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Sensitivity Analysis of Measles Model

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ABSTRACT

The scourge of high-level complications of measles transmission within human population requires immediate attention of all. A situation that cannot be fully avoided must be managed. Therefore, it is highly essential for all hands to be on deck to curb the outbreak of measles globally. In this paper, an extended SEIR model was formulated to capture different stages of diseases in the population. The population was classified into compartments and different parameters were examined with simulation procedure. From the results, needs to boost vaccination of mothers and improved timely treatment of the infected persons were established through sensitivity analysis.

Key words: Complications, Extended SEIR, Compartments, Vaccination, Sensitivity analysis

INTRODUCTION

Measles is a highly contagious and communicable virus disease. It is usually caused by a virus from the paramyxovirus family. The virus infects the respiratory tract and then spreads throughout the body. Measles has symptoms such as excessive coughing, sneezing, and direct interaction with an infected person's nasal or throat secretions. The virus remains active and stays for about 2 h in the air or infected surface.[1] Measles is responsible for approximately 2.6 million deaths each year before the introduction of measles vaccines in 1963 and widespread vaccination. Accelerated immunization has had significant effects in combating deaths due to measles. Global measles mortality between 2000 and 2017 has decreased by 80%, from 545,000 cases to 110,000.

Between 10 and 12 days of exposure to the measles virus, a feverish symptom is developed. At the early stages, coughs, reddish and watery eyes, catarrh, and white spots inside, the cheeks also shows. Usually, face and upper neck rash emerge a few days later. The rash will cover the entire body after 3 days; it takes 5–6 days. Therefore, on average, the rash will infect the whole body within 14 days of exposure to the measles virus (between 7 and 18 days).

Measles deaths have resulted from complications. These complications include encephalitis, blindness, ear infections, severe diarrhea and dehydration, or severe respiratory diseases, such as pneumonia. Severe measles is more likely to occur in malnourished young children, especially those who lack vitamin A, or whose immune systems have been weakened by HIV/AIDS or other diseases.[1]

Supportive care that ensures proper nutrition, adequate fluid intake, and dehydration treatment with an oral rehydration solution recommended by the WHO can prevent some complications of measles. Routine measles vaccination for children and mass immunization campaigns in countries with high rates of cases and deaths are key public health strategies to reduce the global death toll of measles.[1]

In their study, Fred MO, Et'al recommended mass vaccination as a solution to eliminate measles in the world.[2]

Two phases vaccination is said to be appropriate in controlling measles.[3]

This research is aimed at formulating a mathematical model that will be a guide in controlling the spread of measles within human population, obtain stability through disease-free equilibrium and endemic equilibrium for measles model, its reproduction number, and examine

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the behavior of basic reproduction numbers at different parameter values with the sensitivity indices.

Sensitivity Analysis

Sensitivity analysis is the use of certain techniques to determine the effect of individual and/or group parameters on the overall behavior or conduct of a model. Its result clears the uncertainties mostly associated with the values of the parameters and builds confidence in the choice of suitable parameters for a model. How the change in each parameter is determined or results in a similar change in the model's output and discovering the level of impact each parameter posed on the outcome of the model's objectives or dynamics are obtained through sensitivity analysis. There are many techniques or methods of sensitivity analysis of parameters of a model, as in,[4] these include Partial Rank Correlation Coefficient (PRCC), Latin Hypercube Sampling (LHS), Normalized Forward Sensitivity Index, Regression Analysis, One-Way Sensitivity Analysis, Sobol' Method, Sensitivity Heat Map Method, and so on. The backbone of nearly all sensitivity analysis techniques is the normalized forward sensitivity index which is also referred to as elasticity.[5]

METHODOLOGY [FIGURE 1]

N(t) = M(t) + V(t) + S(t) + E(t) + I(t) + T(t) + R(t)

$$M'(t) = V'(t) = S'(t) = E'(t)$$
$$= I'(t) = T'(t) = R'(t) = 0$$

The Variables are defined as follows:
M(t) represents mother to child transmission rates at time t.
V(t) represents vaccination of pregnant/nursing mother against infection at time t

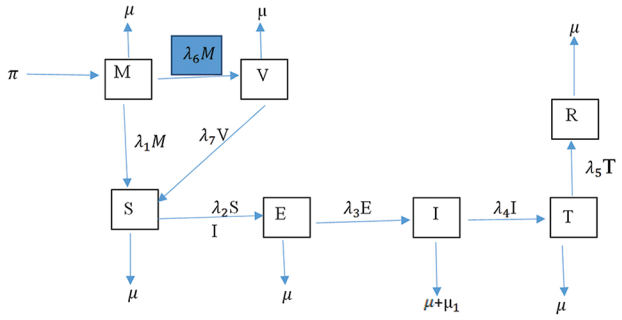

Figure 1: Flow chart of the model

S(t) represents the susceptible population at time t
E(t) represents exposed people in the population at time t.
I(t) represents the infected population at time t
T(t) represents the treatment of those infected at time t
R(t) represents recovered/removed people at time t
While the parameters are defined as follows;
π is the recruitment/birth or entry into population
μ is the natural death rate
μ_1 is the death rate due to the infection.
λ_1 is the movement rate from pregnant mothers to susceptible.
λ_2 is the contact rate from susceptible to Exposed
λ_3 is the movement rate from exposed/latent/incubation Infected stage
λ_4 is the movement rate from infected stage to Treatment stage.
λ_5 is the movement rate from the treatment stage to the recovery or removed stage.
λ_6 is the rate at which pregnant women go for vaccination.
λ_7 is the movement rate from the Vaccination stage to the Susceptible stage.

Model Equations/Rate Values

$$M'(t) = \neq -\lambda_1 M(t) - \lambda_6 M(t) - \mu M(t) \quad (1)$$

$$V'(t) = \lambda_6 M(t) - (\lambda_7 + \mu)V(t) \quad (2)$$

$$S'(t) = \lambda_1 M(t) + \lambda_7 V(t) - (\lambda_2 I + \mu)S(t) \quad (3)$$

$$E'(t) = \lambda_2 SI(t) - (\lambda_3 + \mu)E(t) \quad (4)$$

$$I'(t) = \lambda_3 E(t) - (\lambda_4 + \mu + \mu_1)I(t) \quad (5)$$

$$T'(t) = \lambda_4 I(t) - (\lambda_5 + \mu)T(t) \quad (6)$$

$$R'(t) = \lambda_5 T(t) - \mu R(t) \quad (7)$$

Positivity.
From (1)

$$\frac{dM(t)}{dt} = \pi - (\lambda_1 + \lambda_6 + \mu)M(t)$$

$$\frac{dM(t)}{dt} \geq -(\lambda_1 + \lambda_6 + \mu)M(t)$$

$$\frac{\mathrm{dM(t)}}{M(t)} \geq -(\lambda_1 + \lambda_6 + \mu)\mathrm{dt}$$

$$\int \frac{\mathrm{dM(t)}}{\mathrm{M(t)}} \geq \int -(\lambda_1 + \lambda_6 + \mu)\, dt$$

$$\ln \mathrm{M}(\mathrm{t}) \geq -(\lambda_1 + \lambda_6 + \mu)t + C$$

Where C is a constant of integration

$$\mathrm{M}(\mathrm{t}) \geq \mathrm{e}^{-((\lambda_1+\lambda_6+\mu)t)+\mathrm{C}}$$

$$\mathrm{M}(\mathrm{t}) \geq \mathrm{e}^{-((\lambda_1+\lambda_6+\mu)t)} e^{C}$$

$$\mathrm{M}(\mathrm{t}) \geq M(0)\mathrm{e}^{-((\lambda_1+\lambda_6+\mu)t)} \geq 0$$

From (2)

$$\frac{\mathrm{dV(t)}}{\mathrm{dt}} = \lambda_6 M(t) - (\lambda_7 + \mu)V(t)$$

$$\frac{\mathrm{dV(t)}}{\mathrm{dt}} \geq -(\lambda_7 + \mu)V(t)$$

$$\frac{\mathrm{dV(t)}}{V(t)} \geq -(\lambda_7 + \mu)\mathrm{dt}$$

$$\int \frac{\mathrm{dV(t)}}{V(t)} \geq \int -(\lambda_7 + \mu)dt$$

$$\ln \mathrm{V}(\mathrm{t}) \geq -(\lambda_7 + \mu)t + C$$

$$\mathrm{V}(\mathrm{t}) \geq \mathrm{e}^{-((\lambda_7+\mu)t)+\mathrm{C}}$$

$$\mathrm{V(t)} \geq \mathrm{e}^{-((\lambda_7+\mu)t)} e^{C}$$

$$\mathrm{V(t)} \geq V(0)\mathrm{e}^{-((\lambda_7+\mu)t)} \geq 0$$

From (3)

$$\frac{\mathrm{dS(t)}}{\mathrm{dt}} = \lambda_1 M(t) + \lambda_7 V(t) - (\lambda_2 I + \mu)S(t)$$

$$\frac{\mathrm{dS(t)}}{\mathrm{dt}}) \geq -(\lambda_2 I + \mu)S(t)$$

$$\frac{\mathrm{dS(t)}}{S(t)}) \geq -(\lambda_2 I + \mu)\mathrm{dt}$$

$$\int \frac{\mathrm{dS(t)}}{\mathrm{S(t)}}) \geq \int [-(\lambda_2 I + \mu)]dt$$

$$\ln \mathrm{S(t)} \geq \int [-(\lambda_2 I + \mu)]dt$$

$$\ln \mathrm{S}(\mathrm{t}) \geq -(\lambda_2 I + \mu)t + C$$

$$\mathrm{S}(\mathrm{t}) \geq \mathrm{e}^{-(\lambda_2 I+\mu)t+C}$$

$$\mathrm{S(t)} \geq \mathrm{e}^{-((\lambda_2 I+\mu)t)} e^{C}$$

$$\mathrm{S(t)} \geq S(0)\mathrm{e}^{-((\lambda_2 I+\mu)t)} \geq 0$$

From (4)

$$\frac{\mathrm{dE(t)}}{\mathrm{dt}} = \lambda_2 SI(t) - (\lambda_3 + \mu)E(t)$$

$$\frac{\mathrm{dE(t)}}{\mathrm{dt}}) \geq -(\lambda_3 + \mu)E(t)$$

$$\frac{\mathrm{dE(t)}}{E(t)}) \geq -(\lambda_3 + \mu)\mathrm{dt}$$

$$\int \frac{\mathrm{dE(t)}}{\mathrm{E(t)}} \geq \int -(\lambda_3 + \mu)dt$$

$$\ln \mathrm{E}(\mathrm{t}) \geq -(\lambda_3 + \mu)t + C$$

$$\mathrm{E}(\mathrm{t}) \geq \mathrm{e}^{-(\lambda_3+\mu)t+C}$$

$$\mathrm{E(t)} \geq \mathrm{e}^{-(\lambda_3+\mu)\mathrm{t}} e^{C}$$

$$\mathrm{E(t)} \geq E(0)\mathrm{e}^{-(\lambda_3+\mu)\mathrm{t}} \geq 0$$

From (5)

$$\frac{\mathrm{dI(t)}}{\mathrm{dt}} = \lambda_3 E(t) - (\lambda_4 + \mu + \mu_1)I(t)$$

$$\frac{\mathrm{dI(t)}}{\mathrm{dt}} \geq -(\lambda_4 + \mu + \mu_1)I(t)$$

$$\frac{\mathrm{dI(t)}}{I(t)} \geq -(\lambda_4 + \mu + \mu_1)\mathrm{dt}$$

$$\int \frac{\mathrm{dI(t)}}{\mathrm{I(t)}} \geq \int -(\lambda_4 + \mu + \mu_1)dt$$

$$\ln I(t) \geq -(\lambda_4 + \mu + \mu_1)t + C$$

$$I(t) \geq e^{-(\lambda_4+\mu+\mu_1)t+C}$$

$$I(t) \geq e^{-(\lambda_4+\mu+\mu_1)t} e^{C}$$

$$I(t) \geq I(0)e^{-(\lambda_4+\mu+\mu_1)t} \geq 0$$

From (6)

$$\frac{dT(t)}{dt} = \lambda_4 I(t) - (\lambda_5 + \mu)T(t)$$

$$\frac{dT(t)}{dt} \geq -(\lambda_5 + \mu)T(t)$$

$$\frac{dT(t)}{T(t)} \geq -(\lambda_5 + \mu)dt$$

$$\int \frac{dT(t)}{T(t)} \geq \int -(\lambda_5 + \mu)dt$$

$$\ln T(t) \geq -(\lambda_5 + \mu)t + C$$

$$T(t) \geq e^{-(\lambda_5+\mu)t+C}$$

$$T(t) \geq e^{-(\lambda_5+\mu)t} e^{C}$$

$$T(t) \geq T(0)e^{-(\lambda_5+\mu)t} \geq 0$$

From (7)

$$\frac{dR(t)}{dt} = \lambda_5 T(t) - \mu R(t)$$

$$\frac{dR(t)}{dt} \geq -\propto R(t)$$

$$\frac{dR(t)}{R(t)} \geq -\propto dt$$

$$\int \frac{\partial R(t)}{R(t)} \geq \int -\propto dt$$

$$\ln R(t) \geq -\propto t + C$$

$$R(t) \geq e^{-\propto t+C}$$

$$R(t) \geq e^{-\propto t} e^{C}$$

$$R(t) \geq R(0)e^{-\propto t} \geq 0$$

Therefore, for all $t > 0$, all the solution set {M(t), V(t), S(t), E(t), I(t), T(t), R(t)} of the system of equations are all positive.

Disease Free Equilibrium

At equilibrium $f_i = 0$, I = E = 0

$$f_1 = \pi - (\lambda_1 + \lambda_6 + \mu)M(t) \quad (8)$$

$$f_2 = \lambda_6 M(t) - (\lambda_7 + \mu)V(t) \quad (9)$$

$$f_3 = \lambda_1 M(t) + \lambda_7 V(t) - (\lambda_2 I + \mu)S(t) \quad (10)$$

$$f_4 = \lambda_2 SI(t) - (\lambda_3 + \mu)E(t) \quad (11)$$

$$f_5 = \lambda_3 E(t) - (\lambda_4 + \mu + \mu_1)I(t) \quad (12)$$

$$f_6 = \lambda_4 I(t) - (\lambda_5 + \mu)T(t) \quad (13)$$

$$f_7 = \lambda_5 T(t) - \mu R(t) \quad (14)$$

Let
$K_1 = \lambda_1 + \lambda_6 + \mu$
$K_2 = \lambda_7 + \mu$
$K_3 = \lambda_3 + \mu$
$K_4 = \lambda_4 + \mu + \mu_1$
$K_5 = \lambda_5 + \mu$
From (8)
$M'(t) = V'(t) = S'(t) = E'(t) = I'(t) = T'(t) = R'(t) = 0$
$E = I = 0$

$$\pi - (\lambda_1 + \lambda_6 + \mu)M(t) = 0$$

$$M(0) = \frac{\pi}{K_1} = \frac{\pi}{\lambda_1 + \lambda_6 + \mu}$$

From (9)

$$\lambda_6 M(t) - (\lambda_7 + \mu)V(t) = 0$$

$$V(0) = \frac{\lambda_6 M(t)}{K_1 K_2} = \frac{\lambda_6 \pi}{(\lambda_1 + \lambda_6 + \mu)(\lambda_7 + \mu)}$$

From (10)

$$\lambda_1 M(t) + \lambda_7 V(t) - \mu S(t) = 0$$

$$S(t)=\frac{\lambda_1 M+\lambda_7 V(t)}{\mu}$$

$$\mathrm{S}(0)=\frac{1}{\mu}\{\frac{\pi}{K_1}+\frac{\lambda_6\lambda_7\pi}{K_1K_2}\}$$

$$S(0)=\frac{\lambda_1\pi}{\mu(\lambda_1+\lambda_6+\mu)}+\frac{\lambda_6\lambda_7\pi}{\mu(\lambda_1+\lambda_6+\mu)(\lambda_7+\mu)}$$

$E(0)=0$
$I(0)=0$
$T(0)=0$
$R(0)=0$
It is reasonable enough at this point to conclude that at Disease Free Equilibrium, below is the feasible situation.

$$\{M'(t)=V'(t)=S'(t)=E'(t)=I'(t)=T'(t)$$
$$=R'(t)\}=\{\frac{\pi}{\lambda_1+\lambda_6+\mu},\frac{\lambda_6\pi}{(\lambda_1+\lambda_6+\mu)(\lambda_7+\mu)},$$
$$\frac{\lambda_1\pi}{\mu(\lambda_1+\lambda_6+\mu)}+\frac{\lambda_6\lambda_7\pi}{\mu(\lambda_1+\lambda_6+\mu)(\lambda_7+\mu)},0,0,0,0\}$$

Endemic Equilibrium
This is a situation when society cannot be free from infection as
$E(0)\neq 0$ and $I(0)\neq 0$
Then
From equation (8)

$$\pi-(\lambda_1+\lambda_6+\mu)M(t)=0$$

$$M(0)=\frac{\pi}{K_1}=\frac{\pi}{\lambda_1+\lambda_6+\mu}$$

From (9)

$$\lambda_6 M(t)-(\lambda_7+\mu)V(t)=0$$

$$V(0)=\frac{\lambda_6 M(t)}{K_1K_2}=\frac{\lambda_6\pi}{(\lambda_1+\lambda_6+\mu)(\lambda_7+\mu)}$$

From (11) and (12)

$$f_4=\lambda_2 SI(t)-(\lambda_3+\mu)E(t) \qquad (11)$$

$$f_5=\lambda_3 E(t)-(\lambda_4+\mu+\mu_1)I(t) \qquad (12)$$

Suppose bS = P, $K_3=\lambda_3+\mu$, $K_4=\lambda_4+\mu+\mu_1$,

(*) become
$PI-K_3 E=0$
$\lambda_3 E-K_4 E=0$

$$E=\frac{K_4 I}{\lambda_3}$$

$$PI-\frac{K_3K_4 I}{\lambda_3}=0$$

$$[P-\frac{K_3K_4}{\lambda_3}]I=0$$

$I(0)\neq 0\Rightarrow$

$$P-\frac{K_3K_4}{\lambda_3}=0$$

$$\lambda_2 S=\frac{K_3K_4}{\lambda_3}$$

$$S(0)=\frac{K_3K_4}{\lambda_2\lambda_3}=\frac{(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)}{\lambda_2\lambda_3}$$

From equation (10)

$$\lambda_1 M(t)+\lambda_7 V(t)-(\lambda_2 I+\mu)S(t)=0 \qquad (10)$$

$$I(t)=\frac{\lambda_1 M+i\lambda_7 V(t)-\mu S(t)}{\lambda_2 S}$$

$$I(0)=\frac{\left[\frac{\lambda_1\pi}{K_1}+\frac{\lambda_6\lambda_7\pi}{K_1K_2}-\frac{\mu K_3K_4}{\lambda_2\lambda_3}\right]}{\left[\frac{\lambda_2K_3K_4}{\lambda_2\lambda_3}\right]}$$

$$I(0)=\left[\frac{\lambda_1\lambda_3\pi}{K_1K_3K_4}+\frac{\lambda_3\lambda_6\lambda_7\pi}{K_1K_2K_3K_4}-\frac{\mu K_3K_4}{\lambda_2}\right]$$

$$I(0)=\left[\begin{array}{l}\frac{\lambda_1\lambda_3\pi}{(\lambda_1+\lambda_6+\mu)(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)}\\+\frac{\lambda_3\lambda_6\lambda_7\pi}{(\lambda_1+\lambda_6+\mu)(\lambda_7+\mu)(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)}\\-\frac{\mu(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)}{\lambda_2}\end{array}\right]$$

Recall that,

$$E=\frac{K_4 I}{\lambda_3}$$

$$E(0)=\left[\left[\begin{array}{c}\dfrac{\lambda_1\lambda_3\pi}{(\lambda_1+\lambda_6+\mu)(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)} \\ +\dfrac{\lambda_3\lambda_6\lambda_7\pi}{(\lambda_1+\lambda_6+\mu)(\lambda_7+\mu)(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)}\end{array}\right]\left[\begin{array}{c}\mu(\lambda_3+\mu) \\ -\dfrac{(\lambda_4+\mu+\mu_1)}{\lambda_2}\end{array}\right]\right]\dfrac{\lambda_4+\mu+\mu_1)}{\lambda_3}$$

For T,

$$\lambda_4 I(t)-(\lambda_5+\mu)T(t)=0$$

$$T=\frac{\lambda_4 I}{K_5}$$

$$T(0)=\left[\left[\begin{array}{c}\dfrac{\lambda_1\lambda_3\pi}{(\lambda_1+\lambda_6+\mu)(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)} \\ +\dfrac{\lambda_3\lambda_6\lambda_7\pi}{(\lambda_1+\lambda_6+\mu)(\lambda_7+\mu)(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)} \\ -\dfrac{\mu(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)}{\lambda_2}\end{array}\right]\right]\frac{\lambda_4}{\lambda_5+\mu}$$

For R,

$$\lambda_5 T(t)-\mu R(t)=0$$

$$R(0)=\frac{\lambda_5 T(t)}{\mu}$$

$$R(0)=\frac{\lambda_5}{\mu}\left[\left[\begin{array}{c}\dfrac{\lambda_1\lambda_3\pi}{(\lambda_1+\lambda_6+\mu)(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)} \\ +\dfrac{\lambda_3\lambda_6\lambda_7\pi}{(\lambda_1+\lambda_6+\mu)(\lambda_7+\mu)(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)} \\ -\dfrac{\mu(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)}{\lambda_2}\end{array}\right]\right]\frac{\lambda_4}{\lambda_5+\mu}$$

$$R(0)=\left[\left[\begin{array}{c}\dfrac{\lambda_1\lambda_3\lambda_4\lambda_5\pi}{\mu(\lambda_1+\lambda_6+\mu)(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)(\lambda_5+\mu)}+ \\ \dfrac{\lambda_3\lambda_4\lambda_5\lambda_6\lambda_7\pi}{\mu(\lambda_1+\lambda_6+\mu)(\lambda_7+\mu)(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)(\lambda_5+\mu)} \\ -\dfrac{(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)}{\lambda_2(\lambda_5+\mu)}\end{array}\right]\right]$$

Therefore, at the endemic equilibrium stage,

$$\{M'(t),V'(t),S'(t),E'(t),I'(t),T'(t),R'(t)\}=$$

$$\{\frac{\pi}{\lambda_1+\lambda_6+\mu},\frac{\lambda_6\pi}{(\lambda_1+\lambda_6+\mu)(\lambda_7+\mu)},\frac{(\lambda_3+\mu)}{\lambda_2\lambda_3},$$

$$\left[\left[\begin{array}{c}\dfrac{\lambda_1\lambda_3\pi}{(\lambda_1+\lambda_6+\mu)(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)} \\ +\dfrac{\lambda_3\lambda_6\lambda_7\pi}{(\lambda_1+\lambda_6+\mu)(\lambda_7+\mu)(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)} \\ -\dfrac{\mu(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)}{\lambda_2}\end{array}\right]\right]\frac{\lambda_4+\mu+\mu_1)}{\lambda_3},$$

$$\left[\begin{array}{c}\dfrac{\lambda_1\lambda_3\pi}{(\lambda_1+\lambda_6+\mu)(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)} \\ +\dfrac{\lambda_3\lambda_6\lambda_7\pi}{(\lambda_1+\lambda_6+\mu)(\lambda_7+\mu)(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)} \\ -\dfrac{\mu(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)}{\lambda_2}\end{array}\right],$$

$$\left[\left[\begin{array}{c}\dfrac{\lambda_1\lambda_3\pi}{(\lambda_1+\lambda_6+\mu)(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)}+ \\ \dfrac{\lambda_3\lambda_6\lambda_7\pi}{(\lambda_1+\lambda_6+\mu)(\lambda_7+\mu)(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)} \\ -\dfrac{\mu(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)}{\lambda_2}\end{array}\right]\right]\frac{\lambda_4}{\lambda_5+\mu},$$

$$\left[\left[\begin{array}{l}\dfrac{\lambda_1\lambda_3\lambda_4\lambda_5\pi}{\mu(\lambda_1+\lambda_6+\mu)(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)(\lambda_5+\mu)} \\ +\dfrac{\lambda_3\lambda_4\lambda_5\lambda_6\lambda_7\pi}{\mu(\lambda_1+\lambda_6+\mu)(\lambda_7+\mu)} \\ (\lambda_3+\mu)(\lambda_4+\mu+\mu_1)(\lambda_5+\mu) \\ -\dfrac{(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)}{\lambda_2(\lambda_5+\mu)}\end{array}\right]\right]$$

Basic Reproductive Number (R_0) for the Model

We have two infective classes in the model

$$f_4 = E^{'} = \lambda_2 SI(t) - (\lambda_3+\mu)E(t)$$

$$f_5 = I^{'} = \lambda_3 E(t) - (\lambda_4+\mu+\mu_1)I(t)$$

$$R_0 = \rho FV^{-1}$$

$$F = \begin{pmatrix}\lambda_2 SI \\ 0\end{pmatrix} = \begin{pmatrix}0 & \lambda_2 S \\ 0 & 0\end{pmatrix}$$

$$V = V^{-} - V^{+}$$

$$V^{-} - V^{+} = \begin{pmatrix}(\lambda_3+\mu)E & 0 \\ -\lambda_3 E & (\lambda_4+\mu+\mu_1)I\end{pmatrix}$$

$$V = \begin{pmatrix}(\lambda_3+\mu) & 0 \\ -\lambda_3 & (\lambda_4+\mu+\mu_1)\end{pmatrix}$$

$$V^{-1} = \begin{pmatrix}\dfrac{1}{\lambda_3+\mu} & 0 \\ \dfrac{\lambda_3}{(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)} & \dfrac{1}{\lambda_4+\mu+\mu_1}\end{pmatrix}$$

$$FV^{-1} = \begin{pmatrix}0 & \lambda_2 S \\ 0 & 0\end{pmatrix}\begin{pmatrix}\dfrac{1}{\lambda_3+\mu} & 0 \\ \dfrac{\lambda_3}{(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)} & \dfrac{1}{\lambda_4+\mu+\mu_1}\end{pmatrix}$$

$$= \begin{pmatrix}\dfrac{\lambda_2\lambda_3 S}{(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)} & \dfrac{\lambda_2 S}{\lambda_4+\mu+\mu_1} \\ 0 & 0\end{pmatrix}$$

$$R_0 = \frac{\lambda_2\lambda_3 S}{(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)}$$

$$R_0 = \frac{\lambda_2\lambda_3 S}{(\lambda_3+\mu)(\lambda_4+\mu+\mu_1)}.$$

$$\left[\frac{\lambda_1\pi}{\mu(\lambda_1+\lambda_6+\mu)} + \frac{\lambda_6\lambda_7\pi}{\mu(\lambda_1+\lambda_6+\mu)(\lambda_7+\mu)}\right.$$

Sensitivity Analysis

The process was subjected to some mathematical steps, varying the value of a certain parameter at thesame values of the others.The corresponding values of Basic Reproduction Numbers were obtained and observed to know their effects in the model.

Table 1: Sensitivity of change in the values of parameter π (rate of entry of infected persons into) to the reproduction number R_0

π	λ_1	λ_2	λ_3	λ_4	λ_5	M	μ_2	λ_6	λ_7	R_0	Remarks
0.0001	0.5	0.7	0.125	0.09091	0.125	0.00875	0.125	0.5	0.002	0.0165	Stable
0.0002	0.5	0.7	0.125	0.09091	0.125	0.00875	0.125	0.5	0.002	0.0330	Stable
0.0005	0.5	0.7	0.125	0.09091	0.125	0.00875	0.125	0.5	0.002	0.0826	Stable
0.001	0.5	0.7	0.125	0.09091	0.125	0.00875	0.125	0.5	0.002	0.1652	Stable
0.002	0.5	0.7	0.125	0.09091	0.125	0.00875	0.125	0.5	0.002	0.3304	Stable
0.003	0.5	0.7	0.125	0.09091	0.125	0.00875	0.125	0.5	0.002	0.4957	Stable
0.004	0.5	0.7	0.125	0.09091	0.125	0.00875	0.125	0.5	0.002	0.6009	Stable
0.005	0.5	0.7	0.125	0.09091	0.125	0.00875	0.125	0.5	0.002	0.8260	Stable
0.006	0.5	0.7	0.125	0.09091	0.125	0.00875	0.125	0.5	0.002	0.9913	Stable
0.007	0.5	0.7	0.125	0.09091	0.125	0.00875	0.125	0.5	0.002	1.1566	Unstable

Table 2: Sensitivity of change in the values of parameter *d* (treatment rate) to the reproduction number R_0

π	λ_1	λ_2	λ_3	λ_4	λ_5	M	μ_1	λ_6	λ_7	R_0	Remarks
0.0001	0.5	0.7	0.125	0.0050	0.125	0.00875	0.125	0.5	0.002	0.0268	Stable
0.0001	0.5	0.7	0.125	0.0075	0.125	0.00875	0.125	0.5	0.002	0.0263	Stable
0.0001	0.5	0.7	0.125	0.010	0.125	0.00875	0.125	0.5	0.002	0.0258	Stable
0.0001	0.5	0.7	0.125	0.025	0.125	0.00875	0.125	0.5	0.002	0.0234	Stable
0.0001	0.5	0.7	0.125	0.050	0.125	0.00875	0.125	0.5	0.002	0.0202	Stable
0.0001	0.5	0.7	0.125	0.075	0.125	0.00875	0.125	0.5	0.002	0.0178	Stable
0.0001	0.5	0.7	0.125	0.10	0.125	0.00875	0.125	0.5	0.002	0.0159	Stable
0.0001	0.5	0.7	0.125	0.125	0.125	0.00875	0.125	0.5	0.002	0.0143	Stable
0.0001	0.5	0.7	0.125	0.25	0.125	0.00875	0.125	0.5	0.002	0.0097	Stable
0.0001	0.5	0.7	0.125	0.5	0.125	0.00875	0.125	0.5	0.002	0.0059	Stable

Table 3: Sensitivity of change in the values of parameter g (vaccination rate) to the reproduction number R_0

π	λ_1	λ_2	λ_3	λ_4	λ_5	M	μ_1	λ_6	λ_7	R_0	Remarks
0.0001	0.5	0.7	0.125	0.0050	0.125	0.00875	0.125	0.005	0.002	0.0524	Stable
0.0001	0.5	0.7	0.125	0.0050	0.125	0.00875	0.125	0.0075	0.002	0.0522	Stable
0.0001	0.5	0.7	0.125	0.0050	0.125	0.00875	0.125	0.01	0.002	0.0520	Stable
0.0001	0.5	0.7	0.125	0.0050	0.125	0.00875	0.125	0.025	0.002	0.0505	Stable
0.0001	0.5	0.7	0.125	0.0050	0.125	0.00875	0.125	0.05	0.002	0.0482	Stable
0.0001	0.5	0.7	0.125	0.0050	0.125	0.00875	0.125	0.075	0.002	0.0462	Stable
0.0001	0.5	0.7	0.125	0.0050	0.125	0.00875	0.125	0.10	0.002	0.0443	Stable
0.0001	0.5	0.7	0.125	0.0050	0.125	0.00875	0.125	0.125	0.002	0.0425	Stable
0.0001	0.5	0.7	0.125	0.0050	0.125	0.00875	0.125	0.25	0.002	0.0355	Stable
0.0001	0.5	0.7	0.125	0.0050	0.125	0.00875	0.125	0.5	0.002	0.0268	Stable

All other parameters would remain fixed, while the value of R_0 is computed with the aid of Maple 18. If the increase in a parameter value leads to increase in R_0, then there is need to watch the parameter, but when increase in the value of the parameter reduces R_0. The parameter is efficient at bringing the disease on hold.

SIMULATION AND RESULTS

The transmissibility of every disease could be solely determined by the Performance of the Basic Reproductive Number in response to any change in the parameters of the model. The changes experienced in such a situation are the sensitivity that we are concerned with in this regard. The Tables 1-3 shows the sensitivity of different parameter values determined with the Basic Reproduction Number R_0 of the model under consideration. These values were the results of the analysis carried out with maple 18.

CONCLUSION

As the rate at which infected persons entered, the population increases so the increment in the value of the Reproduction Number, which indicates that the model is highly sensitive to any change in the value of rate of entry. Table 2 demonstrated the fact that the more the efforts at treating the symptomatic people increases, the lower the Reproduction Number. The treatment is proved to be effective. Furthermore, the vaccination boosting reduces the value of the Reproduction Number meaning the vaccine that is efficient at reducing the rate of secondary infections.

It is, therefore, recommended that the vaccination be boosted to ensure the number of infected persons continue fallen until measles disease is get rid of.

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