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RESEARCH ARTICLE

On Topological Fixed Point Iteration Methods and the Optimization Simplex Algorithm in the Flight Attendants' Hiring and Training Problem of the South African Airways

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ABSTRACT

This research aims at generating the topological fixed point iteration scheme for the simplex method of Linear Programming problems in optimization as exemplified in the optimization of the flight attendants' hiring problems of South African Airways Company displayed in the latter part of section two. Review of basic related concepts of the linear programming and flight attendant's problems were discussed in sections one and early part of section two while main results boarding on the iterative schemes we discussed in section three.

Key words: Contraction map, flight attendants, iteration map, linear programming problems, sensitivity analysis, simplex method maximization, simplex method minimization

2010 Mathematics Subject Classification: 46B25, 65K05

INTRODUCTION

The South African Airways

South African Airlines, Inc. (SA) is a major African airline headquartered in Pretoria. It is the Africa's largest airline when measured by fleet size, revenue, scheduled passengers carried, scheduled passenger—kilometers flown, and number of destinations served. South Africa, together with its regional partners, operate an extensive international and domestic network. South African Airlines is a founding member of the world alliance, the third largest airline alliance in the African Airlines and was started in about 1964 through a union of more than eighty small global airlines.[1]

The two organizations from which South African Airlines was originated were Robertson Aircraft Corporation and Colonial Air Transport. The former was first created in Missouri in 1921, with both being merged in 1929 into holding company The Aviation Corporation. This, in turn, was made in 1930 into an operating company and rebranded as South African Airways. In 1934, when new laws and attrition of mail contracts forced many airlines to reorganize, the corporation redid its routes into a connected system and was renamed South African Airlines. Between 1970 and 2000, the company grew into being an international carrier, purchasing Trans World Airlines in 2001.

Flight Attendant

Flight attendant or also known as steward/stewardess or air host/air hostess is a member of an aircrew employed by airlines aboard commercial flights, primarily to ensure the safety and comfort of passengers.

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Collectively called the cabin crew, flight attendants are deployed in the cabins of all commercial flights and additionally may also be present on some private or business jets[2] and government or military aircraft.[3]

History

The first female flight attendant was a 25-year-old registered nurse named Ellen Church.[4] Hired by Africa's Airlines in 1930,[5] she also first envisioned nurses on aircraft. Other airlines followed suit, hiring nurses to serve as flight attendants, then called "stewardesses" or "air hostesses," on most of their flights. In Africa, the job was one of only a few in the 1930s to permit women, which, coupled with the Great Depression, led to large numbers of applicants for the few positions available.

Female flight attendants rapidly replaced male ones, and by 1976, they had all but taken over the role.[6] They were selected not only for their knowledge but also for their characteristics. A 1976 Pretoria Times article described the requirements:

The Africa's Equal Employment Opportunity Commission's (EEOC) first complainants were female flight attendants complaining of age discrimination, weight requirements, and bans on marriages.[7]

In 1968, the EEOC declared age restrictions on flight attendants employment to be illegal sex discrimination under Title VII of the Civil Rights Act of 1964. Also in 1968, the EEOC ruled that sex was not a bona fide occupational requirement to be a flight attendant,[8] The restriction of hiring only women was lifted at all airlines in Beveridge and Schechter[9] due to the decisive court case of Diaz versus Pan Am.[10] By the 1980s., the no- marriage rule was eliminated throughout the Africa's airline industry.[11] The last such broad categorical discrimination, the weight restrictions[12] were relaxed in the 1990s through litigation and negotiations.[13] Airline still often have vision and height requirements and may require flight attendants to pass a medical evaluation.[14]

As there will be 41,030 new airliners by 2036, Boeing expects 839,000 new cabin crew members from 2017 till then: 221,000 in Africa (12%), 298,000 in Asia Pacific (7%), 169,000 in North America (21%), and 151,000 in Europe (19%).[15]

The role of a flight attendant is to "provide routine services and respond to emergencies to ensure the safety and comfort of airline passengers while aboard planes."[16] However, particularly in the South Africa flight attendants often state that they are there "primarily for (the passenger's) safety."[17]

Typically flight attendants require holding a high school diploma or equivalent, and in the South Africa the median annual wage for flight attendants was $50,500 in May 2017, higher than the median for all workers of $37,690.[18]

The number of flight attendants required on flights is mandated by each country's regulations. In South African, for light planes with 19 or fewer seats, or, if weighing more than 7500 pounds, nine or fewer seats, no flight attendant is needed; on larger aircraft, one flight attendant per 50 passenger seats is required.[19]

The majority of flight attendants for most airlines are female, though a substantial number of males have entered the industry since 1980.[20]

Responsibilities

Before each flight attendant attend a safety briefing with the pilots and lead flight attendant. During this briefing, they go over safety and emergency checklists the locations and amounts of emergency equipment and other features specific to that aircraft type. Boarding particulars are verified, such as special needs passengers, small children travelling as unaccompanied or VIPs. Weather conditions are discussed including anticipated turbulence. Before each flight a safety check is conducted to ensure, all equipment such as life — vests, torches (flash lights), and firefighting equipment are on board, in the right quantity, and in proper condition. Any unserviceable or missing items must be reported and rectified before take-off. They must monitor the cabin for any unusual smells or situations. They assist with the loading of carry-on baggage, checking for weight, size and dangerous goods. They make sure those sitting in emergency exit rows are willing and able to assist in an evacuation and move those who

are not willing or able out of the row into another seat. They then must do a safety demonstration or monitor passengers as they watch a safety video. They then must "secure the cabin" ensuring tray tables are stowed, are in their upright positions, armrests down and carryon stowed correctly and seat belts are fastened before take-off. All the service between boarding and takeoff is called Pre Take off Service.[21]
Once up in the air, flight attendants will usually serve drinks and/or food to passengers using an airline service trolley. When not performing customer service duties, flight attendants must periodically conduct cabin checks and listen for any unusual noises or situations. Checks must also be done on the lavatory to ensure the smoke detector has not been disabled or destroyed and to restock supplies as needed. Regular cockpit checks must be done to ensure the health and safety of the pilot(s). They must also respond to call lights dealing with special requests. During turbulence, flight attendants must ensure the cabin is secure. Before landing, all loose items, trays, and rubbish must be collected and secured along with service and galley equipment. All hot liquids must be disposed of. A final cabin check must then be completed before landing. It is vital that flight attendants remain aware as the majority of emergencies occur during takeoff and landing.[22] Upon landing, flight attendants must remain stationed at exits and monitor the airplane and cabin as passengers disembark the plane. They also assist any special needs passengers and small children off the airplane and escort children, while following the proper paperwork and ID process to escort them to the designated person picking them up.
Flight attendants are trained to deal with a wide variety of emergencies and are trained in first aid. More frequent situations may include a bleeding nose, illness, small injuries, intoxicated passengers, aggressive, and anxiety stricken passengers. Emergency training includes rejected takeoffs, emergency landings, cardiac and in-flight medical situations, smoke in the cabin, fires, depressurization, on-board births and deaths, dangerous goods and spills in the cabin, emergency evacuations, hijackings, and water landings.

REVIEW OF THE SIMPLEX METHOD USED IN THIS WORK

Review of Solution of a System of Linear Simultaneous Equation

Before studying the most general method of solving a linear programming problem, it will be useful to review the methods of solving a system of linear equations. Hence, in the present section, we review some of the elementary concepts of linear equations. Consider the following system of n equations in n unknowns.

$$\begin{aligned} &a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1(E_1) \\ &a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2(E_2) \\ &a_{31}x_1 + a_{32}x_2 + \ldots + a_{3n}x_n = b_3(E_3) \\ &\vdots \\ &a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n(E_n) \end{aligned} \tag{1}$$

Assuming that the set of equations possesses a unique solution, a method of solving the system consists of reducing the equations to a form known as canonical form.
It is well known from elementary algebra that the solutions Eqs. (1) will not be altered under the following elementary operations:
(1) Any equations E_r is replaced by the equations KE_r, where k is a non-zero-constant, and
(2) Any equation E_r is replaced by the equation E_r+kE_s, where E_s is any other equation of the system. By making use of these elementary operations. The system of Eqs. (1) can be reduced to a convenient equivalent form as follows. Let us select some variable x_1 and try to eliminate it from all the equations except the j^{th} one (for which a_{ji} is non zero). This can be accomplished by dividing the j^{th} by a_{ji} and subtracting a_{ki} times the result from each of the other equations, k=1, 2,….., j−1+j+1,…, n. The resulting system of equations can be written as.[23]

$$
\begin{aligned}
&a'_{11}x_1 + a'_{12}x_2 + \cdots + a'_{1i-1}x_{i-1} + 0x_i + a'_{1i+1}x_{i+1} + \cdots + a'_{1n}x_n = b'_1 \\
&a'_{21}x_1 + a'_{22}x_2 + \cdots + a'_{2i-1}x_{i-1} + 0x_i + a'_{2i+1}x_{i+1} + \cdots + a'_{2n}x_n = b'_2 \\
&\vdots \\
&a'_{j-1}x_1 + a'_{j-1}x_2 + \cdots + a'_{j-1,i-1}x_{i-1} + 0x_i + a'_{j-1,i+1}x_{i+1} + \cdots + a'_{j-1,n}x_n = b'_{j-1} \\
&a'_jx_1 + a'_jx_2 + \cdots + a'_{j,i-1}x_{i-1} + 1x_i + a'_{j,i+1}x_{i+1} + \cdots + a'_{j,n}x_n = b'_j \\
&a'_{j+1}x_1 + a'_{j+1}x_2 + \cdots + a'_{j+1,i-1}x_{i-1} + 0x_i + a'_{j+1,i+1}x_{i+1} + \cdots + a'_{j+1.n}x_n = b'_{j+1} \\
&a'_{n1}x_1 + a'_{n2}x_2 + \cdots + a'_{n,i-1}x_{i-1} + 0x_i + a'_{n,i+1}x_{i+1} + \cdots + a'_{nn}x_n = b'_n
\end{aligned} \tag{2}
$$

Where the primes indicate that the a'_{ij} and b'_j are changed from the original system. This procedure of eliminating a particular variable from all but one equation is called a pivot operation. The system of (2) produced by the pivot operation have exactly the same solution as the original set of (1). That is, the vector X that satisfies (1) satisfies (2) and vice versa.

Next time, if we take the system of (2) and perform a new pivot operating by eliminating x_s, $s \neq i$, in all the equations except the t^{th} equation, $t \neq j$, the zeros or the 1 in the ith column will not be disturbed. The pivotal operations can be repeated using a different variable and equation each time until the system of (1) is reduced to the form[24]

$$
\begin{aligned}
&1x_1 + 0x_2 + 0x_3 + \ldots + 0x_n = b_1^n \\
&0x_1 + 1x_2 + 0x_3 + \ldots + 0x_n = b_2^n \\
&0x_1 + 0x_2 + 1x_3 + \ldots 0x_n = b_3^n \\
&\vdots \\
&0x_1 + 0x_2 + 0x_3 + \ldots + 1x_n = b_n^n
\end{aligned} \tag{3}
$$

This system of (3) is said to be in conical form and has been obtained after carrying out n pivot operations. From the canonical form, the solution vector can be directly obtained as.[25]

$$x_i = b_i^n, \qquad i = 1, 2\ldots \tag{4}$$

Since the set of (3) has been obtained from (1) only through elementary operations, the system of (3) is equivalent to the system of (1). Thus, the solution given by (4) is desired solution of (1).

Pivotal Reduction of a General System of Equations

Instead of a square system, let us consider a system of m equations in n variables with $n \geq m$. This system of equations is assumed to be consistent[26] so that it will have at least one solution.

$$
\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\
&a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\
&\vdots \\
&a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m
\end{aligned} \tag{5}
$$

The solution vector(s) X that satisfy (5) are not evident from the equations. However, it is possible to reduce this system to an equivalent canonical system from which at least one solution can readily be deduced. If pivotal operations with respect to any set of m variables, say x_1, x_2,…, x_m, are carried, the resulting set of equations[27] can be written as follows:

canonical system with pivotal variables $x_{1,}x_{2,}\ldots,x_m$

$$
\begin{array}{l}
1x_1 + 0x_2 + \ldots + 0x_m + a''_{1,m+1}x_{m+1} + \ldots + a''_{1n}x_n = b''_1 \\
0x_1 + 1x_2 + \ldots + 0x_m + a''_{2,m+1}x_{m+1} + \ldots + a''_{2n}x_n = b''_2 \\
\vdots \\
0x_1 + 0x_2 + \ldots 1x_m + a''_{m,m+1}x_{i+1} + \ldots + a''_{mn}x_n = b''_m
\end{array}
\tag{6}
$$

Pivotal variables	Non-pivotalor dependent variables	Constants

One special solution that can always be deducted from the system of (6) is[28]

$$
\begin{cases} b''_i, & i = 1,2,\ldots,m \\ 0, & i = m+1, m+2,\ldots n \end{cases}
\tag{7}
$$

This solution is called a basic solution since the solution vector contains no more than m nonzero terms. The pivotal variables x_i, $i = 1, 2,\ldots\ldots, m$ are called the basic variables and the other variables x_i, $i = m+1$, $m+2,\ldots\ldots, n$ are called non basic variables. Of course, this is not the only solution, but it is the one most readily deduced from (6). if all b''_i, $i=1,2\ldots,m$, in the solution given (7) are non- negative, it satisfies (3)

in addition to (2),and hence it can be called a basic feasible solution.

It is possible to obtain the other basic solutions from the canonical system of (6). We can perform an additional pivotal operation on the system after it is in canonical form, by choosing a''_{pq} (which is nonzero) as the pivot term, $q \geq m$, and using any row p among (1, 2,…, m). The new system will still be in canonical form but with x_q as the pivotal variable in place of x_p. The variable x_p, which was a basic variable in the original canonical form, will no longer be a basic variable in the new canonical form. This new canonical system yields a new basic solution (which may or may not be feasible) similar to that of (7). It is to be noted that the values of all the basic variables change, in general, as we go from one basic solution to another, but only one zero variable (which is non-basic in the original canonical form) becomes nonzero (which is basic in the new canonical system), and vice versa.[29]

Motivation of the Simplex Method

Given a system in canonical form corresponding to a basic solution, we have seen how to move a neighboring basic solution by a pivot operation. Thus one way to find the basic solutions and pick the one that is feasible and corresponds to the optimal value of the objective function. This can be done because the optimal solution, if one exists, always occurs at an extreme point or vertex of the feasible domain. If there are m equality constraints in n variables with $n \geq m$, a basic solution can be obtained by setting any of the $n - m$ variables equal to zero. The number of basic solutions to be inspected is thus equal to the number of ways in which m variables can be selected from a set of n variables, that is,

$$\binom{n}{m} = \frac{n!}{(n-m)!m!}$$

For example, if $n = 10$ and m=5, we have 252 basic solutions, and if $n = 20$ and m=10, we have 184,756 basic solutions. Usually, we do not have to inspect all these basic solutions since many of them will be infeasible. However, for large values of n and m, this is still a very large number to inspect one by one. Hence, what we really need is a computational scheme that examines a sequence of basic feasible solutions, each of which corresponds to a lower value of f until a minimum is reached. The simplex method of Dantzig is a powerful scheme for obtaining a basic feasible solution; if the solution is not

optimal, the method provides for finding a neighboring basic feasible solution that has a lower or equal value of *f*. The process is repeated until, in a finite number of steps, an optimum is found.
The first step involved in the simplex method is to construct an auxiliary problem by introducing certain variables known as artificial variables into the standard form of the linear programming problem. The primary aim of adding the artificial variables is to bring the resulting auxiliary problem into a canonical form from which the basic feasible solution can be obtained immediately. Starting from the canonical form, the optimal solution of the original linear programming problem is to sought in two phases. The first phase is intended to find a basic feasible solution to the original linear programming problem. It consists of a sequence of PIVOT operations that produce a succession of different canonical forms from which the optimal solution of the auxiliary problem can be found. This also enables us to find a basic feasible solution, if one exists, of the original linear programming problem. The second phase is intended to find the optimal solution of the original linear programming problem; it consists of a second sequence of pivot operations that enables us to move from one basic feasible solution to the next of the original linear programming problem. In this process, the optimal solution of the problem, if one exists, will be identified. The sequence of different canonical forms that is necessary in both the phases of the simplex method is generated according to the simplex algorithm described in the next section. That is,[30] the simplex algorithm forms the main subroutine of the simplex method.

Simplex Algorithm

The starting point of the simplex algorithm is always a set of equations, which includes the objective function along with the equality constraints of the problem in canonical form. Thus, the objective of the simplex algorithm is to find the vector $X \geq 0$ that minimizes the function $f(X)$ and satisfies the equation:

$$\begin{aligned} &1x_1 + 0x_2 + \ldots + 0x_m + a''_{1,m+1}x_{m+1} + \ldots + a''_{1n}x_n = b''_1 \\ &0x_1 + 1x_2 + \ldots + 0x_m + a''_{2,m+1}x_{m+1} + \ldots + a''x_n = b''_2 \\ &\vdots \\ &0x_1 + 0x_2 + \ldots + 1x_m + a''_{m,m+1}x_{m+1} + \ldots + a''_{mn}x_n = b''_m \\ &0x_1 + 0x_2 + \ldots + 0x_m - f + c''_{m+1}x_{m+1} + \cdots + c''_{mn}x_n = -f''_0 \end{aligned} \tag{8}$$

Where a''_{ij}, c''_j, b''_i and f''_0 are constants. Notice that $(-f)$ is treated as a basic variable in the canonical form of (8). The basic solution which can readily be deduced from (8) is

$$\begin{aligned} &x_1 = b''_i, \quad i = 1, 2, \ldots, m \\ &f = f^n_0 \\ &x_i = 0, \quad i = m+1, m+2, \ldots, n \end{aligned} \tag{9}$$

If the basic solution is also feasible, the values of x_i, $i = 1, 2, \ldots\ldots, n$, are non-negative and hence

$$b''_i \geq 0, \ i = 1, 2, \ldots, m \tag{10}$$

In Phase I of the simplex method, the basic solution corresponding to the canonical form obtained after the introduction of the artificial variables will be feasible for the auxiliary problem. As stated earlier, Phase II of these simplex methods starts with a basic feasible solution of the original linear programming problem. Hence, the initial canonical form at the start of the simplex algorithm will always be a basic feasible solution.
We know that[22] the optimal solution of linear programming problem lies at one of the basic feasible solutions. Since the simplex algorithm is intended to move from one basic feasible solution to the other through pivotal operations, before moving to the next basic feasible solution is not the optimal solution. By merely glancing at the numbers.

$$c_j'',\ j = 1, 2, \ldots, n \tag{11}$$

We can tell whether or not the present basic feasible solution is optimal. Theorem 1 provides a means of identifying the optimal point.

Identifying an Optimal Point

Theorem 1[31]: A basic feasible solution is an optimal solution with a minimum objective function f_0'' if all the cost coefficients $c_j'',\ j = m+1, m+2, \ldots, n$ in (8) are nonnegative.
Proof: From the last row of Eqs (2.8), we can write that

$$f_0'' + \sum_{i=m+1}^{n} c_i'' x_i = f \tag{12}$$

Since the variables $x_{m+1}, x_{m+2}, \ldots, x_n$ are presently zero and are constrained to be nonnegative, the only way one of any of them can change is to become positive. But if $c_j'' > 0$ for $i = m+1, m+2, \ldots, n$, then increasing any x_i cannot decrease the value of the objective function f. Since no change in the non-basic variables can cause f to decrease, the present solution must be optimal with the optimal value of f equal to f_0''.

A glance over c_i'' can also tell us if there are multiple optima. Let all $c_i'' > 0$, $i=m+1, m+2, \ldots, k-1, k+1, \ldots 1n$, and let $c_k'' = 0$ for some non-basic variable x_k. Then, if the constraints allow that variable to be made positive (from its present value of zero), no change in f results, and there are multiple optima. It is possible, however, that the variable may not be allowed by the constraints to become positive; this may occur in the case of degenerate solutions. Thus, as a corollary to the discussion above, we can[16] state that a basic feasible solution is the unique optimal feasible solution $c_i'' > 0$ for all non-basic variables x_i, $j = m+1, m+2, \ldots, n$. If, after testing for optimality, the current basic feasible solution is found to be non-optimal, an improved basic solution is obtained from the present canonical form as follows.

Improving a Non-optimal Basic Feasible Solution

From the last row of (8), we can write the objective function as[31]

$$f = f_0'' + \sum_{i=1}^{m} c_i'' x_i + \sum_{j=m+1}^{n} c_j'' x_j = f_0'' \tag{13}$$

for the solution given by (9)
If at least one c_j'' is negative, the value of f can be reduced by making the corresponding $x_j \geq 0$. In other words, the non-basic variable x_j, for which the cost coefficient c_j'' is negative, is to made a basic variable to reduce the value of the objective function. At the same time, due to the pivotal operation, one of the current basic variables will become non-basic and hence the values of the new basic variables are to be adjusted to bring the value of (Tex translation failed). If there are more than one $c_j'' < 0$, the index s of the non-basic variable x_s which is to made basic is chosen such that

$$c_s'' = \text{minimum } c_j'' < 0 \tag{14}$$

The chance of r in the case of a tie, assuming that all $b_i'' > 0$, is arbitrary by any b_i'' for which $a_i'' > 0$ is zero in (11), x_s cannot be increased by any amount. Such a solution is called a degenerate solution.
In the case of a non-degenerate basic feasible solution, a new basic feasible solution can be constructed with a lower value of the objective function as follows. By substituting the value of x_s^* given by (14) into (12) and (13), we obtain

$$x_s = x_s^*$$

$$x_i = b_i'' - a_{is}'' x_s^* \geq 0, \quad i = 1, 2, \ldots, m \text{ and } i \neq r \tag{15}$$

$$x_r = 0$$

$$x_j = 0, \quad j = m+1, m+2, \ldots, n \quad \text{and} \quad j \neq s$$

$$f = f_0'' + c_s'' x_s^* \leq f_0'' \tag{16}$$

which can readily be seen to be feasible solution different from the previous one. Since $a_{rs}'' > 0$ in (14), a single pivot operation on the element a_{rs}'' in the system of (16) will lead to a new canonical form from which the basic feasible solution of (15) can easily be deduced. Furthermore, (16) shows that this basic feasible solution corresponds to a lower objective function value compared to that of (10). This basic feasible solution can again be tested for optimality by seeing whether all $c_i'' > 0$ in the new canonical form. If the solution is not optimal, the entire procedure of moving to another basic feasible solution from the present one has to be repeated. In the simplex algorithm, this procedure is repeated in an iteration manner until the algorithm finds either (1) a class of feasible solutions for which $f \rightarrow -\alpha$ or (2) an optimal basic feasible solutions with all $c_i'' \geq 0, i = 1, 2, \ldots, n$. Since there are only a finite number of ways to choose a set of m basic variables out of n variables, the iteration process of the simplex algorithm will terminate in a finite number of cycles.

Two Phases of the Simplex Method

The problem is to find nonnegative values for the variables $x_1, x_2, \ldots, x_n$ that satisfy[32] the equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= b_m \end{aligned} \tag{17}$$

and minimize the objective function given by

$$c_1x_1 + c_2x_2 + \ldots + c_nx_n = f \tag{18}$$

The general problems encountered in solving this problem are:

1. An initial canonical form may not be readily available. This is the case when the linear programming problem does not have slack variables for some of the equations or when the slack variables have negative coefficients.
2. The problem may have redundancies and/or inconsistencies, and may not be solvable in nonnegative numbers.

The two-phase simplex method can be used to solve the problem.

Phase I of the simplex method uses the simplex algorithm itself to find whether the linear programming problem has a feasible solution. If a feasible solution exists, it provides a basic feasible solution in canonical form ready to initiate phase II of the method. Phase II, in turn, uses the simplex algorithm to find whether the problem has a bounded optimum. If a bounded optimum exists, it finds the basic feasible solution which is optimal. The simplex method[32] is described in the following steps.

1. Arrange the original system of (17) so that all constant terms b_i are positive or zero by changing, where necessary, the signs on both sides of any of the equations.
2. Introduce to this system a set of artificial variables $y_1, y_2, \ldots, y_m$ (which serve as basic variables in Phase I, where each $y_i \geq 0$, so that it becomes

$$\begin{aligned} &a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + y_1 = b_1 \\ &a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + y_2 = b_2 \\ &\vdots \\ &a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n + y_m = b_m \\ &\qquad b_i \geqslant 0 \end{aligned} \tag{19}$$

Note that in (21), for a particular i the $a_{ij}'s$ and the b_i of the negative of what they were in (20) because of step 1.
The objective function of (19) can be written as

$$c_1x_1 + c_2x_2 + \ldots + c_nx_n + (-f) = 0 \tag{20}$$

1. Phase 1 of the Method. Define a quantity w as the sum of the arbitrary variables

$$w = y_1 + y_2 + \ldots + y_m \tag{21}$$

and uses the simplex algorithm to find $x_i \geq 0$ (i=1, 2,……, n) and ≥0 (i=1, 2,……, m) which minimizes w and satisfy (22) and (21). Consequently, consider the array.

$$\begin{aligned} &a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + y_1 = b_1 \\ &a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + y_2 = b_2 \\ &\vdots \\ &a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n + y_m = b_m \\ &c_1x_1 + c_2x_2 + \ldots + c_nx_n + (-f) = 0 \\ &y_1 + y_2 + \ldots + y_m + (-w) = 0 \end{aligned} \tag{22}$$

This array is not in canonical form; however, it can be rewritten as a canonical system with basic variables $y_1, y_2, \ldots, y_m$, $-f$, and $-w$ by subtracting the sum of the first m equations from the last to obtain the new system

$$\begin{aligned} &a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + y_1 = b_1 \\ &a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + y_2 = b_2 \\ &\vdots \\ &a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n + y_m = b_m \\ &c_1x_1 + c_2x_2 + \ldots + c_nx_n + (-f) = 0 \\ &d_1x_1 + d_2x_2 + \ldots + d_nx_n + (-w) = -w_0 \end{aligned} \tag{23}$$

where

$$d_i = (a_{1i+} a_{2i+} \ldots + a_{mi}),\ i = 1,2,\ldots,n \tag{24}$$

$$-w_0 = -(b_1+b_2+\ldots+b_m) \tag{25}$$

Equations (24) provide the initial basic feasible solution that is necessary for starting phase

1. w is called the infeasibility form and has the property that if as a result of phase with a minimum Phase I, with a minimum of $w > 0$, no feasible solution exists for the original linear programming problem stated in (17) and (18), and thus the procedure is terminated. On the other hand, if the minimum of $w = 0$, the resulting array will be in canonical form and hence initiate Phase II by eliminating the w equation as well as the columns corresponding to each of the artificial variables $y_1, y_2, \ldots, y_m$ from the array.
2. Phase II of the method. Apply the simplex algorithm to the adjusted canonical system at the end of Phase I to obtain a solution, if a finite one exists, which optimizes the value of f

3 Main results on topological analysis of the associated fixed point iteration methods

In this iteration method which is a revision of the row operation method of the Tableau format used in section three below, the computation methods here are based on matrix algebra principles. Hence, the general linear programming problem becomes minimize or maximize

$$Z = \sum_{j=1}^{n} c_j x_j$$

subject to

$$\sum_{j=1}^{n} \bar{P}_j x_j = \bar{b} x_j \geq 0 \; j = 1, 2, \ldots, n \ldots \tag{26}$$

for any given basic vector $\bar{X}_j$ so that its corresponding basic $\bar{B}$ and objective vector $\bar{C}_j$ and the simplex iterative method becomes

$$z + \sum_{j=1}^{n} \left(\mathrm{z_j} - \mathrm{c_j}\right) x_{\mathrm{j}} = \bar{C}_B B^{-1} b$$

$$\left(\bar{x}\right) + \sum_{j=1}^{n} \left(\bar{B}^{-1} \bar{p} j\right) x = \left(\bar{B}^{-1} \bar{b}\right) \tag{27}$$

where

$$z - c_j = \bar{C}_B \bar{B}^{-1} \bar{p}_j - z_j \tag{28}$$

$(\bar{V}_j)$ Represent the i^{th} element of the vector $\bar{V}$.

To employ the above iterative method we guarantee ourselves of the following:

a) That the domain of existence of the above simplex method is the metric space (x, p)
b) That the solution of than simplex method converges in the metric space.
c) That the simplex method an initial value problem (27) solvable by (2.8) in the complete metric space is continuous.
d) That the simplex iterative method (28) satisfies the contraction mapping principle.
e) That simplex iterative method is exactly a reformulated Banach fixed point method for solving system of linear equations. The above facts give rise to the following theorem.

Theorem 2. Let (X, p) be a complete matric space in $\mathbb{R}^+$ and T:X→X be a contraction mapping, that is, the contraction factor $k<1$.

Then, there exists uniquely $\bar{x} \in x$ such that $T\bar{x}^* = \bar{x}^*$ and the sequence $\{x_n\}$ of successive approximations generated by

$$x_{j+1} = Tx_j = z + \sum_{j=1}^{n} \left(z_j - c_j\right) x_j = \bar{C}_B B^{-1}$$

$$\bar{x}_B + \underset{j}{=} 1 \sum^{n} \left(\bar{B}^{-1} \bar{P}_j\right) x_j = \bar{B}^{-1} \bar{b}$$

where

$$z - c_j = \bar{C}_B B^{-1} \bar{p}_j - z_j$$

Converges strongly to x^*
where

$$x_j + Z - C_j = x_j + \bar{C}_B \bar{B}^{-1} \bar{P}_j - Z_j \tag{29}$$

and the contraction factor k satisfies $0 \leq K \leq 1$ so that the simplex problem has a solution from the above iteration method with the following algorithm.

1. Choose a basic and non-basic partition (B, N) such that
2. $y^k := B^{-T} c_B$
3. If

$\exists j^k \in \mathbb{N}; s^k_{jk} = c_j k - A^T_{jk} y^k > 0$

then continue else exit because x^k is an optimal solution

4. Let

$$\begin{bmatrix} dx_B \\ dx_N \end{bmatrix} = \begin{bmatrix} -B^{-1} N c_{jk} \\ c_{jk} \end{bmatrix}$$

5. If $dx_B \geq 0$, then terminate because (P) is unbounded
6. Let $\alpha^k := \min\left(-x^k_{B_i} / dx_{B_i} : dx_B < 0\right)$ and choose $i^k \in \left\{i : dx_B < 0, \alpha^k = -x^k_B / dx_B\right\}$
7. $x^{k-1} = x^k - \alpha^k dx$
8. $B := \left(B \setminus \left\{B_i k\right\} \cup \left\{j^k\right\}, N := \left(N \setminus \left\{j^k\right\} \cup B_{i^k}\right)\right)$
9. $k=k+1$
10. Go to 2

Proof: If x^* is the unique fixed point, $x^* = x_0 = T(x_0)$ by the contraction principle.
But let $x_1 = T(x_0)$, then

$$\begin{aligned} x_2 &= T(x_1) = T\left(T\left(x_0\right)\right) = T^2\left(x_0\right) \\ x_3 &= T\left(x_2\right) = T^2\left(T\left(x_2\right)\right) = T^3\left(x_0\right) \\ &\vdots \\ x_n &= T^{n-1}\left(T(x_0\right) = T^n\left(x_0\right) \end{aligned} \tag{30}$$

Hence, we have constructed a sequence $\{x_n\}_{n=0}$ of linear operators for that linear programming simplex matrix problem defined in the metric space (X, p).

We now prove that the above generated sequence is Cauchy. First, we compute $p\ (x_n, x_{n+1}) = p\ (T\ (x_n, x_{n+1}))$ using (30) $\leq KT\ (x_{n-2}, x_{n-1}))$

$$
\begin{aligned}
&= \mathrm{KT}\left(x_{\mathrm{n}-2}, x_{\mathrm{n}-1}\right) \text{since K is a contraction} \\
&= \mathrm{K}^2\mathrm{T}\left(x_{\mathrm{n}-2}, x_{\mathrm{n}-1}\right) \\
&\vdots \\
&= \mathrm{K}^n\mathrm{T}\left(x_0, x_1\right) \\
&\text{i.e. } \mathrm{KT}\left(x_{\mathrm{n}}, x_{\mathrm{n}-1}\right) \leqslant \mathrm{K}^n\mathrm{T}\left(x_0, x_1\right)
\end{aligned}
\tag{31}
$$

We now show that x_n is Cauchy.
Let $m > n$, then

$$\rho\ \left(x_{\mathrm{n}}, x_{\mathrm{m}}\right) \leq\ \rho\ \left(x_{\mathrm{n}}, x_{\mathrm{m}}\right) +\ \rho\ \left(x_{\mathrm{n}-1,\mathrm{m}-1}\right) + \ldots +\ \rho\ \left(x_{\mathrm{n}-\mathrm{k}-1}, x_{\mathrm{m}-\mathrm{k}-1}\right)$$

$$\leq \mathrm{K}^n\mathrm{T}\left(x_0, x_1\right)(1 + \mathrm{K} + \mathrm{K}^2 + \ldots + \mathrm{K}^{n-m-1} + \mathrm{K}^n$$

Since the series on the right hand side is a geometric progression with common ratio <1, its sum to infinity is $\frac{1}{1-k}$. Hence, we have from above that

$$\rho\ \left(x_{\mathrm{n}}, x_{\mathrm{m}}\right) \leq \mathrm{K}^n\mathrm{T}\left(x_0, x_1\right)\left(\frac{1}{1-k}\right) \to 0 \text{ as n} \to \infty$$

Since $k<1$. Hence, the sequence $\{x_n\}$ is Cauchy in (X, p) since X is complete and $\{x_n\}$ converges to point in X.
Let

$$x_n \to x^* \text{ as } \ n \to \infty \tag{32}$$

Since T is a contraction and continuous, it follows from (32) that $\mathrm{T}\ (x_n) \to \mathrm{T}\ (x^*)$ as $n \to \alpha$.
But $\mathrm{T}\ (x_{\mathrm{n}}) = x_{\mathrm{n}+1}$ from (31). So

$$x_{n+1} = \mathrm{T}\left(x_{\mathrm{n}}\right) = \mathrm{T}\left(x^*\right) \tag{33}$$

But limits are unique in a metric space, so from (32) and (33), we obtain that

$$\mathrm{T}(x^*) = x^* \tag{34}$$

Hence, T has a unique fixed point in (X, p). We shall now prove that this fixed point is unique suppose for the contraction there exists $\mathrm{y}^*\ \varepsilon\ \mathrm{X}$ such that

$$y^* = x^* \text{ and } T(y^*) = y^* \tag{35}$$

Then from (34) and (35)

$$\rho\ \left(x^*, \mathrm{y}^*\right) =\ \rho\ \left(T\left(x^*\right), \mathrm{T}\left(\mathrm{y}^*\right)\right) \leq KT\left(x^*, \mathrm{y}^*\right)$$

so that

$$\left(k-1\right)T\left(x^*, \mathrm{y}^*\right) \geq 0 \text{ and } T\left(x^*, \mathrm{y}^*\right) = 0, T =\ \rho$$

We can divide by it to get $k-1\geq 0$ i.e. $k\geq 1$ which is contradiction.
Hence, $x^{*}=y^{*}$ and the fixed point is unique. Therefore,

$$z+\sum_{j=1}^{n}\left(z_j-c_j\right)x_j=\bar{C}_B B^{-1}b$$
$$\bar{x}_B+\sum_{j=1}^{n}\left(\bar{B}^{-1}\bar{P}_j\right)x_j=\bar{B}^{-1}\bar{b} \tag{36}$$

where

$$Z-C_j=x_j+\bar{C}_B\bar{B}^{-1}\bar{P}_j-Z_j$$

And V_j represent the i^{th} element of the vector $\bar{V}$ is by the Banach fixed point method, the simplex iteration formula for the linear programming problem
Minimize or Maximize

$$z=\sum_{j=1}^{n}c_j x_j \tag{37}$$

Subject to

$$\sum_{j=1}^{n}p_j x_j=\bar{b}x_j\geq 0 \quad j=1,2,\ldots n$$

for any given vector $\bar{x}_j$ with corresponding basis $\bar{B}$ and objective vector C_j. It is worthy of note that the Banach fixed point method (36) satisfying the condition $K<1$.
Theorem 3. The necessary and sufficient condition for the linear programming problem (37) to have a unique fixed point is that in the matrix of linear Transformation

$$A=\sum_{i=1}^{n}\begin{bmatrix} z-\sum_{j=1}^{n}c_j \\ \sum_{j=1}^{n}P_{j-}\bar{b}_j\geq 0\end{bmatrix}$$

For any given vector x_j with corresponding basis $\bar{B}$ and objective vector C_j, the original matrix A is diagonal dominant and that $A_\alpha=\max\{|\alpha_{ij}|, 1\leq i,j\leq n\}<1$ in this case, the Banach method called the Picard's method becomes satisfied for use in solving the said problem.

Convergence Analysis

Given the general Linear Programming Problem Minimize or maximize

$$z=\sum_{j=1}^{n}c_j x_j \text{ subject to } \sum_{j=1}^{n}P_j x_j=\bar{b}_j x_j\geq 0 \; j=1,2,3,\ldots,n$$

And for a given basis vector $\bar{X}_B$ and its corresponding basis $\bar{B}_j$ and objective vector $\bar{C}_B$, the general simplex iteration formula given by

$$z+\sum z_j-c_j x_j=\bar{C}_B\bar{B}^{-1}\bar{b}$$
$$\bar{X}_B+\sum B^{-1}P_j x_j=\left(\bar{B}^{-1}\bar{b}\right)$$

where

$$Z_j - c_j = \bar{C}_B \bar{B}^{-1} \bar{p}_j - C_j$$

$(\bar{V}_j)$ Represent the j^{th} element of the vector V; then the Linear Programming problem above is convergent to

$$x_j = \min\left\{ \frac{\bar{B}^{-1}\bar{b}}{\bar{B}^{-1}P_j} B^{-1}P_j > 0 \right\}$$

and the basic variable responsible for minimum ratio leaves the basic solution to become non basic at zero level provided $\left(\bar{B}^{-1}\bar{b}\right) - (\bar{B}^{-1}\bar{p}_j)x_j \geq 0, \forall\, j$.

This condition became realized when from the Z-equation above, an increase in non-basic x_j in the current zero value resulted in an improvement in the value of the Z relative to the current value $\bar{C}_B \bar{B} \bar{b}$ provided $Z_j - C_j$ is strictly negative in the case of maximization and strictly positive in the case of minimization otherwise, X_j cannot improve the solution and must remain non basic at zero level. This condition in optimization is referred to as the optimality and feasibility condition.

4. The simplex method applied in the hiring and training problem of a selected airline company in the South African airways.

Problem Statement

Suppose an South African airline hire s and trains flight attendants over the next 1 year and the requirement expressed as a number of flight attendant flight hours are in 8000 in January, 9000 in February, 8000 in march, 10,000 in April, 9000 in May, and 12,000 in June, also, again 8000 in July, 9000 in August, 8000 in September, 10,000 in October, 9000 in November, and 12,000 in December.

However, the hiring and training must take at least 1 month training before a flight attendant can put on a regular flight. Hence, a trainee must be hired at least 1 month before she is actually needed.

Again a trainee requires 100 in flight experience during the month of training. Hence, for each trainee, 100 less hours are available for flight service by regular flight attendants.

Each experienced flight attendant can work up to 50 h a month and for a given passenger airline company, 60 regular flight attendants available at the beginning of January. If the maximum time available for an experienced flight attendant exceeds a month flying and training requirement, the regular flight attendant work fewer than 150 h, non-laid off.

Each month, approximately 10% of the experienced flight attendants quit their jobs to get married or for other reasons. An experienced flight attendant costs the airline \$100,000, a trainee \$50,000 a month in salary and other benefits.

Model Formulation

Let x_i(i=1,2,…,12) be the number of trainees at the beginning of each month, that is,

$x_{1=}$number of trainees at the beginning of January

$x_{2=}$number of trainees at the beginning of February

⋮

$x_{2=}$number of trainees at the beginning of December

To make financial values not too large we divide \$10,000 and \$50,000 (being the amount paid a regular and trainee respectively) by 103 to obtain 10 and 5 for each.

For the objectives function, we have (60×10)+5x_1 for the month of January since we started with 60 regular flight attendants. Again as it happens that at the end of each month of the regular flight attendants may quit such that 0.9 is left to continue and we have the following.

$February: 0.9\left(60+x_1\right)\times 10+5x_2$

$March: 0.9\left[0.9\left(60+x_1\right)+x_2\right]\times 10+5x_3$

$April: 0.9\left\{0.9\left[0.9\left(60+x_1\right)+x_2\right]+x_3\right\}\times 10+5x_4$

$May: 0.9\left\{0.9\left[0.9\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}\times 10+5x_5$

$June: 0.9\left\{0.9\{0.9\left[0.9[0.9\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}\times 10+5x_6$

$July: 0.9\left\{0.9\{0.9\{0.9\left[0.9[0.9\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}\times 10+5x_7$

$August: 0.9\left\{0.9\{0.9\{0.9\{0.9\left[0.9[0.9\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}+x_7\}\times 10$

$September: 0.9\left\{0.9\left\{0.9\{0.9\{0.9\{0.9\left[0.9[0.9\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\right\}$
$+x_5\}+x_6\}+x_7\}+x_8\}\times 10+5x_9+5x_8$

$October: 0.9\left\{0.9\left\{0.9\left\{0.9\{0.9\{0.9\{0.9\left[0.9[0.9\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\right\}+x_5\right\}$
$+x_6\}+x_7\}+x_8\}+x_9\}\times 10+5x_{10}$

$November: 0.9\left\{0.9\left\{0.9\left\{0.9\left\{0.9\{0.9\{0.9\{0.9\left[0.9[0.9\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\right\}+x_5\right\}+x_6\right\}$
$+x_7\}+x_8\}+x_9\}+x_{10}\}\times 10+5x_{11}$

$December: 0.9\left\{0.9\left\{0.9\left\{0.9\left\{0.9\{0.9\{0.9\{0.9\{0.9\left[0.9[0.9\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\right\}+x_5\right\}+x_6\right\}$
$+x_7\}+x_8\}+x_9\}+x_{10}\}+x_{11}\times 10+5x_{12}$

The above computation can be written in better simplified form as below: First the objective function becomes

$$60\times 10\left(0.9^0+0.9^1+0.9^2+0.9^3+0.9^4+0.9^5+0.9^6+0.9^7+0.9^8+0.9^9+0.9^{10}+0.9^{11}\right)+5x_1$$

$$+10(0.9^1+0.9^2+0.9^3+0.9^4+0.9^5+0.9^6+0.9^7+0.9^8+0.9^9+0.9^{10}+0.9^{11})x_1+5x_2$$

$$+10(0.9^1+0.9^2+0.9^3+0.9^4+0.9^5+0.9^6+0.9^7+0.9^8+0.9^9+0.9^{10})x_2+5x_3$$

$$+10\left(0.9^1+0.9^2+0.9^3+0.9^4+0.9^5+0.9^6+0.9^7+0.9^8+0.9^9\right)x_3+5x_4$$

$$+10\left(0.9^1+0.9^2+0.9^3+0.9^4+0.9^5+0.9^6+0.9^7+0.9^8\right)x_4+5x_5$$

$$+10\left(0.9^1+0.9^2+0.9^3+0.9^4+0.9^5+0.9^6+0.9^7\right)x_5+5x_6$$

$$+10\left(0.9^1+0.9^2+0.9^3+0.9^4+0.9^5+0.9^6\right)x_6+5x_7$$

$$+10\left(0.9^1+0.9^2+0.9^3+0.9^4+0.9^5\right)x_7+5x_8$$

$$+10\left(0.9^1+0.9^2+0.9^3+0.9^4\right)x_8+5x_9$$

$$+10\left(0.9^1+0.9^2+0.9^3\right)x_9+5x_{10}$$

$$+10\left(0.9^1+0.9^2\right)x_{10}+5x_{11}$$

$$+10\left(0.9^1\right)x_{11}+5x_{12}$$

Putting this together, the objective function equation becomes

$$\begin{aligned}f(x)=&\,4474.880544+66.66621192x_1+63.52810596x_2\\&+60.39x_3+55.002579511x_4+50.953279x_5+45.117031x_6+40.8559x_7\\&+35.951x_8+29.39x_9+22.1x_{10}+14x_{11}+5x_{12}\end{aligned}$$

For the constraints, from the problem statement, we have that each experienced flight attendant can work up to 150 h in a month and we have 60 experienced flight attendants available at the beginning of January. And also from the data, we know that a trainee requires 100 h of actual in-flight experience during the month of training. Furthermore, we remember that at the end of each month, 10% of experienced flight attendant quit their job. Then, the constraint is as follows.

$$\text{January}:(150\times 60)+100x_1\geqslant 8000$$

$$\text{February}:150\times 0.9\,(60+x_1)+100x_2\geqslant 9000$$

$$\text{March}:150\times 0.9\left[0.9(60+x_1)+x_2\right]+100x_3\geqslant 8000$$

$$\text{April}:150\times 0.9\left\{0.9\left[0.9(60+x_1)+x_2\right]+x_3\right\}+100x_4\geqslant 10000$$

$$\text{May}:150\times 0.9\left\{0.9\,\{0.9\left[0.9(60+x_1)+x_2\right]+x_3\right\}+x_4\}+100x_5\geqslant 9000$$

$$\text{June}:150\times 0.9\left\{0.9\left\{0.9\left[0.9\left[0.9(60+x_1)\right]+x_2\right]+x_3\right\}+x_4\}+x_5\right\}+100x_6\geqslant 12000$$

$$\text{July}:150\times 0.9\left\{0.9\left\{0.9\left\{0.9\left[0.9\left[0.9\,(60+x_1)+x_2\right]+x_3\right]+x_4\right\}+x_5\right\}+x_6\right\}+100x_7\geqslant 8000$$

$$\text{August}:150\times 0.9\{0.9\{0.9\{0.9\{0.9[0.9\left[0.9(60+x_1)+x_2\right]+x_3\}+x_4\}+x_5\}+x_6\}+x_7\}+100x_8\geqslant 9000$$

$$\text{September}:150\times 0.9\left\{0.9\left\{0.9\left\{0.9\left\{0.9\left\{0.9\,\left\{0.9\left[0.9(60+x_1)+x_2\right]+x_3\right\}x_4\right\}+x_5\right\}+x_6\right\}+x_7\right\}+x_8\right\}$$
$$+100x_9\geqslant 8000$$

$$\text{October}:150\times 0.9\left\{0.9\left\{0.9\left\{0.9\left\{0.9\left\{0.9\,\{0.9[0.9\left[0.9(60+x_1)+x_2\right]+x_3\right\}+x_4\right\}+x_5\right\}+x_6\}+x_7\right\}+x_8\right\}$$
$$+x_9\}+100x_{10}\geqslant 10000$$

$$\text{November:}150\times 0.9\left\{0.9\left\{0.9\left\{0.9\left\{0.9\left\{0.9\,\left\{0.9[0.9[0.9\left[0.9(60+x_1)+x_2\right]+x_3\right\}x_4\right\}+x_5\right\}+x_6\right\}+x_7\right\}+x_8\right\}$$
$$+x_9\}+x_{10}\}+100x_{11}\geqslant 9000$$

$$\text{December}:150\times 0.9\{0.9\left\{0.9\left\{0.9\left\{0.9\left\{0.9\,\{0.9[0.9[0.9[0.9\left[0.9(60+x_1)+x_2\right]+x_3\right\}+x_4\right\}+x_5\right\}+x_6\right\}$$
$$+x_7\}+x_8\}+x_9\}+x_{10}\}+x_{11}\}+100x_{12}\geqslant 12000$$

Hence, the constraints on simplification becomes

$100x_1 \geqslant -1000$

$135x_1 + 100x_2 \geqslant 900$

$121.5x_1 + 135x_2 + 100x_3 \geqslant 710$

$109.35x_1 + 121.5x_2 + 135x_3 + 100x_4 \geqslant 3439$

$98.415x_1 + 109.35x_2 + 121.5x_3 + 135x_4 + 100x_5 \geqslant 3439$

$88.5735x_1 + 98.415x_2 + 109.35x_3 + 121.5x_4 + 135x_5 + 100x_6 \geqslant 6685.59$

$79.7526x_1 + 88.5735x_2 + 98.415x_3 + 109.35x_4 + 121.5x_5$
$+135x_6 + 100x_7 \geqslant 3678.1833$

$49.25572335x_1 + 54.7285815x_2 + 60.809535x_3 + 75.0735x_4 + 87.915x_5$
$+121.5x_6 + 135x_7 + 100x_8 \geqslant 5110.79565$

$44.330151x_1 + 49.25572335x_2 + 54.7285815x_3 + 60.809535x_4 + 75.0735x_5$
$+87.915x_6 + 121.5x_7 + 135x_8 + 100x_9 \geqslant 4499.716085$

$39.8971359x_1 + 44.330151x_2 + 49.25572335x_3 + 54.7285815x_4 + 60.809535x_5 + 75.0735x_6$
$+87.915x_7 + 121.5x_8 + 135x_9 + 100x_{10} \geqslant 4830.391803$

$35.90742225x_1 + 39.8971359x_2 + 44.330151x_3 + 49.25572335x_4 + 54.7285815x_5 + 60.809535x_6$
$+75.0735x_7 + 87.915x_8 + 121.5x_9 + 135x_{10} + 100x_{11} \geqslant 7164.770029$

$32.31668003x_1 + 35.90742225x_2 + 39.8971359x_3 + 44.330151x_4 + 49.25572335x_5 + 54.7285815x_6$
$+60.809535x_7 + 75.0735x_8 + 87.915x_9 + 121.5x_{10} + 135x_{11} + 100x_{12} \geqslant 9448.829303$

Combining the above derived objective function multiplied by 10^3 and the constraints as they are. The developed model for the South African airlines company's flight attendants' hiring problems becomes Model below.

Model 1

Minimize

$F(X) = 4474880.544 + 66666.21192x_1 + 63528.10596x_2 + 60390x_3 + 55002.57911x_4 + 50953.277x_5$
$+45117.031x_6 + 40855.9x_7 + 35951x_8 + 29390x_9 + 22100x_{10} + 14000x_{11} + 5000x_{12}$

Subject to

$100x_1 \geqslant -1000$

$135x_1 + 100x_2 \geqslant 900$

$121.5x_1 + 135x_2 + 100x_3 \geqslant -710$

$109.35x_1 + 121.5x_2 + 135x_3 + 100x_4 \geqslant 3439$

$98.415x_1 + 109.35x_2 + 121.5x_3 + 135x_4 + 100x_5 \geqslant 3095.1$

$88.5735x_1 + 98.415x_2 + 109.35x_3 + 121.5x_4 + 135x_5 + 100x_6 \geqslant 6685.59$

$79.7526x_1 + 88.5735x_2 + 98.415x_3 + 109.35x_4 + 121.5x_5 + 135x_6 + 100x_7 \geqslant 3678.1833$

$49.25572335x_1 + 54.72815x_2 + 60.809535x_3 + 75.0735x_4 + 87.915x_5$
$+121.5x_6 + 135x_7 + 100x_8 \geqslant 5110.79565$

Table 1: Computer spread sheet for solving the flight attendants' problem

Basic	***Z***	***X*** $_1$	***X*** $_2$	***X*** $_3$	***X*** $_4$	***X*** $_5$	***X*** $_6$	***X*** $_7$	***X*** $_8$	***X*** $_9$	***X*** $_{10}$	***X*** $_{11}$	***X*** $_{12}$
Z	1	0	0	3168.5798	0	3677.948	0	5389.108	0	5749.8843	0	0	0
X_1	0	1	0	0	0	0	0	0	0	0	0	0	0
X_2	0	0	0	–74.074074	0	0	0	0	0	0	0	0	0
X_3	0	0	1	0.7407407	0	0	0	0	0	0	0	0	0
X_4	0	0	0	15	0	–74.0741	0	0	0	0	0	0	0
X_5	0	0	0	0.3	1	0.740741	0	0	0	0	0	0	0
X_6	0	0	0	2.24E–15	0	15	0	–74.074074	0	0	0	0	0
X_7	0	0	0	1.95E–17	0	0.3	1	0.7407407	0	0	0	0	0
X_8	0	0	0	–2.2515657	0	–6.85233	0	3.2386831	0	–74.074074	0	0	0
X_9	0	0	0	–3.20E–06	0	2.71E–02	0	0.4176132	1	0.7407407	0	0	0
X_{10}	0	0	0	5.18E–06	0	–0.05541	0	–0.18435	0	0.45	1	0	0
X_{11}	0	0	0	–4.18E–06	0	5.10E–02	0	0.1820219	0	–0.0437222	0	1	0
X_{12}	0	0	0	1.75E–06	0	–2.18E–02	0	–0.1325595	0	–0.164675	0	0	1
	Z	***S*** $_1$	***S*** $_2$	***S*** $_3$	***S*** $_4$	***S*** $_5$	***S*** $_6$	***S*** $_7$	***S*** $_8$	***S*** $_9$	***S*** $_{10}$	***S*** $_{11}$	***S*** $_{12}$
Z	1	94.851365	0	140.5629	0	149.94701	0	158.5757	0	135.1476	62.375	72.5	
X_1	0	0.01	0	0	0	0	0	0	0	0	0	0	
X_2	0	–0.45	1	–0.74074	0	0	0	0	0	0	0	0	
X_3	0	–0.009	0	7.41E–03	0	0	0	0	0	0	0	0	
X_4	0	7.11E–17	0	–0.3	1	–0.7407407	0	0	0	0	0	0	
X_5	0	–7.02E–19	0	–0.006	0	7.41E–03	0	0	0	0	0	0	
X_6	0	2.70E–04	0	6.12E–17	0	–0.3	1	–0.74074	0	0	0	0	
X_7	0	2.70E–06	0	–2.74E–19	0	–0.006	0	741E–03	0	0	0	0	
X_8	0	1.48E–04	0	0.045047	0	0.1158267	0	–0.41761	1	–0.74074	0	0	
X_9	0	1.76E–06	0	1.85E–19	0	5.71E–04	0	–4.82E–03	0	741E–03	0	0	
X_{10}	0	–1.09E–07	0	–2.59E–08	0	–2.43E–04	0	3.00E–04	0	–0.009	0.01	0	
X_{11}	0	2.44E–07	0	3.50E–08	0	–1.74E–04	0	–6.69E–04	0	5.64E–03	–0.0135	0.01	
X_{12}	0	–3.85E–08	0	1.01E–08	0	1.01E–04	0	1.05E–04	0	–0.00224	0.006075	–0.0135	

$$44.330151x_1 + 49.25572335x_2 + 54.72815x_3 + 60.809535x_4 + 75.0735x_5$$
$$+87.915x_6 + 121.5x_7 + 135x_8 + 100x_9 \geqslant 4499.716085$$
$$39.8971359x_1 + 44.330151x_2 + 49.25572335x_3 + 54.72815x_4 + 60.809535x_5$$
$$+75.0735x_6 + 87.915x_7 + 121.5x_8 + 135x_9 + 100x_{10} \geqslant 4830.391803$$
$$35.90742225x_1 + 39.8971359x_2 + 44.330151x_3 + 49.25572335x_4 + 54.72815x_5$$
$$+60.809535x_6 + 75.0735x_7 + 87.915x_8 + 121.5x_9 + 135x_{10} + 100x_{11} \geqslant 7164.770029$$
$$32.31668003x_1 + 35.90742225x_2 + 39.8971359x_3 + 44.330151x_4 + 49.25572335x_5$$
$$+54.72815x_6 + 60.809535x_7 + 75.0735x_8 + 87.915x_9 + 121.5x_{10} + 135x_{11} + 100x_{12} \geqslant 9448$$

We now solve the above problem using the excel simplex package to generate solutions as below Solution Scheme of Model.

THE FLIGHT ATTENDANTS PROBLEM SOLUTION MODEL ANALYSIS

The table presented below is the summary of number of regular and trainee flight attendants for each month of the year as in Table 2.

However, we will continue our summary of the number of regular and trainee flight attendants that the South African airways company will have in each month using the result of the above Table 3. In doing this, we noted the following facts according to the problem statement.

Table 2: Computer solution result for the flight attendants problem of Model 1

X_1	12	12	–5000	1	AA3	0
X_2				0		0
X_3				0		0
X_4				3168.571		1.75E–04
X_5				0		0
X_6				3787.194		–2.18E+00
X_7				0		0
X_8				6051.905		–13.2559
X_9				0		0
X_{10}				6573.259		–16.4675
X_{11}				0		0
X_{12}				0		0
S_1				–5000		100
S_2				94.85156		–3.85E–06
S_3				0		0
S_4				140.563		–1.57E–06
S_5				0		0
S_6				149.4406		1.01E–02
S_7				0		0
S_8				158.0483		1.05E–02
S_9				0		0
S_{10}				146.3326		–0.2237
S_{11}				32		0.6075
S_{12}				140		–1.35
Solution				0		1
				7140189		1773.591

i. At the end of each month approximately 10% of the regular flight attendants quit the job
ii. The cost of payment and other benefits for maintaining regular and trainee flight attendants respectively are $100,000 and $50,000 with all the above, the following Table 4 is generated as below

From the above Table 4, the number of trainees that received $50,000 for the period of 1 year is 131. Hence, we have 13 * $50,000 = $6,550,000$ as their total payment and other benefits whereas for the regular attendants in 1-year period they are 469 while the payment for each of them is $100,000 so that their total cost for the year becomes $46,900,000. Hence, the total amount spent on trainees and regular flight attendants for the period of 1 year is $6,550,000 + $46,900,000 = $53,450,000

SENSITIVITY ANALYSIS

Sensitivity analysis investigates the damage in the optimum solution resulting from making changes in parameters of the LP model [Tables 1,5-7]. It tries to find out how sensitive the optimum solution is to a small change in parameter. These changes often come from:

a. Changes in objective function coefficient.
b. Changes in the right hand side of the constraints.
c. Changes due to additional constraints or variables to the problem.

Suppose from the problem 10% of the experienced flight attendants does not quit their job at the end of each month. We then investigate what will happen to the optimum solution, whether the value of the variables will be affected and how many flight attendants the airline should hire. In view of this, we have that if no flight attendant leaves the job at the end of each month, the following objective function and constraints becomes a reform of the earlier one. For the new objective function, we have

Table 3: Optimal value for hired trainees and quitting trainees each month of the year

Decision variables for each month of the year	Optimal value for hired trainees each month of the year	Optimal value for quitting trainees each month of the year	Meaning
January (X_1)	0	0	No trainee was hired and no regular attendant quitted
February (X_2)	0	0	No trainee was hired and no regular attendant quitted
March (X_3)	0	0	No trainee was hired and no regular attendant quitted
April (X_4)	0	0	No trainee was hired and no regular attendant quitted
May (X_5)	2	0	2 trainees were hired and no regular attendant quitted
June (X_6)	0	0	13 trainees were hired and no regular attendant quitted
July (X_7)	13	0	No trainee was hired and no regular attendant quitted
August (X_8)	0	0	No trainee was hired and no regular attendant quitted
September (X_9)	16	0	16 trainees were hired and no regular attendant quitted
October (X_{10})	0	1	No trainee was hired and 1 regular attendant quitted
November (X_{11})	0	1	No trainee was hired and 1 regular attendant quitted
December (X_{12})	100	1	100 trainees were hired and 1 regular attendant quitted
Z	1773.591		Maximum amount to be spent on hiring and training of flight attendants for 1 year

Table 4: Evaluation of Trainees

Months	No. of trainees hired	No. of regulars at the beginning of the month	No. of regulars at the end of the month	10% that left at the end of the month	No. of regulars remaining which were carried to the next month
January	0	60	60	6	54
February	0	54	54	5.4	48.6≈47
March	0	48.6≈49	48.6≈49	4.86≈49	43.74≈44
April	0	43.74≈44	43.74≈44	4.374≈4	39.366≈39
May	2	39.366≈40	41.366≈41	4.1366≈4	37.2294≈37
June	0	37.2294≈38	37.2294≈37	3.72294≈4	33.50646≈34
July	13	33.50646≈34	46.50646≈47	4.651≈4	41.855814≈42
August	0	41.855814≈42	41.855814≈42	4.1855814≈4	37.6702326≈38
September	16	37.6702326≈38	53.6782326≈54	5.3678≈5	48.31041034≈48
October	0	48.31041≈48	48.31041≈48	4.831≈5	43.4794≈43
November	0	43.4794≈44	43.4794≈43	4.3479≈4	39.1315≈39
December	100	39.1315≈40	139.1315≈139	1.391315≈1	137.740≈138
Total	131		468.59≈469		

$$January: 60\times 10+5x_1$$

$$February: \left(60+x_1\right)\times 10+5x_2$$

$$March: \left[\left(60+x_1\right)+x_2\right]\times 10+5x_3$$

$$April: \left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}\times 10+5x_4$$

$$May: \left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}\times 10+5x_5$$

Table 5: Sensitivity analysis spread sheet for Model 2

Basic	Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
Z	1	0	0	3.33333333	0	10	20	16.6666667	0	3.33333333	0	4.1666667	0
X_1	0	1	0	0	0	0	0	0	0	0	0	0	0
S_2	0	0	0	–66.666667	0	0	0	0	0	0	0	0	0
X_2	0	0	1	0.66666667	0	0	0	0	0	0	0	0	0
S_4	0	0	0	16.6666667	0	–100	–100	–66.666667	0	0	0	0	0
S_5	0	0	0	5.27E–14	0	–50	–150	–100	0	0	0	0	0
S_6	0	0	0	5.27E–14	0	0	–50	–100	0	0	0	0	0
X_4	0	0	0	0.33333333	1	1	1	0.66666667	0	0	0	0	0
S_8	0	0	0	1.76E–14	0	0	0	16.6666667	0	–66.666667	0	0	0
X_8	0	0	0	3.52E–16	0	0	0	0.33333333	1	0.6666667	0	0	0
S_{10}	0	0	0	–1.67E–29	0	0	0	1.76E–14	0	16.666667	0	–66.666667	0
X_{10}	0	0	0	–3.35E–31	0	0	0	3.52E–16	0	0.3333333	1	0.6666667	0
X_{12}	0	0	0	2.63E–48	0	0	0	–5.02E–31	0	5.27E–16	0	0.5	1

Basic	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	Solution
Z	1.00E–01	0	0.133333	0	0	0	0.266667	0	0.133333	0	0.116667	0.05	6716.6667
X_1	0.01	0	0	0	0	0	0	0	0	0	0	0	–10
S_2	–0.5	1	–0.66667	0	0	0	0	0	0	0	0	0	1166.66667
X_2	–0.01	0	6.67E–03	0	0	0	0	0	0	0	0	0	3.333333
S_4	–1.04E–17	0	–0.33333	1		0	–0.66667	0	0	0	0	0	1.70E–13
S_5	–4.52E–32	0	4.54E–16	0	1	0	–1	0	0	0	0	0	1000
S_6	–4.52E–32	0	4.54E–16	0	0	1	–1	0	0	0	0	0	4000
X_4	2.08E–19	0	–6.67E–03	0	0	0	6.67E–03	0	0	0	0	0	3.40E–15
S_8	–1.51E–32	0	1.51E–16	0	0	0	–0.33333	1	–0.66667	0	0	0	1000
X_8	–3.01E–34	0	3.03E–18	0	0	0	–6.67E–03	0	6.67E–03	0	0	0	3.42E–30
S_{10}	–1.77E–48	0	–3.60E–33	0	0	0	1.51E–16	0	–0.333333	1	–0.66667	0	1333.33333
X_{10}	–3.54E–50	0	–7.19E–35	0	0	0	3.03E–18	0	–6.67E–03	0	6.67E–03	0	6.6666667
X_{12}	4.86E–65	0	4.98E–50	0	0	0	–1.08E–34	0	4.54E–18	0	–0.01	0.01	30

Table 6: Computer result for the sensitivity analysis model 2

X_1	12	12	–5	1	AA3	0
X_2				0		0
X_3				0		0
X_4				3.333333		2.63E–46
X_5				0		0
X_6				10		0
X_7				20		0
X_8				16.66667		–5.02E–29
X_9				0		0
X_{10}				3.333333		5.27E–14
X_{11}				0		0
X_{12}				1.666667		50
S_1				–5		100
S_2				1.00E–01		4.86E–63
S_3				0		0
S_4				0.133333		4.98E–48
S_5				0		0
S_6				0		0
S_7				0		0
S_8				0.266667		–1.08E–32
S_9				0		0
S_{10}				0.133333		4.54E—16
S_{11}				0		0
S_{12}				0.166667		–1
Solution				0		1
				6566.667		3000

Table 7: Sensitivity analysis summary

Month	No. of trainees hired	No. of regular at the beginning of the month	No. of regulars at the end of the month	0% of those that left at the end of the month	No. of regular remaining which will be carried over to the next month
January	0	60	60	0	60
February	0	60	60	0	60
March	0	60	60	0	60
April	0	60	60	0	60
May	0	60	60	0	60
June	0	60	60	0	60
July	0	60	60	0	60
August	0	60	60	0	60
September	0	60	60	0	60
October	0	60	60	0	60
November	50	60	110	0	110
December	100	110	210	0	210
Total	150	770	920		920

$$June:\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}\times 10+5x_6$$

$$July:\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}\times 10+5x_7$$

$$August:\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}+x_7\}\times 10+5x_8$$

$$September: \left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}+x_7\}+x_8\}\times 10+5x_9$$

$$October: \left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}+x_7\}+x_8\}+x_9\}\times 10+5x_{10}$$

$$November: \left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}+x_7\}+x_8\}+x_9\}+x_{10}\}\times 10+5x_{11}$$

$$December: \left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}+x_7\}+x_8\}+x_9\}+x_{10}\}+x_{11}\}\times 10+5x_{12}.$$

Putting this together, we have

$$60\times 10+5x_1$$

$$\left(60+x_1\right)\times 10+5x_2$$

$$\left[\left(60+x_1\right)+x_2\right]\times 10+5x_3$$

$$\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}\times 10+5x_4$$

$$\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}\times 10+5x_5$$

$$\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}\times 10+5x_6$$

$$\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}\times 10+5x_7$$

$$\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}+x_7\}\times 10+5x_8$$

$$\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}+x_7\}+x_8\}\times 10+5x_9$$

$$\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}+x_7\}+x_8\}+x_9\}\times 10+5x_{10}$$

$$\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}+x_7\}+x_8\}+x_9\}+x_{10}\}\times 10+5x_{11}$$

$$\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}+x_7\}+x_8\}+x_9\}+x_{10}\}+x_{11}\}\times 10+5x_{12}.$$

Hence, simplifying the above, we obtain that

$$p=f\left(X\right)=7200+115x_1+105x_2+95x_3+85x_4+75x_5+65x_6+55x_7+45x_8+35x_9+25x_{10}+15x_{11}+5x_{12}$$

Then, we now center on the constraints to have the following

$$For\ January-150\times 60+100x_1\geq 8000$$

$$For\ February-150\left(60+x_1\right)+100x_2\geq 9000$$

$$For\ March-150\times\left[\left(60+x_1\right)+x_2\right]+100x_3\geq 8000$$

$$For\ April-150\times\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+100x_4\geq 10000$$

$$For\ May-150\times\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4+\}+100x_5\geq 9000$$

$$For\ June-150\times\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+100x_6\geq 12000$$

$$For\ July-150\times\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}+100x_7\geq 8000$$

$$For\ August-150\times\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}+x_7\}+100x_8\geq 9000$$

$$For\ September-150\times\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}+x_7\}+x_8\}+100x_9\geq 8000$$

$$For\ October-150\times\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}+x_7\}+x_8\}+x_9\}+100x_{10}\geq 10,000$$

$$For\ November-150\times\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}+x_7\}+x_8\}+x_9\}+x_{10}\}+100x_{11}\geq 9000$$

$$For\ December-150\times\left\{\left[\left(60+x_1\right)+x_2\right]+x_3\right\}+x_4\}+x_5\}+x_6\}+x_7\}+x_8\}+x_9\}+x_{10}\}+x_{11}\}+100x_{12}\geq 12000.$$

Putting

$$p = f(X) = 7200 + 115x_1 + 105x_2 + 95x_3 + 85x_4 + 75x_5 + 65x_6 + 55x_7 + 45x_8 + 35x_9 + 25x_{10} + 15x_{11} + 5x_{12}$$

together, the above constraints, we now have sensitivity analysis formulations as in Model 1 below we have.

Model 2 (Sensitivity Analysis Model)

Minimize

$$P = f(X) = 7200 + 115x_1 + 105x_2 + 95x_3 + 85x_4 + 75x_5 + 65x_6 + 55x_7 + 45x_8 + 35x_9 + 25x_{10} + 15x_{11} + 5x_{12}$$

Subject to

$$100x_1 \geq -1000$$

$$150x_1 + 100x_2 \geq 0$$

$$150x_1 + 150x_2 + 100x_3 \geq -1000$$

$$150x_1 + 150x_2 + 150x_3 + 100x_4 \geq -1000$$

$$150x_1 + 150x_2 + 150x_3 + 150x_4 + 100x_5 \geq 0$$

$$150x_1 + 150x_2 + 150x_3 + 150x_4 + 150x_5 + 100x_6 \geq 3000$$

$$150x_1 + 150x_2 + 150x_3 + 150x_4 + 150x_5 + 150x_6 + 100x_7 \geq -1000$$

$$150x_1 + 150x_2 + 150x_3 + 150x_4 + 150x_5 + 150x_6 + 150x_7 + 100x_8 \geq 0$$

$$150x_1 + 150x_2 + 150x_3 + 150x_4 + 150x_5 + 150x_6 + 150x_7 + 150x_8 + 100x_9 \geq -1000$$

$$150x_1 + 150x_2 + 150x_3 + 150x_4 + 150x_5 + 150x_6 + 150x_7 + 150x_8 + 150x_9 + 100x_{10} \geq 1000$$

$$150x_1 + 150x_2 + 150x_3 + 150x_4 + 150x_5 + 150x_6 + 150x_7 + 150x_8 + 150x_9 + 150x_{10} + 100x_{11} \geq 0$$

$$150x_1 + 150x_2 + 150x_3 + 150x_4 + 150x_5 + 150x_6 + 150x_7 + 150x_8 + 150x_9 + 150x_{10} + 150x_{11} + 100x_{12} \geq 3000$$

SENSITIVITY ANALYSIS RESULT

From the solution result of the sensitivity analysis problem, we see that only 150 trainees were to be hired within the 1 year period. Since each trainee costs $50,000 then the cost for the 150 will be 150 * $50,000 = $7,500,000 so that we have which implies that this difference is now the total cost for maintaining the regular flight attendants which is greater than that spent for trainees thereby corresponding to the observation in our earlier problem solution indicating that the amount spent on maintaining regular attendants is far more than that on trainees.

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