

## RESEARCH ARTICLE

## Application of Multi-server Queue Model (m/m/c) for Waiting Lines Management in Banking System

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### ABSTRACT

Queues or waiting lines arise when the demand for service exceeds the capacity of a service facility. One of the major challenges bank customers encounter in banks is the waiting lines in automated teller machines (ATMs). This study formulated a Multi-Server Queue Model (M/M/C) for Queue Management in Banking ATM. The performance level of a typical bank ATM has been effectively investigated using the M/M/S queuing model. It was observed that the busy time of the machine is 2.6 h while the idle time is 7.4 h in the 10 h of banking time which is attributed to the availability of many servers in the system. The utilization factor is 0.26 or 26.0% shows that the service delivery of the machine is very efficient and there is no urgent need for an additional server. The researcher thereby recommended that banks should consider the use of the queue model to test the performance of waiting lines in the ATMs.

**Key words:** Multi-server, Queue model, Waiting line, Banking system

### INTRODUCTION

The act of joining a line or waiting is referred to as a queue. Customers (arrivals) wanting service must wait because the number of servers available exceeds the number of servers available, or the facility does not perform smoothly or takes longer than the time allotted to serve a client.<sup>[1]</sup> It is a common occurrence in gas stations, supermarkets, and banks, among other places.

Electronic banking is one of the many technological achievements in the banking industry. In the banking business, an automated teller machine (ATM) is one of various electronic banking channels. In the banking business, ATMs are one of the most crucial service facilities.<sup>[2]</sup> ATMs first introduced in Nigeria in 1989 and have since gained widespread acceptance and usage. Nigeria exchange group is a company based in Nigeria.<sup>[3]</sup> More than half of respondents in a study stated that preference for the situation becomes compounded during festive periods and month finishes, when demand for cash is at its peak.

According<sup>[4]</sup> to they described, queuing theory is a research that aims to assist business owners in analyzing the percentage of time customers wait for services to be delivered to them and improving the percentage of services delivered in this way. Erlang, a Danish mathematician, conducted the first investigation on queuing theory, which led to the development of the world-renowned Erlang telephone model. He looked at the phone network system and sought to figure out what influence variable service demands had on call volume and the use of automatic dialing equipment.<sup>[5]</sup>

Queuing theory, often known as congestion theory, is an area of operational research that investigates the relationship between demand for a service system and the delays experienced by its users.<sup>[6]</sup> Other researchers<sup>[7,8]</sup> worked on improving performance inside the banking hall.

The application of queuing theory in banking operations is still in its early stages of research, with notable studies being conducted in various areas.<sup>[1]</sup> Salami *et al.*<sup>[9]</sup> investigated strategies for Chinese commercial banks to increase their efficiency in terms of customer queuing in bank halls. The focus was on using queuing theory to investigate the queue problem, which was based on supposed data with an arrival rate of 32 and a service rate of 20.

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Kumaran *et al.*<sup>[10]</sup> did a study on three ATM facilities from three different banks that were evaluated at Veliore Institute of Technology using simulation and queuing theory. Simulation was used to collect data, which was done throughout a daily free period and on weekends. They used transformed data to calculate client wait times in the queue and in the system, after which they compared consumer satisfaction with the services offered by various ATM facilities. In the study of<sup>[4]</sup> a method for calculating queue length and waiting time was established. The appropriate queue models are utilized to estimate queue length, which is then used to determine gateway server memory size. In Nigeria's financial system, ATMs and ATM queues are relatively new.

The present study sought to investigate the application of multi-server queue model (M/M/C) for waiting lines management of Union Bank (Union Bank) Bida branch ATMs.

## MATERIALS AND METHODS

### Research Method

The research method used in this study was quantitative and no qualitative data were collected. Because the goal of this study is to look into the performance of a Union Bank ATM located on Bida Road in Bida Niger State. The development of a queuing model for the analysis of the queuing system at Union Bank ATM service point, as well as the establishment of a strategy to tackle the problem of customer arrival rate, was among the methodologies employed in this research effort.

### Method of Data Collection

The input data were manually collected with the help of an electronic instrument (e-stopwatch) and a study assistant using direct observation. The information for this study was gathered at the Union Bank Bank PIC ATM service station on Bida Road Bida, Niger State, for a period of 5 working days, from 7:30 a.m. to 7:00 p.m., respectively. Direct observation of consumers was used to collect data, and their arrival time, "the time service begins," and "the time service finishes" were all collected and entered in real-time onto a form created specifically for this experiment. To calculate, we used data that were well-fitting. According to queue theory, the following assumptions were made for the queuing system. They are:

1. Arrivals follow a Poisson probability distribution and are drawn from an unlimited population
2. No balking or renegeing occurs because there is only one waiting line and each arrival waits to be serviced regardless of the length of the line
3. Queue discipline is first come first serve (FCFS)
4. With an average of customers per unit of time, service time follows an exponential distribution
5. The average rate of service is higher than the average rate of arrival
6. The system can accommodate an infinite number of customers
7. Service providers do not go faster because the line is longer; service providers do not go faster because the line is longer.

### Method of Data Analysis

Based on the data collected from the bank ATM service point, we conduct an analysis.

- i. The time arrival of each customer
- ii. The inter-arrival time between customers
- iii. The time service commenced and ends for each customer using the M/M/S model and a single queue.

To appropriately assess the data recorded and generate the performance measurements, an analytical approach/method or queuing theory (formula based) was used.

### Formulation of Model from Kendall's Notation

Let us introduce a notation created by Kendall to define a queuing system before we begin our research of simple queuing systems. Let's call it a system if A/B/m/K/n/D

- A: Distribution function of the inter arrival times
- B: Distribution function of the service times
- m: number of servers
- K: Capacity of the system, the maximum number of customers in the system including the one being serviced
- n: Population size, number of sources of customers,
- D: Service discipline.

M, which stands for Markovian or memoryless, is used to denote exponentially distributed random

variables. They are also omitted if the population size and capacity are unlimited, and the service discipline is FIFO.

With the value of  $s=5$ , the Kendall notation becomes M/M/5, which exactly fits the queuing model utilized in this research to analyze the performance metrics of the queue system at Union Bank Bida. M/M/r/K/n also refers to a system in which consumers enter from a finite-source with n elements and stay for an exponentially dispersed period, service times are exponentially distributed, service is delivered according to the arrival of the request by r servers, and the system capacity is K.

### Queuing Model with Single Queue and Multiple ATMs [(M/M/S): ( $\infty$ /FCFS)] M/M/S Model

The model shown here can capture all of the characteristics involved in a multi-server queuing system for an infinite calling population with a first-come and first-served multiple server queuing system ( $\infty$ FCFS). In this example, multiple and identical servers are connected in a parallel line to deliver the same service to clients. When a customer arrives, he is placed in a single server queue and remains there until he is served. Here,  $\mu$  is the service rate of one server, but we have s number of servers; therefore,  $s\mu$  will be the service rate. If there are n consumers in the queue at any given time, one of the following two scenarios may occur:

- i. If  $n < s$  (number of customers in the system is less than the number of servers), then there will be no queue. However,  $(s-n)$  numbers of servers are not busy. The combined service rate will be:  $\mu_n = n\mu; n < s$
- ii. If  $n \geq s$  (number of customers in the system is more than or equal to the number of servers) then all servers will be busy and the maximum number of customers in the queue will be  $(n-s)$ . The combined service rate will be:  $\mu_n = s\mu; n \geq s$ . Thus, we have  $\lambda_n = \lambda$  for all  $n \geq 0$ .  
 $\mu_n = \begin{cases} n\mu & n < s \\ s\mu & n \geq s \end{cases}$ . Figure 1 shows the diagram queuing system model.

### The Basic Indexes/Parameters of the Queuing system

- $n$  = Number of customers in the system at time t

- $\lambda$  = Mean arrival rate (number of arrival per unit of time)
- $\mu$  = Mean Service rate per busy server (number of customers served per unit of time)
- $\rho$  = Expected fraction of time for which server is busy
- $s$  = number of service channel channels (service facilities or servers).

### Some Queuing Notations used in this Multi-Server Queuing Model

- i.  $n$  = Number of total customers in the system (in queue plus in service)
- ii.  $\lambda$  = Mean arrival rate [1/(average number of customers arriving in each queue in a system in 1 h)]
- iii.  $\mu$  = Mean service rate [1/(average number of customers being served at a server per hour)]
- iv.  $s$  = Number of parallel servers (service units in queue system)
- v.  $s\mu$  = Servicing rate when  $s > 1$  in a system
- vi.  $\rho$  = System intensity or load, utilization factor ( $\rho = \lambda/s\mu =$ ) (the expected factor of time the server is busy that is service capability being utilized on the average arriving customers)
- vii.  $L_s$  = Length of system
- viii.  $L_q$  = Length of queue
- ix.  $W_s$  = Waiting time in the system
- x.  $W_q$  = Waiting time in the queue
- xi.  $P_n$  = Probability of having n customers in the queuing system
- xii.  $P_0(t)$  = Probability that there are no customers in queuing system at time t
- xiii.  $P_s$  = The probability that on arrival, a customer must wait for services.

### Operating Characteristics of a Multiple Server Queue Model

- a. The probability  $P_n$  of n customers in the queuing system is given by

$$P_n = \begin{cases} \frac{\rho^n}{n!} P_0; n \leq s \\ \frac{\rho^n}{s!s^{n-s}} P_0; n > s \end{cases} \quad (1)$$

- b. The probability that the ATM is idle ( $P_0$ ) that is, the probability of no customer in the ATM

$$P_0 = \left[ \sum_{n=0}^{s-1} \frac{1}{n!} \left( \frac{\lambda}{\mu} \right)^n + \frac{1}{s!} \left( \frac{\lambda}{\mu} \right)^s \left( \frac{\lambda\mu}{s\mu - \lambda} \right) \right]^{-1}$$

- c. Expected number of customers waiting in the queue (i.e., queue length)

$$L_q = \left[ \frac{1}{(s-1)!} \left( \frac{\lambda}{\mu} \right)^s \frac{\lambda\mu}{(s\mu - \lambda)^5} \right] P_0$$

- d. Expected number of customers in a system

$$L_s = L_q + \frac{\lambda}{\mu}$$

- e. Expected waiting time of a customer in the queue

$$W_q = \left[ \frac{1}{(s-1)!} \left( \frac{\lambda}{\mu} \right)^s \frac{\lambda\mu}{(s\mu - \lambda)^5} \right] P_0 = \frac{L_q}{\lambda}$$

- f. Expected waiting time that a customer spends in a system

$$W_s = W_q + \frac{1}{\mu}$$

- g. Utilization factor, that is, the fraction of time servers is busy

$$\rho = \frac{\lambda}{s\mu}$$

- h. The probability that on arrival a customer must wait for service

$$P_s = \frac{1}{s!} \left( \frac{\lambda}{\mu} \right)^s \left( \frac{s\mu}{s\mu - \lambda} \right) P_0$$

**RESULTS AND DISCUSSION**

This section discusses that the analysis of data obtained on daily observation taking at ATM stand of Union Bank, Bida. Table 1 shows the summary of data collected on inter-arrival time, service time for ATM 1, ATM 2, ATM 3, ATM 4, and ATM 5 as well as total number of customers.

From the Table 1, we can obtain the following

1. Average inter-arrival time for each day =  $\frac{\text{total inter - arrival for each day}}{\text{total no. of customer for each day}}$

**Table 1:** Summary of data collected on total inter-arrival time and total service time of the ATMs for period of 5 working day (7:30 am–7:00 pm)

ATM	Day					Total
	1	2	3	4	5	
1	499	526	405	364	352	2146
2	435	422	402	431	407	2097
3	375	406	418	397	404	2000
4	454	412	436	429	418	2149
5	465	439	403	427	474	2208
Inter-Arrival Time (min)	1089	1069	954	1072	933	5117
Total No. of customers	742	692	733	854	768	3789

Hence, average inter-arrival time for Monday is =  $\frac{1089}{742} = 1.468$

Average inter-arrival time for Tuesday is =  $\frac{1069}{692} = 1.545$

Average inter-arrival time for Wednesday is =  $\frac{954}{733} = 1.302$

Average inter-arrival time for Thursday is =  $\frac{1072}{854} = 1.255$

Average inter-arrival time for Friday is =  $\frac{933}{7688} = 1.215$

2. The average service time for each server is =  $\frac{\text{total service time for each day}}{\text{total no. of customer for each day}}$

a. The average service time for ATM 1 is =  $\frac{\text{total service time for each day}}{\text{total no. of customer for each day}}$

therefore, Average service time for Monday is =  $\frac{499}{742} = 0.673$

Average service time for Tuesday is =  $\frac{526}{692} = 0.760$

Average service time for Wednesday is =  $\frac{405}{733} = 0.553$

$$\text{Average service time for Thursday is } = \frac{364}{854} = 0.426$$

$$\text{Average service time for Friday is } = \frac{352}{768} = 0.458$$

$$\text{b. The average service time for ATM 2 is } = \frac{\text{total service time for each day}}{\text{total no. of customer for each day}}$$

$$\text{therefore, average service time for Monday is } = \frac{435}{742} = 0.586$$

$$\text{Average service time for Tuesday is } = \frac{422}{692} = 0.610$$

$$\text{Average service time for Wednesday is } = \frac{402}{733} = 0.548$$

$$\text{Average service time for Thursday is } = \frac{431}{854} = 0.505$$

$$\text{Average service time for Friday is } = \frac{407}{768} = 0.530$$

$$\text{c. The average service time for ATM 3 is } = \frac{\text{total service time for each day}}{\text{total no. of customer for each day}}$$

$$\text{therefore, average service time for Monday is } = \frac{375}{742} = 0.505$$

$$\text{Average service time for Tuesday is } = \frac{406}{692} = 0.587$$

$$\text{Average service time for Wednesday is } = \frac{41}{733} = 0.570$$

$$\text{Average service time for Thursday is } = \frac{397}{854} = 0.465$$

$$\text{Average service time for Friday is } = \frac{404}{768} = 0.526$$

$$\text{d. The average service time for ATM 4 is } = \frac{\text{total service time for each day}}{\text{total no. of customer for each day}}$$

$$\text{Therefore, average service time for Monday is } = \frac{454}{742} = 0.612$$

$$\text{Average service time for Tuesday is } = \frac{412}{692} = 0.595$$

$$\text{Average service time for Wednesday is } = \frac{436}{733} = 0.595$$

$$\text{Average service time for Thursday is } = \frac{429}{854} = 0.502$$

$$\text{Average service time for Friday is } = \frac{418}{768} = 0.544$$

$$\text{e. The average service time for ATM 5 is } = \frac{\text{total service time for each day}}{\text{total no. of customer for each day}}$$

$$\text{therefore, average service time for Monday is } = \frac{465}{742} = 0.627$$

$$\text{Average service time for Tuesday is } = \frac{439}{692} = 0.634$$

$$\text{Average service time for Wednesday is } = \frac{403}{733} = 0.550$$

$$\text{Average service time for Thursday is } = \frac{427}{854} = 0.500$$

$$\text{Average service time for Friday is } = \frac{474}{768} = 0.617$$

$$\text{3. Obtaining the total average service time } = \frac{\text{sum of average servicetime for each day}}{\text{number of day}}$$

$$\text{a. Obtaining the total average service time for ATM1}$$

$$= \frac{0.673 + 0.600 + 0.553 + 0.426 + 0.458}{5} = 0.574$$

$$\text{b. Obtaining the total average service time for ATM2}$$

$$= \frac{0.56 + 0.610 + 0.548 + 0.505 + 0.530}{5} = 0.556$$

c. Obtaining the total average service time for ATM3

$$= \frac{0.505 + 0.587 + 0.570 + 0.465 + 0.526}{5} = 0.531$$

d. Obtaining the total average service time for ATM4

$$= \frac{0.612 + 0.595 + 0.595 + 0.502 + 0.544}{5} = 0.570$$

e. Obtaining the total average service time for ATM5

$$= \frac{0.627 + 0.634 + 0.550 + 0.500 + 0.617}{5} = 0.586$$

4. Obtaining the total average service time for system

$$\begin{aligned} \text{sum of average} & 0.574 + 0.556 + \\ \text{servicetime for} & 0.531 + 0.570 + \\ \text{each ATM} & 0.56 \\ \text{number of ATM} & 5 \end{aligned} = 0.563$$

5. Obtaining the average inter arrival time for system

$$\begin{aligned} \text{sum of inter} & 1.468 + 1.545 + 1 \\ \text{arrival time} & .302 + 1.255 + 1.215 \\ \text{number of days} & 5 \end{aligned} = 1.375$$

From the expression, it can be deduce that average service time is 0.56 while average inter arrival time is 1.375

Then, the service rate and arrival rate are calculated as;

$$\begin{aligned} \text{Service rate } (\mu) &= \\ \frac{1}{\text{average service time}} &= \frac{1}{0.563} = 1.776 \cong 1.78 \end{aligned}$$

customers per minute = 106.8 cust/h

$$\begin{aligned} \text{average inter arrival time } (\lambda) &= \\ \frac{1}{\text{average inter arrival time}} &= \frac{1}{1.375} = 0.737 \cong 0.74 \\ \text{customers per minute} &= 44.4 \text{ cust/h.} \end{aligned}$$

Since the major queue parameters are already known, we can proceed to obtain the performance measure of the system such as;

1. Utilization factor

$$\rho = \frac{\lambda}{s\mu} = \frac{0.74}{5 \times 1.78} = 0.262 \text{ or } 26.2\%.$$

2. Probability of number of customer in the system;

$$P_o = \left[ \sum_{n=0}^{s-1} \frac{1}{n!} \left( \frac{\lambda}{\mu} \right)^n + \frac{1}{s!} \left( \frac{\lambda}{\mu} \right)^s \left( \frac{\lambda\mu}{s\mu - \lambda} \right) \right]^{-1}$$

$$P_o = \left[ \sum_{n=0}^4 \frac{1}{0!} \left( \frac{0.74}{1.78} \right)^0 + \frac{1}{5!} \left( \frac{0.74}{1.78} \right)^5 \left( \frac{5 \times 1.78}{5 \times 1.78 - 0.74} \right) \right]^{-1}$$

$$P_o = \left[ \begin{aligned} & \frac{1}{0!} \left( \frac{0.74}{1.78} \right)^0 + \frac{1}{1!} \left( \frac{0.74}{1.78} \right)^1 + \frac{1}{2!} \left( \frac{0.74}{1.78} \right)^2 + \\ & \sum_{n=0}^4 \frac{1}{3!} \left( \frac{0.74}{1.78} \right)^3 + \frac{1}{4!} \left( \frac{0.74}{1.78} \right)^4 \\ & + \frac{1}{5!} \left( \frac{0.74}{1.78} \right)^5 \times \left( \frac{5 \times 1.78}{5 \times 1.78 - 0.74} \right) \end{aligned} \right]^{-1}$$

$$P_o = \left[ \sum_{n=0}^4 1 + 0.41573 + 0.086416 + 0.011975 + 0.001245 + 0.00011287 \right]^{-1}$$

$$P_o = \left[ \frac{1 + 0.41573 + 0.086416 + 0.011975 + 0.001245 + 0.00011287}{0.001245 + 0.00011287} \right]^{-1}$$

$$P_o = [1.515478911]^{-1}$$

$$P_o = \frac{1}{1.515478911}$$

$$P_o = 0.659857$$

3. The expected number of customers waiting in the queue

$$L_q = \left[ \frac{1}{(s-1)!} \left( \frac{\lambda}{\mu} \right)^s \frac{\lambda\mu}{(s\mu - \lambda)^5} \right] P_o$$

$$L_q = \left[ \frac{1}{(5-1)!} \left( \frac{0.74}{1.78} \right)^5 \frac{0.74 \times 1.78}{(5 \times 1.78 - 0.74)^5} \right] \times 0.659857$$

$$\rho = \frac{\lambda}{s\mu}$$

$$L_q = \left[ \frac{1}{(24)} (0.41573)^5 \frac{1.3172}{(8.9 - 0.74)^5} \right] \times 0.659857$$

$$L_q = [0.000517 \times 3.64083E - 05] \times 0.659857$$

$$L_q = 1.243072163361630E - 08$$

4. Expected number of customers in a system

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$L_s = 1.243072163361630E - 08 + \frac{0.74}{1.78}$$

$$= 0.43529413 \text{ customers}$$

5. Expected waiting time of a customer in the queue

$$W_q = \left[ \frac{1}{(s-1)!} \left( \frac{\lambda}{\mu} \right)^s \frac{\lambda\mu}{(s\mu - \lambda)^5} \right] P_0 = \frac{L_q}{\lambda}$$

$$W_q = \frac{1.243072163361630E - 08}{0.74}$$

$$W_q = 1.679827247785980E - 08 \text{ min}$$

6. Expected waiting time that a customer spends in a system

$$W_s = W_q + \frac{1}{\infty}$$

$$W_s = 1.679827247785980E - 08 + \frac{1}{1.78}$$

$$W_s = 0.562524627 \text{ min}$$

7. The probability  $P_n$  of n customers in the queuing system is given by

$$P_n = \begin{cases} \frac{\rho^n P_0; n \leq s}{s!s^{n-s} P_0; n > s} & \text{where } \rho = \frac{\lambda}{s\mu} \end{cases} \quad (2)$$

Probability when n 5, n = 0, 1, 2, ... 5.

At n = 0

$$P_n = \frac{\rho^n}{n!} P_0 = \frac{(0.262)^0}{0!} (0.659857) = 0.659857$$

At n = 1

$$P_1 = \frac{\rho^1}{1!} P_0 = \frac{(0.262)^1}{1!} (0.659857) = 0.172882534$$

At n = 2

$$P_2 = \frac{\rho^2}{2!} P_0 = \frac{(0.262)^2}{2!} (0.659857) = 0.045295224$$

At n = 3

$$P_3 = \frac{\rho^3}{3!} P_0 = \frac{(0.262)^3}{3!} (0.659857) = 0.011867349$$

At n = 4

$$P_4 = \frac{\rho^4}{4!} P_0 = \frac{(0.262)^4}{4!} (0.659857) = 0.003109245$$

At n = 5

$$P_5 = \frac{\rho^5}{5!} P_0 = \frac{(0.262)^5}{5!} (0.659857) = 0.000814622$$

The probability when n > 5 i.e. n = 6, 7, ... 10

At n = 6

$$P_6 = \frac{\rho^n}{s!s^{n-s}} P_0 = \frac{\rho^6}{5!5^{6-5}} P_0 = \frac{(0.262)^6}{5!5} (0.659857) = 0.000814622$$

At n = 12

$$P_{12} = \frac{\rho^n}{s!s^{n-s}} P_0 = \frac{\rho^{12}}{12!12^{12-5}} P_0 = \frac{(0.262)^{12}}{12!5^7} (0.659857) = 0.000814622$$

8. The probability that on arrival a customer must wait for service:

$$P_s = \frac{1}{s!} \left( \frac{\lambda}{\mu} \right)^s \left( \frac{s\mu}{s\mu - \lambda} \right) P_0$$

$$P_s = \frac{1}{5!} \left( \frac{0.74}{1.78} \right)^5 \left( \frac{5 \times 1.78}{5 \times 1.78 - 0.74} \right) P_0$$

$$P_s = \frac{1}{120} \left( \frac{0.74}{1.78} \right)^5 \left( \frac{5 \times 1.78}{5 \times 1.78 - 0.74} \right) \times 0.659857$$

$$P_s = 9.2488E-07$$

9. Busy time of the system:

To calculate the busy time of the machine, we multiply the banking hours of the ATM machines used by utilization factor, that is,

$$B = \text{Banking hours of ATM} \times \frac{\lambda}{s\mu}$$

$$B = 100.262 = 2.62 \text{ h}$$

10. Idle time of the system:

To calculate the idle time of the machine, we subtract busy time from banking hours of the ATM  
 $I = \text{Banking hours of ATM} - \text{Busy time}$   
 $I = 10 - 2.62 = 7.38 \text{ h}$

Figure 2 disclosed the probability of n customer in the system.

Table 2 shows the summary of values of parameters and queue formula.

### Discussion of Result

Considering the analytical solution, the capacity of the system under study is 1617 customers and the arrival rate is 0.74 while the service rate is 1.78. This shows that the service rate of the system is greater than the arrival rate, this implies that customers would not have to queue up so much waiting to be served. Probability that the servers are idle is 0.6599 which implies a probability of 65.99% idle server and 34.01% busy server and the utilization factor or traffic intensity is 0.262. The expected number in the waiting line is 0.0904.

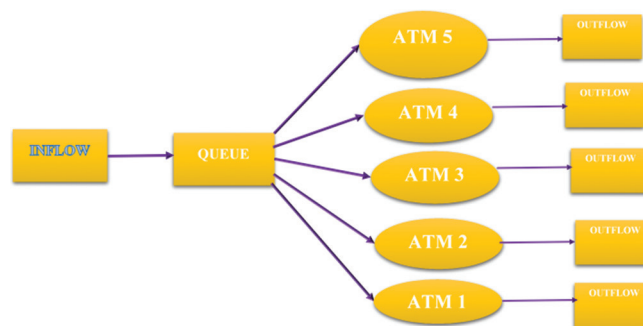


Figure 1: Design queuing system model for the study

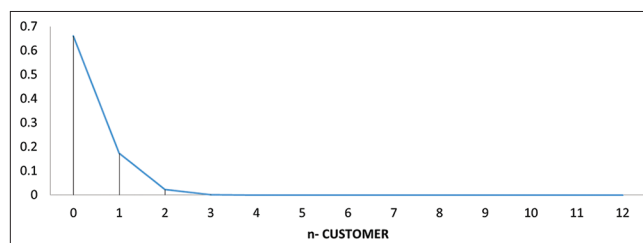


Figure 2: Probability of n customer in the system

Table 2: Summary of values of parameters and queue formula

Probability	Value
Arrival rate $\lambda$	0.737
Service rate $\mu$	1.776
Utilization factor $\rho$	0.262
Expected number of customers in system $L_s$	0.43529413
Average Length of queue $L_q$	1.2430721336130E-08
Expected waiting time the in system $W_s$	0.55524627
Expected waiting time the in queue $W_q$	1.679827247785980E-08
Probability of zero customers in the system $P_0$	0.43529413
Probability that customers must wait for service on arrival $P_s$	9.2488E-07

The expected number in the system is 0.43529413. The expected waiting time in the queue is 1.679827247785980E-08 min and the expected waiting time in the system is 0.55524627 min.

### CONCLUSION

The performance level of the Union Bank ATM has been effectively investigated using the M/M/S queuing model. It was observed that the busy time of the machine is 2.62 hours while the idle time is 7.38 h in the 10 h of banking time which attributed to available of much server in the system. The utilization factor is 0.262 or 26.2% shows that the service delivery of the machine is very efficient and there is no urgent need for an additional server.

### RECOMMENDATION

Based on the conclusion of the study, it is recommended that banks should consider the use



of queue model to test and control waiting lines in the ATMs.

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