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RESEARCH ARTICLE

Solving Two Topical Problems

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ABSTRACT

The Goldbach–Euler binary problem is formulated as follows: Any even number, starting from 4, can be represented as the sum of two primes. The ternary Goldbach problem is formulated as follows: Every odd number >7 can be represented as the sum of three odd primes, which was finally solved in 2013. In 1995, Olivier Ramare proved that any even number is the sum of no more than 6 primes. The second problem is about the infinity of twin primes. The author carries out the proof by the methods of elementary number theory.

Key words: In, Number, Problems, Solving, Theory

INTRODUCTION, LITERATURE REVIEW AND SCOPE OF WORK

In 1742, the Prussian mathematician Christian Goldbach sent a letter to Leonard Euler, in which he made the following conjecture: Every odd number >5 can be represented as the sum of three prime numbers.[1-4] Euler became interested in the problem and put forward a stronger conjecture: Every even number greater than two can be represented as the sum of two prime numbers. The first statement is called the ternary Goldbach problem, the second the binary Goldbach problem (or Euler problem). In 1995, Olivier Rameur proved that any even number is the sum of at most 6 prime numbers. From the validity of the ternary Goldbach conjecture (proved in 2013 by year), it follows that any even number is a sum of at most 4 numbers.[6] As of July 2008, Goldbach's binary conjecture has been tested for all even numbers not exceeding 1.2 × 1018.[2] The binary Goldbach conjecture can be reformulated as statement about the unsolvability of a Diophantine equation of the 4th some special kind.[5,6] The question of whether there are infinitely many twin primes has been one of the most open questions in theory numbers for many years. This is the content of the twin prime conjecture, which states that there are infinitely many primes p such that that p+2 is also prime. In 1849, de Polignac advanced more the general conjecture that for every natural number k, there exist infinitely many primes p such that p+2k is also simple.[7] The case k = 1 of the de.

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CONTENT (MAIN PART) H

Task 1. Binary Goldbach–Euler problem

Lemma 1. Any even number starting from 12 is representable as a sum four odd prime numbers.

1. For the first even number 12 = 3+3+3+3.

We allow justice for the previous N> 5:

$$p_1+p_2+p_3+p_4=2N \quad (1)$$

We will add to both parts on 1

$$p_1+p_2+p_3+p_4+1=2N+1 \quad (2)$$

Where on the right the odd number also agrees

$$p_1+p_2+p_3+p_4+1=p_5+p_6+p_7 \quad (3)$$

Having added to both parts still on 1

$$p_1+p_2+p_3+p_4+2=p_5+p_6+p_7+1 \quad (4)$$

We will unite $p_{6+}p_{7+1}$ again, we have some odd number, which according to finally solved ternary Goldbach problem we replace with the sum of three simple and as a result we receive:

$$p_1+p_2+p_3+p_4+2=p_5+p_6+p_7+p_8 \quad (5)$$

at the left the following even number is relative, and on the right the sum four prime numbers.

$$p_1+p_2+p_3+p_4+2N \quad (6)$$

Thus obvious performance of an inductive mathematical method. As was to be shown.

Lemma 2.

The sum of six odd primes, starting from 18 – any even number.

According to the solved Goldbach ternary problem:

$$p_1+p_2+p_3=2N_1+1 \quad (7)$$

$$p_4+p_5+p_6 = 2N_2+1 \quad (8)$$

and (7)+(8):

$$p_1+p_2+p_3+p_4+p_5+p_6 = 2(N_1+N_2+1) \quad (9)$$

where $N=N_1+N_2+1$ and N = 9,10,....∞

Corollary 1. Possible value of one of the four primes odd equals in the sum of 2N from 3 to 2N-9 inclusive.

This follows from the finally solved Goldbach's ternary problem.[8]

Theorem 1. The sum of two odd prime numbers is any even number, starting from 6.

The sum of four odd primes:

$$p_1+p_2+p_3+p_4 = 2N \quad (10)$$

The sum of six odd primes:

$$p_5+p_6+p_7+p_8+p_9+p_{10} = 2N \quad (11)$$

and:

$$p_1+p_2+p_3+p_4 = p_5+p_6+p_7+p_8+p_9+p_{10} \quad (12)$$

Then:

$$p_1+p_2 = p_5+p_6+p_7+p_8+p_9+p_{10} - (p_3+p_4) \quad (13)$$

Let p_3, p_4 any odd prime numbers. Difference of six primes and the sums of the two indicated primes is even numbers from 4 to infinity since the sum of six is any even number, starting from 18. Thus, the right side of (13) is even numbers from 4 to infinity, which is equal to the sum of two odd primes and 2+2=4.

If p_1+p_2 not equal to any even number, then (13) and (12) become inequality, which is impossible, for which it is necessary that the sum of four prime numbers or and the sum of six prime numbers is not equal to a specific even number. Thus:

$$p_1+p_2 = 2N \quad (14)$$

Corollary 2. The difference of two odd prime numbers is any even number.

From what follows:

$$p_1+p_2 = p_3+p_4+2 \quad (15)$$

$$p_1-p_3=p_4-p_2+2 \quad (16)$$

and we assert:

$$p_1-p_2 =2N \quad (17)$$

where N=1,2.....∞

Corollary 3. If the sum of four simple odd, then the sum two-even number from 6 to 2N-6 inclusive.

What follows from the solved Goldbach–Euler conjecture.

Corollary 4. The difference of the sums of pairs of two primes is any even number.

According to Lemma 1 and Corollary 3, we have:

$$p_1+p_2+p_3+p_4+2 = p_5+p_6+p_7+p_8 \quad (18)$$

$$p_1+p_2-p_7-p_8 +2= p_5+p_6 - p_3-p_4 \quad (19)$$

Q.E.D

Task 2. Twin primes are infinite

Note: under prime numbers, so as not to repeat, further implied odd prime number

Theorem 2. Starting from 16, even numbers are the sum of two odd primes not less than two different representations.

For task 1:

$$p_1+p_2+p_3+p_4 = p_5+p_6 = 2N \quad (1)$$

Based on Corollary 1, let $p_1 = p_5$

$$2N-p_1 = p_6 \quad (2)$$

$$2N-p_2 \neq p_7 \quad (3)$$

(2)+(3):

$$p_1+p_2+p_6+p_7 \neq 4N \quad (4)$$

which contradicts Lemma 1 and is inevitable:

$$p_1+p_5=p_6+p_2=2N \quad (5)$$

where $N \geq 8$
Let:

$$p_5= p_2+p_3+p_4 \quad (6)$$

$$p_6 \neq p_1+p_3+p_4 \quad (7)$$

where $p_1 \neq p_2$, then:

$$p_6-p_1 \neq p_3+p_4 \quad (8)$$

and according to Corollary 2:

$$p_6-p_1=2N_2 \quad (9)$$

and Theorem 1:

$$p_3+p_4=2N_2 \quad (10)$$

Hence, it follows that (8) for any sum of two primes has the possibility to be equality and (7) is an inevitable equality. Note that in this case:

$$p_5-p_2=p_6-p_1=p_3+p_4 \quad (11)$$

$$p_5+p_1=p_6+p_2=2N \quad (12)$$

where $N \geq 8$
3+3+5+3+3+5 = 22, and taking into account two common ones, then in four simple ones, it is 16. Thus, any even number, starting from 16, is the sum of two primes, as shown above.
Q.E.D.
Corollary 5. The number of twins is infinite.
Corollary 4 is a special case of the above theorem
Let p_1, p_2 a pair of twins. Then according to (12) p_5, p_6 inevitably next set of twins. Next, instead of p_1, p_2, we substitute in (12) p_5, p_6 we have the next pair, etc. Hence, the process is endless and there is no finite pair of twins!

CONCLUSION

The solution of these problems opens up opportunities for solving a number of problems in number theory.

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