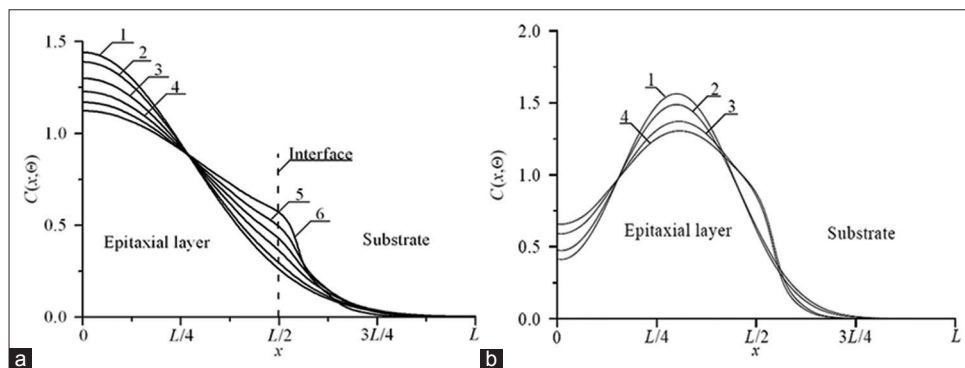
$$C(x,y,z,t) = \sum_{i=0}^{\infty} \varepsilon_L^i \sum_{j=1}^{\infty} \xi^j C_{ij}(x,y,z,t).$$

Using the relation into Equations (1) and condition (2) leads to obtaining equations for the functions  $C_{ij}(x,y,z,t)$  ( $i \geq 1, j \geq 1$ ), boundary and initial conditions for them. The equations are presented in the Appendix. Solutions of the equations have been calculated by standard approaches.<sup>[24,25]</sup> The solutions are presented in the Appendix.

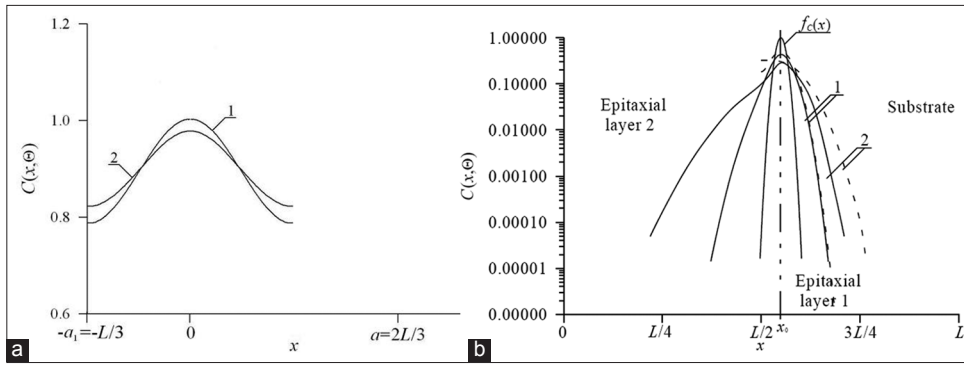
We analyzed distributions of concentrations of dopant and radiation defects in space and time analytically using the second-order approximations on all parameters, which have been used in appropriate series. Usually, the second-order approximations are enough good approximations to make qualitative analysis and to obtain quantitative results. All analytical results have been checked by numerical simulation.

## DISCUSSION

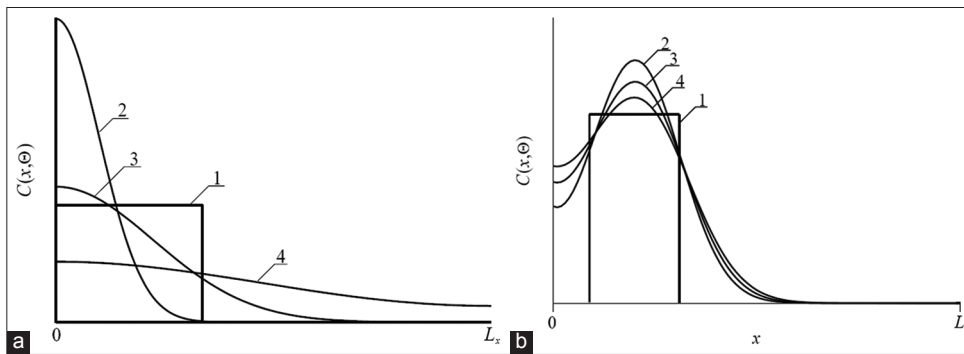
In this section, we analyzed spatiotemporal distributions of the concentrations of dopants. Figure 2 shows typical spatial distributions of the concentrations of dopants in neighborhood of interfaces of heterostructures. We calculate these distributions of the concentrations of dopants under the following condition: Value of dopant diffusion coefficient in doped area is larger than the value of dopant diffusion coefficient in nearest areas. In this situation, one can find increasing of compactness of field-effect transistors with increasing of homogeneity of distribution of the concentration of dopant at 1 time. Changing relation between values of dopant diffusion coefficients leads to opposite result [Figure 3]. It should be noted that framework the considered approach one should optimize annealing of dopant and/or radiation defects. To do the optimization, we used recently introduced criterion.<sup>[26-34]</sup> The optimization based on approximation real distribution by step-wise function  $y(x,y,z)$  [Figure 4]. Farther, the required values of optimal annealing time have been calculated by minimization the following mean squared error.



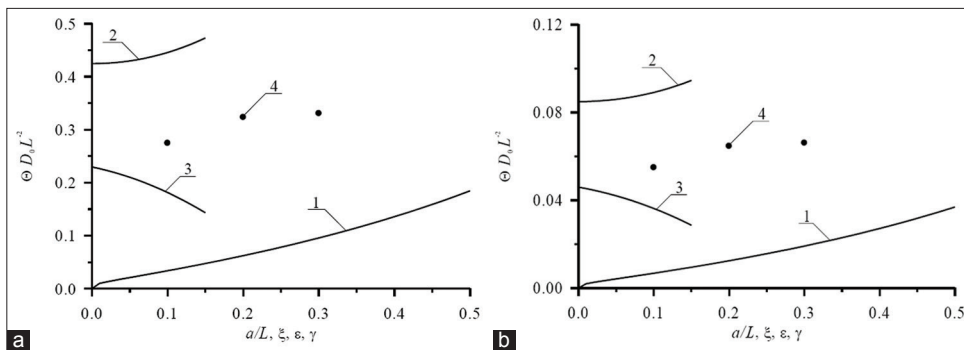
**Figure 2:** (a) Dependences of concentration of dopant, infused in heterostructure from Figure 1, on coordinate in direction, which is perpendicular to interface between epitaxial layer substrate. Difference between the values of dopant diffusion coefficient in layers of heterostructure increases with increasing of the number of curves. Value of dopant diffusion coefficient in the epitaxial layer is larger than the value of dopant diffusion coefficient in the substrate. (b) Dependences of the concentration of dopant, implanted in heterostructure from Figure 1, on coordinate in direction, which is perpendicular to interface between epitaxial layer substrate. Difference between the values of dopant diffusion coefficient in layers of heterostructure increases with increasing of the number of curves. Value of dopant diffusion coefficient in the epitaxial layer is larger than the value of dopant diffusion coefficient in the substrate. Curve 1 corresponds to homogeneous sample and annealing time  $\Theta = 0.0048 (L_x^2 + L_y^2 + L_z^2)/D_0$ . Curve 2 corresponds to homogeneous sample and annealing time  $\Theta = 0.0057 (L_x^2 + L_y^2 + L_z^2)/D_0$ . Curves 3 and 4 correspond to heterostructure from Figure 1; annealing times  $\Theta = 0.0048 (L_x^2 + L_y^2 + L_z^2)/D_0$  and  $\Theta = 0.0057 (L_x^2 + L_y^2 + L_z^2)/D_0$ , respectively



**Figure 3:** (a) Distributions of the concentration of dopant infused in average section of epitaxial layer of heterostructure from Figure 1 in direction parallel to interface between epitaxial layer and substrate of heterostructure. Difference between the values of dopant diffusion coefficients increases with increasing of the number of curves. Value of dopant diffusion coefficient in this section is smaller than the value of dopant diffusion coefficient in nearest sections. (b) Calculated distributions of implanted dopant in epitaxial layers of heterostructure. Solid lines are spatial distributions of implanted dopant in system of two epitaxial layers. Dashed lines are spatial distributions of implanted dopant in one epitaxial layer. Annealing time increases with increasing of number of curves



**Figure 4:** (a) Distributions of the concentration of infused dopant in depth of heterostructure from Figure 1 for different values of annealing time (curves 2-4) and idealized step-wise approximation (curve 1). Increasing of the number of curve corresponds to increasing of annealing time. (b) Distributions of the concentration of implanted dopant in depth of heterostructure from Figure 1 for different values of annealing time (curves 2-4) and idealized step-wise approximation (curve 1). Increasing of the number of curve corresponds to increasing of annealing time



**Figure 5:** (a) Dimensionless optimal annealing time of infused dopant as a function of several parameters. Curve 1 describes the dependence of the annealing time on the relation  $a/L$  and  $\xi=\gamma=0$  for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 describes the dependence of the annealing time on value of parameter  $\epsilon$  for  $a/L=1/2$  and  $\xi=\gamma=0$ . Curve 3 describes the dependence of the annealing time on value of parameter  $\gamma$  for  $a/L=1/2$  and  $\epsilon=\xi=0$ . Curve 4 describes the dependence of the annealing time on value of parameter  $\gamma$  for  $a/L=1/2$  and  $\epsilon=\xi=0$ . (b) Dimensionless optimal annealing time of implanted dopant as a function of several parameters. Curve 1 describes the dependence of the annealing time on the relation  $a/L$  and  $\xi=\gamma=0$  for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 describes the dependence of the annealing time on value of parameter  $\epsilon$  for  $a/L=1/2$  and  $\xi=\gamma=0$ . Curve 3 describes the dependence of the annealing time on value of parameter  $\xi$  for  $a/L=1/2$  and  $\epsilon=\gamma=0$ . Curve 4 describes the dependence of the annealing time on value of parameter  $\gamma$  for  $a/L=1/2$  and  $\epsilon=\xi=0$

$$U = \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} [C(x, y, z, \Theta) - \psi(x, y, z)] dz dy dx \quad (12)$$

We show optimal values of annealing time as functions of parameters in Figure 5. It is known that standard step of manufactured ion-doped structures is annealing of radiation defects. In the ideal case, after finishing the annealing dopant achieves interface between the layers of heterostructure. If the dopant has no enough time to achieve the interface, it is practicably to anneal the dopant additionally. Figure 5b shows the described dependences of optimal values of additional annealing time for the same parameters as for Figure 5a. Necessity to anneal radiation defects leads to smaller values of optimal annealing of implanted dopant in comparison with optimal annealing time of infused dopant.

## CONCLUSIONS

In this paper, we introduce an approach to increase the integration rate of element of a hybrid comparator with the first dynamic amplifying stage and the second quasi-dynamic latching stage. The approach gives us possibility to decrease area of the elements with smaller increasing of the element's thickness.

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## APPENDIX

Equations for the functions  $\tilde{I}_{ijk}(\chi, \eta, \varphi, \vartheta)$  and  $\tilde{V}_{ijk}(\chi, \eta, \varphi, \vartheta)$ ,  $i \geq 0, j \geq 0, k \geq 0$  and conditions for them

$$\frac{\partial \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right]$$

$$\frac{\partial \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right];$$

$$\frac{\partial \tilde{I}_{i00}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{i00}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{i00}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{i00}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] + \sqrt{\frac{D_{0I}}{D_{0V}}}$$

$$\times \left\{ \frac{\partial}{\partial \chi} \left[ g_I(\chi, \eta, \varphi, T) \frac{\partial \tilde{I}_{i-100}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \right] + \frac{\partial}{\partial \eta} \left[ g_I(\chi, \eta, \varphi, T) \frac{\partial \tilde{I}_{i-100}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \right] + \right.$$

$$\left. + \frac{\partial}{\partial \varphi} \left[ g_I(\chi, \eta, \varphi, T) \frac{\partial \tilde{I}_{i-100}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \right] \right\}, i \geq 1,$$

$$\frac{\partial \tilde{V}_{i00}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{i00}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{i00}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{i00}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] + \frac{\partial}{\partial \chi} [g_V(\chi, \eta, \varphi, T)$$

$$\times \frac{\partial \tilde{V}_{i-100}(\chi, \eta, \varphi, \vartheta)}{\partial \chi}] \sqrt{\frac{D_{0V}}{D_{0I}}} + \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial}{\partial \eta} \left[ g_V(\chi, \eta, \varphi, T) \frac{\partial \tilde{V}_{i-100}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \right] + \frac{\partial}{\partial \varphi} [g_V(\chi, \eta, \varphi, T)$$

$$\times \frac{\partial \tilde{V}_{i-100}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi}] \sqrt{\frac{D_{0V}}{D_{0I}}}, i \geq 1,$$

$$\frac{\partial \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right]$$

$$- [1 + \varepsilon_{I,V} g_{I,V}(\chi, \eta, \varphi, T)] \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta)$$

$$\frac{\partial \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right]$$

$$- [1 + \varepsilon_{I,V} g_{I,V}(\chi, \eta, \varphi, T)] \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta);$$

$$\frac{\partial \tilde{I}_{020}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{020}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{020}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{020}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right]$$

$$- [1 + \varepsilon_{I,V} g_{I,V}(\chi, \eta, \varphi, T)] [\tilde{I}_{010}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta) + \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta)]$$

$$\frac{\partial \tilde{V}_{020}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{020}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{020}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{020}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right]$$

$$- [1 + \varepsilon_{I,V} g_{I,V}(\chi, \eta, \varphi, T)] [\tilde{I}_{010}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta) + \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta)];$$

$$\frac{\partial \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right]$$

$$- [1 + \varepsilon_{I,I} g_{I,I}(\chi, \eta, \varphi, T)] \tilde{I}_{000}^2(\chi, \eta, \varphi, \vartheta)$$

$$\frac{\partial \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right]$$

$$- [1 + \varepsilon_{I,I} g_{I,I}(\chi, \eta, \varphi, T)] \tilde{V}_{000}^2(\chi, \eta, \varphi, \vartheta);$$

$$\frac{\partial \tilde{I}_{110}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{110}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{110}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{110}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] + \sqrt{\frac{D_{0I}}{D_{0V}}}$$

$$\times \left\{ \frac{\partial}{\partial \chi} \left[ g_I(\chi, \eta, \varphi, T) \frac{\partial \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \right] + \frac{\partial}{\partial \eta} \left[ g_I(\chi, \eta, \varphi, T) \frac{\partial \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \right] + \frac{\partial}{\partial \varphi} \left[ g_I(\chi, \eta, \varphi, T) \right. \right.$$

$$\left. \times \frac{\partial \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \right\} - [\tilde{I}_{100}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta) + \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{100}(\chi, \eta, \varphi, \vartheta)]$$

$$\times [1 + \varepsilon_{I,I} g_{I,I}(\chi, \eta, \varphi, T)]$$

$$\frac{\partial \tilde{V}_{110}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{110}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{110}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{110}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right]$$

$$+ \sqrt{\frac{D_{0V}}{D_{0I}}} \left\{ \frac{\partial}{\partial \chi} \left[ g_V(\chi, \eta, \varphi, T) \frac{\partial \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \right] + \frac{\partial}{\partial \eta} \left[ g_V(\chi, \eta, \varphi, T) \frac{\partial \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \right] \right\}$$

$$+ \frac{\partial}{\partial \varphi} \left[ g_V(\chi, \eta, \varphi, T) \frac{\partial \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \right] \Bigg\} - [1 + \varepsilon_{V,V} g_{V,V}(\chi, \eta, \varphi, T)]$$

$$\times [\tilde{V}_{100}(\chi, \eta, \varphi, \vartheta) \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) + \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta) \tilde{I}_{100}(\chi, \eta, \varphi, \vartheta)];$$

$$\frac{\partial \tilde{I}_{002}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{002}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{002}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{002}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right]$$

$$- [1 + \varepsilon_{I,I} g_{I,I}(\chi, \eta, \varphi, T)] \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta) \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta)$$

$$\frac{\partial \tilde{V}_{002}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{002}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{002}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{002}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right]$$

$$- [1 + \varepsilon_{V,V} g_{V,V}(\chi, \eta, \varphi, E)] \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta);$$

$$\frac{\partial \tilde{I}_{101}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{101}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{101}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{101}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right]$$

$$+ \sqrt{\frac{D_{0I}}{D_{0V}}} \left\{ \frac{\partial}{\partial \chi} \left[ g_I(\chi, \eta, \varphi, T) \frac{\partial \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \right] + \frac{\partial}{\partial \eta} \left[ g_I(\chi, \eta, \varphi, T) \frac{\partial \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \right] \right\}$$

$$+ \frac{\partial}{\partial \varphi} \left[ g_I(\chi, \eta, \varphi, T) \frac{\partial \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \right] \Bigg\} - [1 + \varepsilon_I g_I(\chi, \eta, \varphi, T)] \tilde{I}_{100}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta)$$

$$\frac{\partial \tilde{V}_{101}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{101}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{101}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{101}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right]$$

$$+ \sqrt{\frac{D_{0V}}{D_{0I}}} \left\{ \frac{\partial}{\partial \chi} \left[ g_V(\chi, \eta, \varphi, T) \frac{\partial \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \right] + \frac{\partial}{\partial \eta} \left[ g_V(\chi, \eta, \varphi, T) \frac{\partial \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \right] \right\}$$

$$+ \frac{\partial}{\partial \varphi} \left[ g_V(\chi, \eta, \varphi, T) \frac{\partial \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \right] \Bigg\} - [1 + \varepsilon_V g_V(\chi, \eta, \varphi, T)] \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{100}(\chi, \eta, \varphi, \vartheta);$$

$$\frac{\partial \tilde{I}_{011}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{011}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{011}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{011}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] - \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta)$$

$$\times [1 + \varepsilon_{I,I} g_{I,I}(\chi, \eta, \varphi, T)] \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) - [1 + \varepsilon_{I,V} g_{I,V}(\chi, \eta, \varphi, T)] \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta)$$

$$\frac{\partial \tilde{V}_{011}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{011}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{011}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{011}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] - \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta)$$

$$\times [1 + \varepsilon_{V,V} g_{V,V}(\chi, \eta, \varphi, T)] \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta) - [1 + \varepsilon_{I,V} g_{I,V}(\chi, \eta, \varphi, T)] \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta);$$

$$\left. \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \right|_{\chi=0} = 0, \left. \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \right|_{\chi=1} = 0, \left. \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \right|_{\eta=0} = 0, \left. \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \right|_{\eta=1} = 0,$$

$$\left. \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \right|_{\varphi=0} = 0, \left. \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \right|_{\varphi=1} = 0 \quad (i \geq 0, j \geq 0, k \geq 0);$$

$$\tilde{\rho}_{000}(\chi, \eta, \varphi, 0) = f_{\rho}(\chi, \eta, \varphi) / \rho^*, \tilde{\rho}_{ijk}(\chi, \eta, \varphi, 0) = 0 \quad (i \geq 0, j \geq 0, k \geq 0).$$

Solutions of the above equations could be written as

$$\tilde{\rho}_{000}(\chi, \eta, \varphi, \vartheta) = \frac{1}{L} + \frac{2}{L} \sum_{n=1}^{\infty} F_{n\rho} c(\chi) c(\eta) c(\varphi) e_{n\rho}(\vartheta),$$

$$\text{where } F_{n\rho} = \frac{1}{\rho^*} \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) f_{n\rho}(u, v, w) d w d v d u, c_n(\chi) = \cos(\pi v \chi),$$

$$e_{nI}(\vartheta) = \exp(-\pi^2 n^2 \vartheta \sqrt{D_{0V}/D_{0I}}), e_{nV}(\vartheta) = \exp(-\pi^2 n^2 \vartheta \sqrt{D_{0I}/D_{0V}});$$

$$\tilde{I}_{i00}(\chi, \eta, \varphi, \vartheta) = -2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n c_n(\chi) c(\eta) c(\varphi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 s_n(u) \int_0^1 c_n(v) \int_0^1 \frac{\partial \tilde{I}_{i-100}(u, v, w, \tau)}{\partial u}$$

$$\times c_n(w) g_I(u, v, w, T) d w d v d u d \tau - 2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n c_n(\chi) c(\eta) c(\varphi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 c_n(u) \int_0^1 s_n(v)$$

$$\times \int_0^1 c_n(w) g_I(u, v, w, T) \frac{\partial \tilde{I}_{i-100}(u, v, w, \tau)}{\partial v} d w d v d u d \tau - 2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n c_n(\chi) c(\eta) c(\varphi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau)$$

$$\times \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 s_n(w) g_I(u, v, w, T) \frac{\partial \tilde{I}_{i-100}(u, v, w, \tau)}{\partial w} d w d v d u d \tau, i \geq 1,$$

$$\tilde{V}_{i00}(\chi, \eta, \varphi, \vartheta) = -2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n c_n(\chi) c(\eta) c(\varphi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 s_n(u) \int_0^1 c_n(v) \int_0^1 g_V(u, v, w, T)$$

$$\begin{aligned} & \times c_n(w) \frac{\partial \tilde{V}_{i-100}(u, \tau)}{\partial u} d w d v d u d \tau - \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n c_n(\chi) c(\eta) c(\varphi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 c_n(u) \int_0^1 s_n(v) \\ & \times 2\pi \int_0^1 c_n(w) g_V(u, v, w, T) \frac{\partial \tilde{V}_{i-100}(u, \tau)}{\partial v} d w d v d u d \tau - 2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n c_n(\chi) c(\eta) c(\varphi) e_{nV}(\vartheta) \\ & \times \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 s_n(w) g_V(u, v, w, T) \frac{\partial \tilde{V}_{i-100}(u, \tau)}{\partial w} d w d v d u d \tau, \quad i \geq 1, \end{aligned}$$

where  $s_n(\chi) = \sin(\pi v \chi)$ ;

$$\begin{aligned} \tilde{\rho}_{010}(\chi, \eta, \varphi, \vartheta) &= -2 \sum_{n=1}^{\infty} c_n(\chi) c_n(\eta) c_n(\varphi) e_{n\rho}(\vartheta) \int_0^{\vartheta} e_{n\rho}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(w) \\ & \times [1 + \varepsilon_{I,V} g_{I,V}(u, v, w, T)] \tilde{I}_{000}(u, v, w, \tau) \tilde{V}_{000}(u, v, w, \tau) d w d v d u d \tau; \\ \tilde{\rho}_{020}(\chi, \eta, \varphi, \vartheta) &= -2 \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} c_n(\chi) c_n(\eta) c_n(\varphi) e_{n\rho}(\vartheta) \int_0^{\vartheta} e_{n\rho}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(w) [1 + \varepsilon_{I,V} \\ & \times g_{I,V}(u, v, w, T)] [\tilde{I}_{010}(u, v, w, \tau) \tilde{V}_{000}(u, v, w, \tau) + \tilde{I}_{000}(u, v, w, \tau) \tilde{V}_{010}(u, v, w, \tau)] d w d v d u d \tau; \end{aligned}$$

$$\begin{aligned} \tilde{\rho}_{001}(\chi, \eta, \varphi, \vartheta) &= -2 \sum_{n=1}^{\infty} c_n(\chi) c_n(\eta) c_n(\varphi) e_{n\rho}(\vartheta) \int_0^{\vartheta} e_{n\rho}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(w) \\ & \times [1 + \varepsilon_{\rho,\rho} g_{\rho,\rho}(u, v, w, T)] \tilde{\rho}_{000}^2(u, v, w, \tau) d w d v d u d \tau; \end{aligned}$$

$$\begin{aligned} \tilde{\rho}_{002}(\chi, \eta, \varphi, \vartheta) &= -2 \sum_{n=1}^{\infty} c_n(\chi) c_n(\eta) c_n(\varphi) e_{n\rho}(\vartheta) \int_0^{\vartheta} e_{n\rho}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(w) \\ & \times [1 + \varepsilon_{\rho,\rho} g_{\rho,\rho}(u, v, w, T)] \tilde{\rho}_{001}(u, v, w, \tau) \tilde{\rho}_{000}(u, v, w, \tau) d w d v d u d \tau; \end{aligned}$$

$$\begin{aligned} \tilde{I}_{110}(\chi, \eta, \varphi, \vartheta) &= -2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n c_n(\chi) c_n(\eta) c_n(\varphi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 s_n(u) \int_0^1 c_n(v) \int_0^1 c_n(u) \\ & \times g_I(u, v, w, T) \frac{\partial \tilde{I}_{i-100}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - 2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n c_n(\chi) c_n(\eta) c_n(\varphi) e_{nI}(\vartheta) \\ & \times \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 c_n(u) \int_0^1 s_n(v) \int_0^1 c_n(u) g_I(u, v, w, T) \frac{\partial \tilde{I}_{i-100}(u, v, w, \tau)}{\partial v} d w d v d u d \tau - 2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \\ & \times \sum_{n=1}^{\infty} n e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 s_n(u) g_I(u, v, w, T) \frac{\partial \tilde{I}_{i-100}(u, v, w, \tau)}{\partial w} d w d v d u d \tau \end{aligned}$$

$$\begin{aligned}
 & \times c_n(\chi)c_n(\eta)c_n(\varphi) - 2 \sum_{n=1}^{\infty} c_n(\chi)e_{nI}(\vartheta)c_n(\eta)c_n(\varphi) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(v) [1 + \varepsilon_{I,V} \\
 & \times g_{I,V}(u,v,w,T)] [\tilde{I}_{100}(u,v,w,\tau) \tilde{V}_{000}(u,v,w,\tau) + \tilde{I}_{000}(u,v,w,\tau) \tilde{V}_{100}(u,v,w,\tau)] d w d v d u d \tau \\
 \\
 & \tilde{V}_{110}(\chi,\eta,\varphi,\vartheta) = -2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n c_n(\chi)c_n(\eta)c_n(\varphi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 s_n(u) \int_0^1 c_n(v) \int_0^1 c_n(u) \\
 & \times g_V(u,v,w,T) \frac{\partial \tilde{V}_{i-100}(u,v,w,\tau)}{\partial u} d w d v d u d \tau - 2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n c_n(\chi)c_n(\eta)c_n(\varphi) e_{nV}(\vartheta) \\
 & \times \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 c_n(u) \int_0^1 s_n(v) \int_0^1 c_n(u) g_V(u,v,w,T) \frac{\partial \tilde{V}_{i-100}(u,v,w,\tau)}{\partial v} d w d v d u d \tau - 2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \\
 & \times \sum_{n=1}^{\infty} n e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 s_n(u) g_V(u,v,w,T) \frac{\partial \tilde{V}_{i-100}(u,v,w,\tau)}{\partial w} d w d v d u d \tau \\
 & \times c_n(\chi)c_n(\eta)c_n(\varphi) - 2 \sum_{n=1}^{\infty} c_n(\chi)e_{nI}(\vartheta)c_n(\eta)c_n(\varphi) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 [1 + \varepsilon_{I,V} g_{I,V}(u,v,w,T)] \\
 & \times c_n(w) [\tilde{I}_{100}(u,v,w,\tau) \tilde{V}_{000}(u,v,w,\tau) + \tilde{I}_{000}(u,v,w,\tau) \tilde{V}_{100}(u,v,w,\tau)] d w d v d u d \tau ; \\
 \\
 & \tilde{I}_{101}(\chi,\eta,\varphi,\vartheta) = -2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n c_n(\chi)c_n(\eta)c_n(\varphi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 s_n(u) \int_0^1 c_n(v) \int_0^1 g_I(u,v,w,T) \\
 & \times c_n(w) \frac{\partial \tilde{I}_{001}(u,v,w,\tau)}{\partial u} d w d v d u d \tau - 2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n c_n(\chi)c_n(\eta)c_n(\varphi) e_{nI}(\vartheta) \\
 & \times \int_0^1 s_n(v) \int_0^1 c_n(w) g_I(u,v,w,T) \frac{\partial \tilde{I}_{001}(u,v,w,\tau)}{\partial v} d w d v d u d \tau - 2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n e_{nI}(\vartheta) c_n(\chi)c_n(\eta)c_n(\varphi) \\
 & \times \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 s_n(w) g_I(u,v,w,T) \frac{\partial \tilde{I}_{001}(u,v,w,\tau)}{\partial w} d w d v d u d \tau - 2 \sum_{n=1}^{\infty} c_n(\chi)c_n(\eta)c_n(\varphi) \\
 & \times e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(w) [1 + \varepsilon_{I,V} g_{I,V}(u,v,w,T)] \tilde{I}_{100}(u,v,w,\tau) \tilde{V}_{000}(u,v,w,\tau) d w d v d u d \tau \\
 \\
 & \tilde{V}_{101}(\chi,\eta,\varphi,\vartheta) = -2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n c_n(\chi)c_n(\eta)c_n(\varphi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 s_n(u) \int_0^1 c_n(v) \int_0^1 g_V(u,v,w,T)
 \end{aligned}$$

$$\begin{aligned} & \times c_n(w) \frac{\partial \tilde{V}_{001}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - 2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n c_n(\chi) c_n(\eta) c_n(\varphi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 c_n(u) \\ & \times \int_0^1 s_n(v) \int_0^1 c_n(w) g_I(u, v, w, T) \frac{\partial \tilde{I}_{001}(u, v, w, \tau)}{\partial v} d w d v d u d \tau - 2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n e_{nI}(\vartheta) c_n(\chi) c_n(\eta) c_n(\varphi) \\ & \times \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 s_n(w) g_V(u, v, w, T) \frac{\partial \tilde{V}_{001}(u, v, w, \tau)}{\partial w} d w d v d u d \tau - 2 \sum_{n=1}^{\infty} c_n(\chi) c_n(\eta) c_n(\varphi) \\ & \times e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(w) [1 + \varepsilon_{I,V} g_{I,V}(u, v, w, T)] \tilde{I}_{100}(u, v, w, \tau) \tilde{V}_{000}(u, v, w, \tau) d w d v d u d \tau ; \\ \tilde{I}_{011}(\chi, \eta, \varphi, \vartheta) & = -2 \sum_{n=1}^{\infty} c_n(\chi) c_n(\eta) c_n(\varphi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(w) \{ \tilde{I}_{000}(u, v, w, \tau) \\ & \times [1 + \varepsilon_{I,I} g_{I,I}(u, v, w, T)] \tilde{I}_{010}(u, v, w, \tau) + [1 + \varepsilon_{I,V} g_{I,V}(u, v, w, T)] \tilde{I}_{001}(u, v, w, \tau) \tilde{V}_{000}(u, v, w, \tau) \} d w d v d u d \tau \\ \tilde{V}_{011}(\chi, \eta, \varphi, \vartheta) & = -2 \sum_{n=1}^{\infty} c_n(\chi) c_n(\eta) c_n(\varphi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(w) \{ \tilde{I}_{000}(u, v, w, \tau) \\ & \times [1 + \varepsilon_{I,I} g_{I,I}(u, v, w, T)] \tilde{I}_{010}(u, v, w, \tau) + [1 + \varepsilon_{I,V} g_{I,V}(u, v, w, T)] \tilde{I}_{001}(u, v, w, \tau) \tilde{V}_{000}(u, v, w, \tau) \} d w d v d u d \tau . \end{aligned}$$

Equations for functions  $\Phi_{\rho_i}(x, y, z, t)$ ,  $i \geq 0$  to describe concentrations of simplest complexes of radiation defects.

$$\frac{\partial \Phi_{I_0}(x, y, z, t)}{\partial t} = D_{0\Phi I} \left[ \frac{\partial^2 \Phi_{I_0}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 \Phi_{I_0}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 \Phi_{I_0}(x, y, z, t)}{\partial z^2} \right]$$

$$+ k_{I,I}(x, y, z, T) I^2(x, y, z, t) - k_I(x, y, z, T) I(x, y, z, t)$$

$$\frac{\partial \Phi_{V_0}(x, y, z, t)}{\partial t} = D_{0\Phi V} \left[ \frac{\partial^2 \Phi_{V_0}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 \Phi_{V_0}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 \Phi_{V_0}(x, y, z, t)}{\partial z^2} \right]$$

$$+ k_{V,V}(x, y, z, T) V^2(x, y, z, t) - k_V(x, y, z, T) V(x, y, z, t);$$

$$\frac{\partial \Phi_{I_i}(x, y, z, t)}{\partial t} = D_{0\Phi I} \left[ \frac{\partial^2 \Phi_{I_i}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 \Phi_{I_i}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 \Phi_{I_i}(x, y, z, t)}{\partial z^2} \right]$$

$$+ D_{0\Phi I} \left\{ \frac{\partial}{\partial x} \left[ g_{\Phi I}(x, y, z, T) \frac{\partial \Phi_{I_{i-1}}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ g_{\Phi I}(x, y, z, T) \frac{\partial \Phi_{I_{i-1}}(x, y, z, t)}{\partial y} \right] \right\}$$

$$+ \frac{\partial}{\partial z} \left[ g_{\Phi l}(x, y, z, T) \frac{\partial \Phi_{li-1}(x, y, z, t)}{\partial z} \right] \Bigg\}, i \geq 1,$$

$$\frac{\partial \Phi_{Vi}(x, y, z, t)}{\partial t} = D_{0\Phi V} \left[ \frac{\partial^2 \Phi_{Vi}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 \Phi_{Vi}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 \Phi_{Vi}(x, y, z, t)}{\partial z^2} \right]$$

$$+ D_{0\Phi V} \left\{ \frac{\partial}{\partial x} \left[ g_{\Phi V}(x, y, z, T) \frac{\partial \Phi_{Vi-1}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ g_{\Phi V}(x, y, z, T) \frac{\partial \Phi_{Vi-1}(x, y, z, t)}{\partial y} \right] \right\}$$

$$+ \frac{\partial}{\partial z} \left[ g_{\Phi V}(x, y, z, T) \frac{\partial \Phi_{Vi-1}(x, y, z, t)}{\partial z} \right] \Bigg\}, i \geq 1;$$

Boundary and initial conditions for the functions take the form

$$\frac{\partial \Phi_{\rho i}(x, y, z, t)}{\partial x} \Bigg|_{x=0} = 0, \frac{\partial \Phi_{\rho i}(x, y, z, t)}{\partial x} \Bigg|_{x=L_x} = 0, \frac{\partial \Phi_{\rho i}(x, y, z, t)}{\partial y} \Bigg|_{y=0} = 0, \frac{\partial \Phi_{\rho i}(x, y, z, t)}{\partial y} \Bigg|_{y=L_y} = 0,$$

$$\frac{\partial \Phi_{\rho i}(x, y, z, t)}{\partial z} \Bigg|_{z=0} = 0, \frac{\partial \Phi_{\rho i}(x, y, z, t)}{\partial z} \Bigg|_{z=L_z} = 0, i \geq 0; \Phi_{\rho} 0(x, y, z, 0) = f_{\Phi \rho}(x, y, z),$$

$$\Phi_{\rho i}(x, y, z, 0) = 0, i \geq 1.$$

Solutions of the above equations could be written as

$$\Phi_{\rho 0}(x, y, z, t) = \frac{1}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{n\Phi \rho} c_n(x) c_n(y) c_n(z) e_{n\Phi \rho}(t) + \frac{2}{L} \sum_{n=1}^{\infty} n c_n(x) c_n(y) c_n(z) \times e_{\Phi \rho, n}(t) \int_0^t e_{\Phi \rho, n}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) [k_{I, l}(u, v, w, T) I^2(u, v, w, \tau)$$

$$- k_I(u, v, w, T) I(u, v, w, \tau)] d w d v d u d \tau,$$

where

$$F_{n\Phi \rho} = \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) f_{\Phi \rho}(u, v, w) d w d v d u, e_{n\Phi \rho}(t) = \exp \left[ -\pi^2 n^2 D_{0\Phi \rho} t (L_x^{-2} + L_y^{-2} + L_z^{-2}) \right],$$

$$c_n(x) = \cos(\pi v \xi / L_x);$$

$$\Phi_{\rho i}(x, y, z, t) = -\frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n c_n(x) c_n(y) c_n(z) e_{\Phi \rho, n}(t) \int_0^t e_{\Phi \rho, n}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} g_{\Phi \rho}(u, v, w, T) \times c_n(w) \frac{\partial \Phi_{I \rho, i-1}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n c_n(x) c_n(y) c_n(z) e_{\Phi \rho, n}(t) \int_0^t e_{\Phi \rho, n}(-\tau)$$



$$\begin{aligned} & \times \int_0^t e_{\Phi_{\rho,n}}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) g_{\Phi_{\rho}}(u, v, w, T) \frac{\partial \Phi_{I_{\rho}, i-1}(u, v, w, \tau)}{\partial v} d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n \\ & \times e_{\Phi_{\rho,n}}(t) \int_0^t e_{\Phi_{\rho,n}}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) \frac{\partial \Phi_{I_{\rho}, i-1}(u, v, w, \tau)}{\partial w} g_{\Phi_{\rho}}(u, v, w, T) d w d v d u d \tau \\ & \times c_n(x) c_n(y) c_n(z), i \geq 1, \end{aligned}$$

where  $s_n(x) = \sin(\pi v x/L_x)$ .

Equations for the functions  $C_{ij}(x, y, z, t)$  ( $i \geq 0, j \geq 0$ ), boundary and initial conditions could be written as

$$\begin{aligned} \frac{\partial C_{00}(x, y, z, t)}{\partial t} &= D_{0L} \frac{\partial^2 C_{00}(x, y, z, t)}{\partial x^2} + D_{0L} \frac{\partial^2 C_{00}(x, y, z, t)}{\partial y^2} + D_{0L} \frac{\partial^2 C_{00}(x, y, z, t)}{\partial z^2}, \\ \frac{\partial C_{i0}(x, y, z, t)}{\partial t} &= D_{0L} \left[ \frac{\partial^2 C_{i0}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 C_{i0}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 C_{i0}(x, y, z, t)}{\partial z^2} \right] \\ &+ D_{0L} \frac{\partial}{\partial x} \left[ g_L(x, y, z, T) \frac{\partial C_{i-10}(x, y, z, t)}{\partial x} \right] + D_{0L} \frac{\partial}{\partial y} \left[ g_L(x, y, z, T) \frac{\partial C_{i-10}(x, y, z, t)}{\partial y} \right] \\ &+ D_{0L} \frac{\partial}{\partial z} \left[ g_L(x, y, z, T) \frac{\partial C_{i-10}(x, y, z, t)}{\partial z} \right], i \geq 1; \\ \frac{\partial C_{01}(x, y, z, t)}{\partial t} &= D_{0L} \frac{\partial^2 C_{01}(x, y, z, t)}{\partial x^2} + D_{0L} \frac{\partial^2 C_{01}(x, y, z, t)}{\partial y^2} + D_{0L} \frac{\partial^2 C_{01}(x, y, z, t)}{\partial z^2} \\ &+ D_{0L} \frac{\partial}{\partial x} \left[ \frac{C_{00}^{\gamma}(x, y, z, t)}{P^{\gamma}(x, y, z, T)} \frac{\partial C_{00}(x, y, z, t)}{\partial x} \right] + D_{0L} \frac{\partial}{\partial y} \left[ \frac{C_{00}^{\gamma}(x, y, z, t)}{P^{\gamma}(x, y, z, T)} \frac{\partial C_{00}(x, y, z, t)}{\partial y} \right] \\ &+ D_{0L} \frac{\partial}{\partial z} \left[ \frac{C_{00}^{\gamma}(x, y, z, t)}{P^{\gamma}(x, y, z, T)} \frac{\partial C_{00}(x, y, z, t)}{\partial z} \right], \\ \frac{\partial C_{02}(x, y, z, t)}{\partial t} &= D_{0L} \frac{\partial^2 C_{02}(x, y, z, t)}{\partial x^2} + D_{0L} \frac{\partial^2 C_{02}(x, y, z, t)}{\partial y^2} + D_{0L} \frac{\partial^2 C_{02}(x, y, z, t)}{\partial z^2} \\ &+ D_{0L} \left\{ \frac{\partial}{\partial x} \left[ C_{01}(x, y, z, t) \frac{C_{00}^{\gamma-1}(x, y, z, t)}{P^{\gamma}(x, y, z, T)} \frac{\partial C_{00}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ C_{01}(x, y, z, t) \frac{C_{00}^{\gamma-1}(x, y, z, t)}{P^{\gamma}(x, y, z, T)} \right. \right. \\ &\times \left. \left. \frac{\partial C_{00}(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ C_{01}(x, y, z, t) \frac{C_{00}^{\gamma-1}(x, y, z, t)}{P^{\gamma}(x, y, z, T)} \frac{\partial C_{00}(x, y, z, t)}{\partial z} \right] \right\} \\ &\times \left. \frac{\partial C_{00}(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ C_{01}(x, y, z, t) \frac{C_{00}^{\gamma-1}(x, y, z, t)}{P^{\gamma}(x, y, z, T)} \frac{\partial C_{00}(x, y, z, t)}{\partial z} \right] \Bigg\} + D_{0L} \left\{ \frac{\partial}{\partial x} \left[ \frac{C_{00}^{\gamma}(x, y, z, t)}{P^{\gamma}(x, y, z, T)} \right. \right. \end{aligned}$$

$$\times \left. \left[ \frac{\partial C_{01}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{C_{00}^\gamma(x, y, z, t)}{P^\gamma(x, y, z, T)} \frac{\partial C_{01}(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ \frac{C_{00}^\gamma(x, y, z, t)}{P^\gamma(x, y, z, T)} \frac{\partial C_{01}(x, y, z, t)}{\partial z} \right] \right\};$$

$$\frac{\partial C_{11}(x, y, z, t)}{\partial t} = D_{0L} \frac{\partial^2 C_{11}(x, y, z, t)}{\partial x^2} + D_{0L} \frac{\partial^2 C_{11}(x, y, z, t)}{\partial y^2} + D_{0L} \frac{\partial^2 C_{11}(x, y, z, t)}{\partial z^2}$$

$$+ \left\{ \frac{\partial}{\partial x} \left[ C_{10}(x, y, z, t) \frac{C_{00}^{\gamma-1}(x, y, z, t)}{P^\gamma(x, y, z, T)} \frac{\partial C_{00}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ C_{10}(x, y, z, t) \frac{C_{00}^{\gamma-1}(x, y, z, t)}{P^\gamma(x, y, z, T)} \right. \right.$$

$$\left. \left. \times \frac{\partial C_{00}(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ C_{10}(x, y, z, t) \frac{C_{00}^{\gamma-1}(x, y, z, t)}{P^\gamma(x, y, z, T)} \frac{\partial C_{00}(x, y, z, t)}{\partial z} \right] \right\} D_{0L}$$

$$+ D_{0L} \left\{ \frac{\partial}{\partial x} \left[ \frac{C_{00}^\gamma(x, y, z, t)}{P^\gamma(x, y, z, T)} \frac{\partial C_{10}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{C_{00}^\gamma(x, y, z, t)}{P^\gamma(x, y, z, T)} \frac{\partial C_{10}(x, y, z, t)}{\partial y} \right] \right.$$

$$\left. \left. + \frac{\partial}{\partial z} \left[ \frac{C_{00}^\gamma(x, y, z, t)}{P^\gamma(x, y, z, T)} \frac{\partial C_{10}(x, y, z, t)}{\partial z} \right] \right\} + D_{0L} \left\{ \frac{\partial}{\partial x} \left[ g_L(x, y, z, T) \frac{\partial C_{01}(x, y, z, t)}{\partial x} \right] \right.$$

$$\left. \left. + \frac{\partial}{\partial y} \left[ g_L(x, y, z, T) \frac{\partial C_{01}(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ g_L(x, y, z, T) \frac{\partial C_{01}(x, y, z, t)}{\partial z} \right] \right\};$$

$$\left. \frac{\partial C_{ij}(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \left. \frac{\partial C_{ij}(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \left. \frac{\partial C_{ij}(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \left. \frac{\partial C_{ij}(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0,$$

$$\left. \frac{\partial C_{ij}(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \left. \frac{\partial C_{ij}(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, i \geq 0, j \geq 0;$$

$$C_{00}(x, y, z, 0) = f_C(x, y, z), C_{ij}(x, y, z, 0) = 0, i \geq 1, j \geq 1.$$

Functions  $C_{ij}(x, y, z, t)$  ( $i \geq 0, j \geq 0$ ) could be approximated by the following series during solutions of the above equations.

$$C_{00}(x, y, z, t) = \frac{F_{0C}}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t).$$

$$\text{Here, } e_{nC}(t) = \exp \left[ -\pi^2 n^2 D_{0C} t \left( \frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2} \right) \right], F_{nC} = \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} f_C(u, v, w) c_n(w) dw dv du;$$

$$C_{i0}(x, y, z, t) = -\frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} g_L(u, v, w, T)$$

$$\begin{aligned} & \times c_n(w) \frac{\partial C_{i-10}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \\ & \times \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(v) g_L(u, v, w, T) \frac{\partial C_{i-10}(u, v, w, \tau)}{\partial v} d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n F_{nC} e_{nC}(t) \\ & \times c_n(x) c_n(y) c_n(z) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(v) g_L(u, v, w, T) \frac{\partial C_{i-10}(u, v, w, \tau)}{\partial w} d w d v d u d \tau, i \geq 1; \\ C_{01}(x, y, z, t) &= -\frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) \\ & \times \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \\ & \times \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial v} d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n e_{nC}(t) \\ & \times F_{nC} c_n(x) c_n(y) c_n(z) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial w} d w d v d u d \tau; \\ C_{02}(x, y, z, t) &= -\frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) \\ & \times C_{01}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{nC} c_n(x) c_n(y) \\ & \times n c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} C_{01}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial v} \\ & \times c_n(w) d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \\ & \times \int_0^{L_z} s_n(w) C_{01}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial w} d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n c_n(x) \\ & \times F_{nC} c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) C_{01}(u, v, w, \tau) \frac{\partial C_{00}(u, v, w, \tau)}{\partial u} \\ & \times \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \end{aligned}$$

$$\begin{aligned}
 & \times \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) C_{01}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial v} d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n \\
 & \times F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) C_{01}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \\
 & \times \frac{\partial C_{00}(u, v, w, \tau)}{\partial w} d w d v d u d \tau - \frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \\
 & \times n \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{01}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} c_n(x) e_{nC}(t) \\
 & \times F_{nC} c_n(y) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{01}(u, v, w, \tau)}{\partial v} d w d v d u d \tau \\
 & \times n c_n(z) - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) \\
 & \times \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{01}(u, v, w, \tau)}{\partial w} d w d v d u d \tau ; \\
 C_{11}(x, y, z, t) = & - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) \\
 & \times g_L(u, v, w, T) \frac{\partial C_{01}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \\
 & \times \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) g_L(u, v, w, T) \frac{\partial C_{01}(u, v, w, \tau)}{\partial v} d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z^2} \\
 & \times \sum_{n=1}^{\infty} n e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) g_L(u, v, w, T) \frac{\partial C_{01}(u, v, w, \tau)}{\partial w} d w d v d u d \tau \\
 & \times F_{nC} c_n(x) c_n(y) c_n(z) - \frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \\
 & \times n \int_0^{L_z} c_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{10}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) \\
 & \times c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{10}(u, v, w, \tau)}{\partial v} d w d v d u d \tau
 \end{aligned}$$

$$\begin{aligned}
 & - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \\
 & \times \frac{\partial C_{10}(u, v, w, \tau)}{\partial w} d w d v d u d \tau - \frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \\
 & \times \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) C_{10}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n \\
 & \times F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial v} \\
 & \times C_{10}(u, v, w, \tau) d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \\
 & \times \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) C_{10}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial w} d w d v d u d \tau
 \end{aligned}$$