ISSN 2581-3463

AJMS

Asian Journal of Mathematical Sciences

RESEARCH ARTICLE

A Study on Selected Mathematical Theories for Decision-Making Problems

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Received: 10-11-2021; Revised: 01-12-2021; Accepted: 05-01-2022

ABSTRACT

The most appropriate mathematical theory for dealing with uncertainty in decision making problems, the theory of fuzzy set developed by Zedeh in 1965. Later in 1999, Molodstov introduced another one namely, soft set theory for modelling vagueness and uncertainty in decision making problems. In this paper, we study some mathematical tools such as fuzzy set, soft set and fuzzy soft set for solving decision making problems.

Key words: Decision making, Fuzzy set, Fuzzy soft set, Soft set

INTRODUCTION

Decision makers in any discipline are always challenged to make the best decisions. To successfully run and flow through the normal operations of their organization and to improve the existing processes, decision makers usually depend on their expertise and the available data. The golden rule is that the best decision can be made with a better understanding of the system. Therefore, a prior study of the system under consideration is vital, before any decision is taken. The traditional decision-making approach is based on continuous improvement and on the experience of the decision maker. However, this approach is inferior in many areas including the amount of time required, the cost, and uncertainty in the result.

Designing and working on these systems, especially in a complex scenario, is extremely risky, mainly due to the high number of resources involved, the uncertainty resulting from these activities occurring at different moments and on the distinct probability of simultaneously required resources. There are many mathematical approaches or theories to deal with uncertainty cases in decision making problems. Some of the mathematical theories are fuzzy set, soft set, fuzzy soft set, and so on.[1-6]

The theory of fuzzy set provides a host attractive aggregation connective for integrating membership values describing uncertain information. In 1965, Zadeh[1] proposed and defined the operations of fuzzy set theory in his publication. A commonly used appropriate method in handling uncertainties and representing incomplete and unreliable data is soft computing, which was born as a direct result of the establishment of soft set theory by Molodtsov.[7] The definition by Molodtsov applies for a single universe and the relationship between associated parameters. These extensions all augment itself to Zadeh's[1] concept of fuzzy sets in modeling uncertainty in decision making problems.

Soft set is a mathematical tool free from the problems arising due to the inadequacy of parameters and was introduced by D. Molodstov in 1999.[7] Molodtsov insisted that soft set could be used in any parametrization we prefer such as words and sentences, real numbers, and functions. Maji *et al*.[2] presented the application of soft sets in optimization problems in their article. Consequently, many authors are working on soft set theory.[3,8-11] Maji presented the concept of Fuzzy Soft set (fs-sets) by embedding the ideas of fuzzy set theory. Recently, many scholars study the properties and applications of fuzzy soft set theory. Roy and Maji gave some applications of

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fs-sets. Gogoi and Dutta, presented an applications of fuzzy soft set theory in day to day problems.[5,12-15] In this paper, we take up the concept of fuzzy set soft set, and fuzzy soft sets.[16] Decision making problems is discussed using these mathematical tools.

PRELIMINARIES

Here, we recollect the basic definitions of fuzzy set, soft set, and fuzzy soft set which deal with optimization problems in this paper.

Definition 2.1.[1] Let U be a universal set. A fuzzy set X over U is a set defined by membership function μ_x representing a mapping from U to [0, 1]. Then, the value μ_x (x) for the fuzzy set X is called the membership value. The fuzzy set X on U can be represented as follows:

$X = \{(\mu_x(x)/x): x \in U, \mu_x(x) \in [0; 1]\}$

Definition 2.2.[7] Let U be an initial universe and E be a set of parameters. Let P(U) denote the power set of U and A, B $\subseteq$ E. A soft set F_A over U is a set defined by a function f_A representing a mapping F_A from E to P(U) such that $f_A = \emptyset$, if $x \notin A$. Here f_A is called approximate function of the soft set F_A. A soft set over U can be represented by the set of ordered pairs,

$F_A = \{(x, f_A(x)); x \in E; f_A(x) \in P(U)\}$

Example 2.3. Suppose that U = $\{u_1; u_2; u_3; u_4; u_5\}$ is the universe contains five cars under consideration from a dealer and E = $\{x_1; x_2; x_3; x_4; x_5\}$ is the set of parameters, where x_1 = modern, x_2 = low cost, x_3 = spacious, x_4 = small, and x_5 = charming. A customer to select a car from the dealer, can construct a soft set F_A that describes the characteristic of cars according to own requirements. Assume that

$f_A(x_1) = \{u_1; u_2\}, f_A(x_2) = \{u_1; u_3\}, f_A(x_3) = \{u_2; u_4\}, f_A(x_4) = \{u_1; u_5\}, f_A(x_5) = \{u_1; u_3; u_5\}$ then the soft set F_A is written by

$F_A = \{(x_1; \{u_1; u_2\}); (x_2; \{u_1; u_3\}); (x_3; \{u_2; u_4\}); (x_4; \{u_1; u_5\}); (x_5; \{u_1; u_3; u_5\})\}$.

Definition 2.4.[5] Let U be an initial universe and F(U) be the fuzzy sets over U. Let E be the set of all parameters and A ⊆ E, then a fuzzy soft set γ_A on the universe U is defined by the set of ordered pairs as follows:

$\Gamma_A = \{(x,\gamma_A(x)): x \in E; \gamma_A(x) \in F(U)\}$, where

Γ_A: E → F(U) such that $\gamma_A(x) = \emptyset$, if $x \notin A$, and $\forall x \in E$

Also, $\Gamma A(x) = \{(\mu\gamma_A(x)(u)/u): u \in U; \mu\gamma_A(u) \in [0; 1]\}$ is a fuzzy set over U represented as FS(U).

The application by fuzzy soft aggregation operator is another successful method to isolate the preferences based on a set of parameters. We revisit N Cagman's fuzzy soft aggregation algorithm.[17]

- Step 1. Constructs a fuzzy soft set Γ_{E1} with fuzzy approximate function

γ_{E1}: E1 → F(U) over U, where U = $\{v_1, v_2,\ldots v_n\}$.

- Step 2. Calculate the cardinal set $c\Gamma_{E1}$, which is defined as

$c\Gamma_{E1} = \{\mu c\Gamma_{E1}(e_1)/e_1: e_1 \in E_1\}$ over Γ_{E1}, where $\mu_c\Gamma E_1(e_1) = |\gamma_{E1}\ (e1)|/|U|$

- Step 3. Calculate the aggregate fuzzy set Γ^*_{E1} of Γ_{E1}, where

$\Gamma^* E_1 = \{\mu_\Gamma{}^* E_1(v)/v \in U\}$ is a fuzzy set over U and

$\mu_{\Gamma^*} E_1(v) = 1/|E|\sum_{e\in E} (\mu c\Gamma E1\ (e_1)\ \mu\gamma E1\ (eE1)\ (v))$

- Step 4. Find the best alternative from the above column matrix that has the highest membership grade by Max $\mu\Gamma_{E1}$ (v).

MATHEMATICAL TOOLS FOR DECISION MAKING

The key to success in complex environments is to acquire knowledge based on facts and experimentations. The importance and impact of taking decisions based on facts are considerable. However, to ensure a smooth and efficient decision-making, the interpretation of these facts is also a key issue. It is also customary to present the conclusion of decision makers in a correct and comprehensive manner to get feedback. Thus, the management of decision making is shifting from a preference based one to an evidence-based approach, which has many implications by which decisions are taken in domains such as healthcare.

Application of Fuzzy set in Decision Making

The theory of fuzzy set provides a host attractive aggregation connective for integrating membership values describing uncertain information. In 1965, Zadeh[1] proposed and de fined the operations of fuzzy set theory.

A fuzzy set can be defined by assigning to each possible individual in the universe of discourse, a value of it representing its grade of membership in the fuzzy set. Each membership grades are very often represented by real numbers in the [0,1]. The nearer the value of an element to unity, the higher the grade of its membership. The fuzzy set theory has a wider scope of applicability than classical set theory in solving various problems.

The decision-maker does not think the commonly used distribution like probability is always appropriate, especially when the data are vague, relating to human behavior, imprecise system data, or when the data or information could not be defined and deemed well due to limited knowledge and deficiency in its understanding. Such types of uncertainty are categorized as fuzziness which can be further classified into vagueness. Vagueness here is associated with the difficulty of making sharp or precise distinctions that are fuzzy theory deals with the situation where the information or data cannot be valued sharply or cannot be described clearly in linguistic term, such as preference-related information. This type of fuzziness is usually represented by membership function which the decision-maker's subjectivity and preference on the objects. Ambiguity is associated with the situation in which the choice between two or more alternatives is left unspecified, and the occurrence of each alternative is unknown owing to deficiency in knowledge and tools.

Solve Decision-Making Problem using Soft Set

In classical mathematics, the existent tools for modeling and reasoning are crisp in nature. However, on the other hand, the modern world in different domains such as engineering, medical science, and economics are driven by uncertainty and uncertain data. As traditional tools for modeling fails for uncertain data, many theories such as probability theory, fuzzy sets, and rough sets[1,6,18] was introduced to deal with uncertainties. But still, these theories had difficulties due to lack of parameterization tool. Later, Molodtsov[2] introduced new concept soft sets as a better mathematical tool for dealing with uncertainties, which is free from the previous difficulties.

In this section, we study different parameters to solve decision making problems using soft set. Here, we are going to illustrate the applications of soft set theory in decision making problem. For this purpose, we considered the following situations. Let C = $\{c_1, c_2, c_3, c_4, c_5, c_6\}$ be the set of products (different cars), comprising of the universal set and

E = {Good looking, Cost, Quality, Comfort ability, Color, Spaciousness, Good design} be the set of parameters.

Suppose a customer Mrs. A (Say) interested to buy a car on the basis of parameters like Good looking, Cost, Quality, Comfort ability, Color which the subset of E. That means out of available cars in C, she selects that car which qualifies with all or maximum number of parameters of the soft set.

This is given by

(F, E) = {Goodlooking = $\emptyset$, Cost = $\{c_1, c_2, c_3, c_4, c_5, c_6\}$; Quality = $\{c_1, c_2, c_3\}$,

Comfort ability = $\{c_4, c_5, c_6\}$, Color = $\{c_1, c_5, c_6\}$,

Spaciousness = $\{c_1, c_2, c_5, c_6\}$.

The parameters, we wish to investigate can be taken from a subset of parameters E^1 and choice parameters can be altered for analyzing the cars under consideration. Different sets of analysis with different sets of choice parameters can be done to arrive at conclusions. It is to be understood that the choice parameters are all subsets E^1 of E.

Representation of a soft set

Tabular representation is a useful tool to show information system in rough set approach. The same concept can be applied for soft sets too. The tabular representation of the soft set we considered above is shown in Table 1.

A representation in the form of 1's and 0's are useful in creating knowledge representations in computer systems. The mapping only classifies the objects into two simple classes (Yes or No). We can say that if $c_i \in$ (F, E'), then $c_{ij} = 1$, otherwise $c_{ij} = 0$, where c_{ij} are the entries in Table 1.

Thus, we are able to create a knowledge representation system using soft sets.

Choice value of an object

Definition 3.1. Let (c_{ij}) be the of members reduct soft set. Choice Value of $c_i \in C$ is denoted by ch_i and is defined as

$ch_i = \sum_j (c_{ij})$

Algorithm 1

1. Input the soft set (F, E')

Table 1: Soft set tabular representation of the problem

C	Good looking	Cost	Quality	Comfort ability	Color
1	0	1	1	1	0
2	0	1	1	1	0
3	1	1	1	1	0
4	1	1	0	1	0
5	0	1	0	1	1
6	1	1	0	1	1

Table 2: Reduced soft set tabular representation of the problem

C	Good looking	Cost	Quality	Comfort ability	Choice Value
1	0	1	1	1	3
2	0	1	1	1	3
3	1	1	1	1	4
4	1	1	0	1	3
5	0	1	0	1	2
6	1	1	0	1	3

2. Given that the choice parameters set P, where $P \subseteq E$
3. Compute all reduct soft sets of (F; P)
4. Choose one reduct soft set of (F; P) say (F, Q)
5. Find the value of m, for which c_m = maxchi.

Then conclude that c_m is the maximal choice object, which confirms the most suitable car for buying. Incorporating the choice values, one of the reduce soft set can be represented as Table 2:

Here max $c_{hi} = c_3$. Decision: Mrs. A can buy the car c_3.

Solve Decision-Making Problem Using Fuzzy Soft Set

In our real-life, we often face some problems in which the right or accurate decision-making is highly essential. To obtain the best feasible solution or optimal solution of the real-life problems we have to consider different parameters relating to the optimal solution. For this purpose, we can use the best mathematical tool called Fuzzy soft set theory. The application by fuzzy soft aggregation operator is another method to isolate the preferences based on a set of parameters. We define and revisit fuzzy soft aggregation operator for a more efficient decision-making method.

Application

We take up the issue of finding the most attractive city from a group of cities. The related parameters are given as fuzzy inputs.

We use the notations U_c = universe of cities, and p_i = set of parameters. Let U_c = {c – Chennai, b - Bangalore; p - Pune; d - Delhi; k - Kolkata; h - Hyderabad; f - Faridabad; m–Mumbai, a - Agra; v -Visakhapatnam}and p_i = {p_1 - “Cultural,” p_2 - “Tourist Destination,” p_3 - “Spectacular Buildings,” p_4 – “Stunninglakes”}

As part of data collection, these parameter values are accumulated by various sensors located in the respective cities we mentioned above.

The collected values are analyzed by means of computer programs, which will identify the level of attraction (acceptable levels) in each city based on prior research. A fuzzy soft set-based system is set up to find the aggregate values of the recorded parameters by the application of soft set aggregation process. As per the aggregation process explained in section,[18] we find the aggregate attraction levels for each city as follows: Initially we fuzzify the recorded inputs from sensors and feed it to the automated system.

i. A sample fuzzy soft set for attractive values can be defined as follows:

Γ_{Pi} = {(p_1,{0:5/c, 0:6/p, 0:7/k}), (p_2; {0:2/b, 0:4/d, 0:6/h, 0:8/m, 1/v}),(p_3,{0:1/c, 0:3/f, 0:7/a}), (p_4; {1/d, 0:9/k, 0:6/m})

ii. The cardinal set of fuzzy soft set $c\Gamma_{Pi}$ is obtained as

iii. The attractive values for each city with their parameters can be represented as a the fuzzy soft matrix.

$$A_{Pi=}\begin{array}{c c} & \begin{matrix} p1 & p2 & p3 & p4 \end{matrix} \\ \begin{matrix} c \\ b \\ p \\ d \\ k \\ h \\ f \\ m \\ a \\ v \end{matrix} & \begin{pmatrix} 0.5 & 0 & 0.1 & 0 \\ 0 & 0.2 & 0 & 0 \\ 0.6 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 1 \\ 0.7 & 0 & 0 & 0.9 \\ 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \\ 0 & 0.8 & 0 & 0.6 \\ 0 & 0 & 0.7 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

iv. Compute transpose of cardinal fuzzy soft set

$$A^{T}{}_{c\ Pi} = \begin{pmatrix} 0.18 \\ 0.3 \\ 0.11 \\ 0.25 \end{pmatrix}$$

v. The following column matrix $A_{\Gamma^{*}pi}$ represents aggregate attractive levels in each of cities

$$A^{*}_{pi} = 1/\left|E\right| A_{P} A^{T}_{cPi} = \begin{array}{c} c \\ b \\ p \\ d \\ k \\ h \\ f \\ m \\ a \\ v \\ \end{array} \begin{pmatrix} 0.010 \\ 0.006 \\ 0.010 \\ 0.037 \\ 0.035 \\ 0.006 \\ 0.003 \\ 0.039 \\ 0.007 \\ 0.030 \\ \end{pmatrix}$$

Then we obtain
Γ^{*}_{pi}= {0:010/c, 0:006/b, 0:010/p, 0:037/d, 0:035/k, 0:006/h, 0:003/f, 0:039/m, 0:007/a, 0:030/v}.

vi. The maximum aggregate value represents the highest attractive index in the respective cities. The lowest aggregate value is indicative of the low attractive index. Finally, we get the highest membership grade max $\mu*_{Pi}$ (c) = 0.039, which means that the city "m- Mumbai" has the highest attractive index, while "f – Faridabad" has the lowest attractive index. Based on standard acceptable attractive levels in each city, we can find the minimum threshold to determine the attractive index of the city by means of mapping.

CONCLUSION

In this paper, the mathematical tools mainly we concentrate are fuzzy set, soft set, and fuzzy soft set theories, are very much interesting and useful for solving real life problems. These tools help us to take decision making in critical situations.

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