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RESEARCH ARTICLE

A Comparative Mathematical Study on the Area of a Trapezoid

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ABSTRACT

In this study, we have found a completely new formula to calculate the area of a trapezoid. Due to the difficulties to determine the area of trapezoid (AOT), our proposed method has thus indicated possible alternative way for finding the AOT which initiates further research and development on a new mathematical formula under study.

Key words: Area, height, new formula, trapezoid

INTRODUCTION

From the very past decade, the area of a land is determined through the adding of two triangles[1] for a trapezoid system. This is much complicated and conditional as well.

In another case, for a given four sides of a trapezoid, the ordinary formula[2] only used when we have the height of that system. However, this is more complicated sometimes. In this sense, we have studied a lot to get of this issue. An area of a given trapezoid system also calculated with the help of our proposed method to understand in a better way and numerical evidence as well.

In 2014, Manizade and Mason[3] presented an interesting way out to find an area of a trapezoid by the contrast of the ordinary method. Herein, we also make a calculation for calculating the area of trapezoid (AOT) in a better way, including without zero limitation.

Moreover, our proposed method displays an alternative and interesting way to determine the AOT in the absence of height finding. In view of the above-mentioned importance on the new formula, there is a use to investigate and the method for AOT calculation in this modern age and also for future research.

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RESULTS AND DISCUSSION

Methodology

PRTQ is a trapezoid [Figure 1], in which PQ and RT are parallel sides; PR and QT are oblique sides. We cut the RS equal to PQ from RT side, that is, PQ = RS

Now, PQ || RT *i.e.* PQ || RS, and PQ = RS. Hence, PQSR is a parallelogram. PR || QS and PR = QS.

Let, RT = a, PQ = b, PR = c, QT = d. so, ST = RT-RS = a-b = e (let), height QN = h (let).

The area of the trapezoid is $(1/2) \times (a+b)\times h$; according to ordinary method.

The area of the trapezoid is $(S/4D) \times \sqrt{\{(S_1^2-D^2)(D^2-D_1^2)\}}$; according to proposed method.

Where, $S = a+b$, $S_1 = c+d$, $D = a-b$, $D_1 = c-d$.

Proof: In triangle QST, QN is height, let QN = h,

The half perimeter of QST,

$$s = \{(a-b) +d+c\}/2$$
$$=(c+d+e)/2;\ a-b=e$$

The area of the triangle QST is

$$\sqrt{\{s(s-a)(s-b)(s-c)\}}$$
$$= \sqrt{[\{(c+d+e)/2\}\{(c+d+e)/2-c\}\{(c+d+e)/2-d\}\{(c+d+e)/2-e\}]}$$
$$= (1/4) \times\sqrt{\{(c+d+e)(c+d-e)(e+c-d)(e-c+d)\}}$$
$$= (1/4) \times\sqrt{[\{(c+d)^2-e^2\}\{e^2-(c-d)^2\}]}$$
$$= (1/4) \times\sqrt{\{(S_1^2-D^2)(D^2-D_1^2)\}},\ \text{where } c+d = S_1,\ e = a-b = D,\ c-d = D_1$$

Now, if the height of triangle is h then,

$$(1/2)\times h\times (a-b) = (1/4) \times\sqrt{\{(S_1^2-D^2)(D^2-D_1^2)\}}$$
$$\text{Or, } h = \{1/(2D)\} \times\sqrt{\{(S_1^2-D^2)(D^2-D_1^2)\}}$$

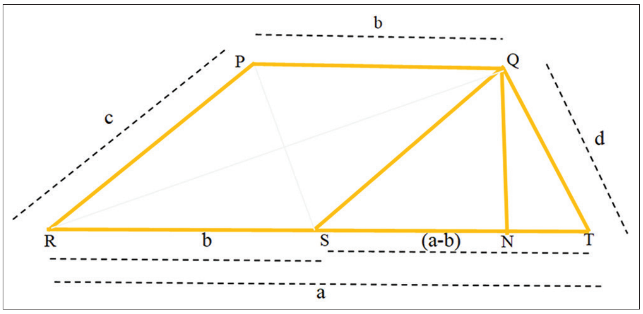

Figure 1: PQTR trapezoid

Now, the area of the trapezoid is $(1/2) \times (a+b) \times h$

$$= (1/2) \times S \times \{1/(2D)\} \times \sqrt{\{(S_1^2-D^2)(D^2-D_1^2)\}}$$

$$= (S/4D) \times \sqrt{\{(S_1^2-D^2)(D^2-D_1^2)\}}$$

Interestingly herein, we are going to proof the whole discussion with the help of an example which gives a numerical evidence for our proposed method and bitterness of the ordinary method as well.

Let, the two parallel sides of a trapezoid are 91 cm and 51 cm, the transverses sides are 37 cm and 13 cm. We have to find out the area of the trapezoid.

Solution in ordinary method

Let, ABCD is a trapezoid [Figure 2], in which the parallel side AB is 91cm and DC is 51 cm and the other two sides, AD is 37 cm and BC is 13 cm.

Let, the height DF = CE= h cm and EB = x,

$$AF = AB-BE-EF$$
$$=91-x-51$$
$$=(40-x) \text{ cm}$$

In right-angled triangle EBC, $BC^2 = CE^2+EB$,

$$\text{Or, } 13^2 = h^2+x^2$$
$$\text{Or, } h^2 = 13^2-x^2 \quad (1)$$

Again, right-angled triangle ADF, $AD^2 = AF^2+DF^2$

$$\text{Or, } 37^2 = (40-x)^2+h^2$$
$$\text{Or, } h^2 = 37^2-(40-x)^2 \quad (2)$$

Now, we get from the equation (1) and (2), 13^2-x^2

$$= 37^2-(40-x)^2$$
$$\text{Or, } 80x = 400$$
$$\text{Or, } x = 5$$
$$\text{Now, } h^2 = 13^2-5^2$$
$$\text{Or, } h = 12$$

The area of the trapezoid $= (1/2) \times (AB+CD) \times h$

$$= (1/2) \times (91+51) \times 12$$
$$= 852 \text{ cm}^2$$

Again, by applying the proposed method

Here [Figure 3], the summation of the parallel two sides is, S = AB + CD

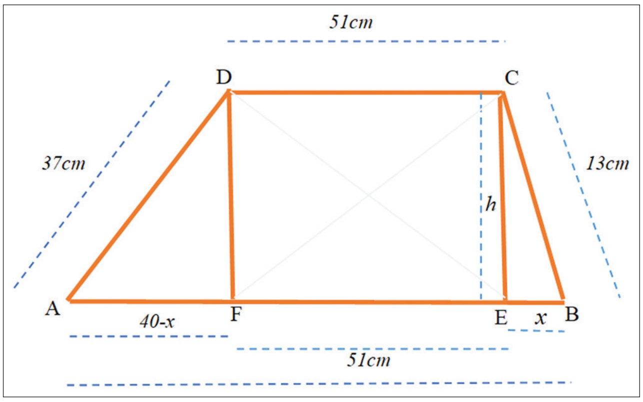

Figure 2: ABCD trapezoid

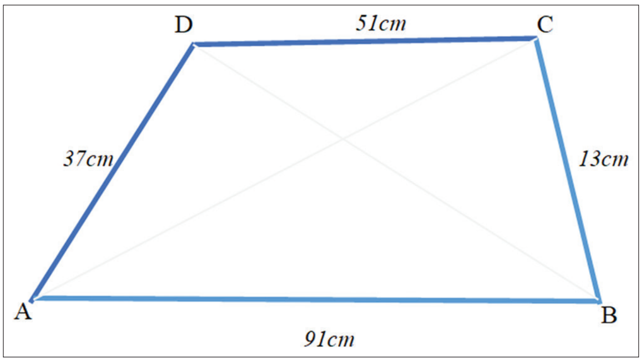

Figure 3: ABCD trapezoid

$$= 91 + 51$$
$$= 142 \text{ cm}$$

The difference, D = AB - CD, where AB > CD

$$= 40 \text{ cm}$$

The summation of the two oblique sides is, S_1=AD + CB

$$=37+13$$
$$=50 \text{ cm}$$

The difference D_1 = AD - CB, AD >CB

$$= 37–13$$
$$= 24 \text{ cm}$$

So, the area of the trapezoid is $(S/4D) \times \sqrt{\{(S_1^2-D^2)(D^2-D_1^2)\}}$

$$= \{142/(4\times40)\} \times \sqrt{\{(50^2-40^2)(40^2-24^2)\}}$$
$$= (142/160) \times \sqrt{(900 \times 1024)}$$
$$= (142/160) \times 960$$
$$= 852 \text{ cm}^2$$

CONCLUSION

Before this study, there is no alternative method to determine the AOT in such an effectively and easiest way; actually, we initiatively discussed in the results and discussions section. Due to the massive difficulties for using the ordinary method, the newly proposed method will provide much initiative to research and mathematical community, an interesting new work, where the value of AOT actually used as well.

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REFERENCES

1. The Mathematical Gazette. Vol. 87; 2003. p. 324-6. Available from: https://www.en.wikipedia.org/wiki/Heron%27s_formula. 2018:23, 25-30.
2. The Mathematics Teacher. Vol. 53; 1960. p. 106-8. Available from: https://www.mathopenref.com/trapezoidarea.html. 2019:5, 109-111.
3. Manizade AG, Mason MM. Developing the area of a trapezoid. Math Teach 2014;l107:508-14.