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RESEARCH ARTICLE

Some Results on Modified Metrization Theorems

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ABSTRACT

In this paper, we have established the some results on modified topological metric spaces.

Key words: Topological spaces, basis, metrizable spaces
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INTRODUCTION

In this discussion of some equivalence metrization theorems, modified some sequence theorems and modified double sequence theorems have been studied by Nigata.[1] We also defined metric topologies, before that, however, we want to give a name to those topological spaces.

Definition of T_1 spaces

A T_1 – space is a topological space in which given any pair of disjoint points, each has a neighborhood which does not contain the other.
It is obvious that any subspace of T_1 – space is also a T_1 – space.

Definition

A topological space (X, T) is said to be metrizable if there is a metric d on X that generates T, topologies are metric topologies.

Theorem 1

If a topological space τ then
1.1 is a T_1-space
1.2 has a neighborhood basis of $\{U_n(p): n=1,2\ldots\ldots\}$
1.3 $\{q \notin U_n(p)\} \Rightarrow H_n(q) \cap H_n(p) = \phi$
1.4 $\{q \in H_n(p)\} \Rightarrow H_n(q) \subset U_n$ (p) then τ is metrizable.

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Theorem 2

If a topological space τ then
2.1 is a T_1-space
2.2 for every $p \in \tau$ then there exists a neighborhood basis $\{V_n(p): n = 1,2,3\ldots\}$
2.3 given that $V_n(p)$ there exits $m > n$ and $m = m(n,p)$ such that $V_m(q) \cap V_m(p) \neq \phi \Rightarrow V_m(p)$ then τ is metrizable.

Proof: To show that the conditions of Theorem 1, imply the conditions of Theorem 2, we have established only (2.3) of Theorem 2.
If (2.3) does not hold.

Let q $\notin U_n$ (p) and $H_n(q) \cap H_n(p) \neq \varphi$ (2.4)

Let $s \in H_n(q) \Rightarrow H_n(s) \subset U_n(q)$ and $s \in H_n(q) \Rightarrow q \in H_n(s)$ which implies
$H_n(q) \subset U_n(s)$ also s $\in H_n(p) \Rightarrow p \in H_n(s)$ which implies as $H_n(p) \Rightarrow U_n(s)$

Therefore, q $\in H_n(s) \subset U_n(p)$ (2.5)

Which is a contradiction of (2.3) is established and therefore the proof is completed.
We studied by the proof given by Martin[3] that is contradiction of Theorem 1 imply conditions of Theorem 2. We have only to establish (2.3).
Proof: Without loss of geniality we assume that

$$U_{n+1}(p) \subset U_n(p) \quad (2.6)$$

For all $n \in N$ and $p \in H$.

$$\text{Set } V_n(p) = H_1(p) \cap H_2(p) \cap \ldots\ldots H_n(p) \quad (2.7)$$

For all $n \in N$ and $p \in H$.

The sequence $\{U_n(p)\}$ and $\{Vn(p)\}$ will satisfy the conditions of (2.2), (2.3), and (2.4).
By (2.2) there exists m $m > n$ with

$$U_m(p) \subset V_n(p) \tag{2.8}$$

Similarly, there exists $k > m$ such that

$$U_m(p) \subset V_n(p) \tag{2.9}$$

Suppose $V_k(q) \cap V_k(p) \neq \varphi$ (2.10)
By (2.3) which implies that
$q \in U_k(p)$ But (2.8) we have $q \in V_m(p)$ from (2.4).

$$V_m(q) \subset U_m(p) \tag{2.11}$$

Combining (2.7), (2.8), and (2.10) we have $V_k(q) \subset V_n(p)$ which proves (2.3).
From (2.4), we have $V_n(p) \subset U_n(p)$ if a neighborhood $U(p)$ of p is given science from the existence of n such that $U_n(p) \subset U(p)$ and hence $V_n(p) \subset U(p)$. Thus, $V_n(p)$ is neighborhood basis at p., i.e. (2.2) is proved.

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