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**RESEARCH ARTICLE**

# Application of new Decomposition Method (AKDM) for the solution of SIR model and Lotka-voltra (Prey-Predator) model

***Mulugeta Andualem**

*Department of Mathematics, Bonga University, Bonga, Ethiopia*

**Corresponding Author: mulugetaandualem4@gmail.com**

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## ABSTRACT

Solving nonlinear problems analytically are somewhat difficult because of their variety of natures. Therefore, researchers are always searching for new techniques (analytic, approximate or numerical) to solve such nonlinear initial value, boundary value, or mixed problems. Based on such a motivation, in this work, we have developed a new method known as Andualem and Khan Decomposition Method (AKDM) for the solution of (SIR) model and Lotka-voltra model. We also used mat lab to analysis the series solution of both models graphically.

**Keywords:** AK transform, SIR model, Lotka-voltra model, Adomain decomposition method, System of differential equation

## INTRODUCTION

An equation that consists of derivatives is called a differential equation. Differential equations have applications in all areas of science and engineering. Mathematical formulation of most of the physical and engineering problems lead to differential equations [1]. Most of the problems that arise in the real world are modelled by nonlinear differential equation. A mathematical model is a description of a real situation using mathematical concepts [2-4]. The process of creating a mathematical model for a given problem is called mathematical modeling. Mathematical modeling involves using mathematics to describe a problem-context and determine meaningful solutions to the problem. Many mathematical models relate to real life problems and that are interdisciplinary in nature [5].There are lots of different techniques to solve differential equations numerically. In order to solve the differential equations, the integral transform was extensively used and thus there are several works on the theory and application of integral transform such as the Laplace, Fourier, Mellin, and Hankel, Fourier Transform, Sumudu Transform, Elzaki Transform and ZZ transform Aboodh Transform[6-9].
New integral transform, named as AK Transformation [10] introduce by Mulugeta Andualem and Ilyas Khan [2022], AK transform was successfully applied to fractional differential equations. AK is an integral transform which helpful in solving differential equations, due to the properties simplify its computation. The majority of this study here deals with for the solution of (SIR) model and Lotka-voltra model using the combination of AK and Adomian Decomposition (AD). The organization of the paper is mentioned in the following format: In the first part we describe the new method and fundamental

theorems of AK transform method and Adomain decomposition method in order to solve the nonlinear differential equations. Secondly, some examples are presented to illustrate the efficiency of AK transform. Finally, we give the conclusion.

## 1. AK Transform

A new transform called the AK Transformof the function $y(t)$ belonging to a class $A$, where:

$$A = \left\{ y(t): \exists\, N, \eta_1, \eta_2 > 0, |y(t)| < Ne^{\frac{|t|}{\eta_i}}, if\ t \in (-1)^i \times [0\ \infty) \right\}$$

The AK transform of $y(t)$denoted by $M_i[y(t)] = \bar{y}(t,s,\beta)$and is given by:

$$\bar{y}(t,s,\beta) = M_i\{y(t)\} = s\int_0^\infty y(t)e^{-\frac{s}{\beta}t}\,dt \tag{1}$$

**Some Properties AK transform**

1) If $y(t) =$ any constant $(c)$, then
$$M_i\{c\} = s\int_0^\infty ce^{-\frac{s}{\beta}t}\,dt = cs\int_0^\infty e^{-\frac{s}{\beta}t}\,dt = cs\left[-\frac{\beta}{s}e^{-\frac{s}{\beta}t}\right]_0^\infty = c[\beta] = c\beta$$
2) If $y(t) = t$, then using integration by part, we have
$$M_i\{t\} = s\int_0^\infty te^{-\frac{s}{\beta}t}\,dt = s\left[\left[-\frac{\beta}{s}te^{-\frac{s}{\beta}t}\right]_0^\infty + \frac{\beta}{s}\int_0^\infty e^{-\frac{s}{\beta}t}\,dt\right] = \frac{\beta^2}{s}$$
3) If $y(t) = t^n$, then
$$M_i[t^n] = \frac{n!\,\beta^{n+1}}{s^n} = \Gamma(n+1)\frac{\beta^{n+1}}{s^n}$$
4) If $y(t) = e^{at}$, then using substitution the method substitution, we have
$$M_i\{e^{at}\} = s\int_0^\infty e^{at}e^{-\frac{s}{\beta}t}\,dt = s\int_0^\infty e^{-t\left(-a+\frac{s}{\beta}\right)}\,dt$$
$$= -\frac{s\beta}{s-a\beta}\left[e^{-t\left(-a+\frac{s}{\beta}\right)}\right]_0^\infty = \frac{s\beta}{s-a\beta}$$
5) If $y(t) = \sin t$, then
$$M_i[\sin t] = \frac{s\beta^2}{\beta^2+s^2}$$

**AK transform on first and second derivatives are mentioned as below**:

$$M_i\left[\frac{df(t)}{dt}\right] = \int_0^\infty s\frac{df(t)}{dt}e^{-\frac{s}{\beta}t}dt = s\lim_{\eta\to\infty}\int_0^\eta e^{-\frac{s}{\beta}t}\frac{df(t)}{dt}dt$$

Using integration by part, we have

$$M_i\left[\frac{df(t)}{dt}\right] = \lim_{\eta\to\infty}\left(s[e^{-\frac{s}{\beta}t}f(t)]_0^\eta\right) + \frac{s^2}{\beta}\int_0^\infty e^{-\frac{s}{\beta}t}f(t)dt$$

$$= -sf(0) + \frac{s}{\beta}\bar{f}(s,\beta)$$

$$= \frac{s}{\beta}\bar{f}(s,\beta) - sf(0) \quad (2)$$

Next, to find $M_i\left[\frac{d^2f(t)}{dt^2}\right]$, let $\frac{df(t)}{dt} = h(t)$then by using equation (2) we have

$$M_i\left[\frac{d^2f(t)}{dt^2}\right] = M_i\left[\frac{dh(t)}{dt}\right] = \int_0^\infty s\frac{dh(t)}{dt}e^{-\frac{s}{\beta}t}dt = s\lim_{\eta\to\infty}\int_0^\eta e^{-\frac{s}{\beta}t}\frac{dh(t)}{dt}dt$$

$$M_i\left[\frac{d^2f(t)}{dt^2}\right] = \frac{s}{\beta}M_i[h(t)] - sh(0)$$

Where, $\bar{h}(s,\beta)$ is AK transform of $h(t)$. Since $h(t) = \frac{df(t)}{dt}$ consequently we have $M_i[h(t)] = \bar{h}(s,\beta) = \frac{s}{\beta}\bar{f}(s,\beta) - sf(0)$

$$M_i\left[\frac{d^2f(t)}{dt^2}\right] = \frac{s}{\beta}\left(\frac{s}{\beta}\bar{f}(s,\beta) - sf(0)\right) - s\frac{df(0)}{dt}$$

$$M_i\left[\frac{d^2f(t)}{dt^2}\right] = \frac{s^2}{\beta^2}\bar{f}(s,\beta) - \frac{s^2}{\beta}f(0) - s\frac{df(0)}{dt}$$

## 2. Application of AK decomposition for nonlinear DEs

The AK Adomian Decomposition Method is a powerful tool to search for solution of various nonlinear problems. In this section we tried to solve SIR and Lotka-voltra model which is system of nonlinear first order ordinary differential equations using the combination of AK transform method and adomain decomposition method.

Consider the general form of first order inhomogeneous nonlinear differential equation equations is given below

$$Lu(t) + Ru(t) + Nu(t) = g(t) \quad (3)$$

with initial conditions $u(0) = b$

Where:

$u$is the unknown function

$L$ The higher order ordinary differential equations (in our case first order ordinary differential equations) and is given by $L = \frac{d}{dt}$

$R$is the reminder of the differential operator

$g(t)$ is nonhomogeneous term and

$N(u)$is the nonlinear term.

Now, taking the AK transform both sides of eq. (3), we get

$$M_i[Lu(t)] + M_i[Ru(t)] + M_i[Nu(t)] = M_i[g(t)]$$

Using the differentiation property of AK transform we get

$$\frac{s}{\beta}\bar{u}(s,\beta) - su(0) + M_i[Ru(t)] + M_i[Nu(t)] = M_i[g(t)]$$

Substituting the given initial condition from equation

$$\frac{s}{\beta}\bar{u}(s,\beta) - sb + M_i[Ru(t)] + M_i[Nu(t)] = M_i[g(t)]$$

Multiplying both sides by $\frac{\beta}{s}$

$$\bar{u}(s,\beta) - \beta b\frac{\beta}{s}M_i[Ru(t)] + \frac{\beta}{s}M_i[Nu(t)] = \frac{\beta}{s}M_i[g(t)]$$

$$\bar{u}(s,\beta) = \frac{\beta}{s}M_i[g(x,t)] + \beta b - \frac{\beta}{s}M_i[Ru(t)] - \frac{\beta}{s}M_i[Nu(t)] \quad (4)$$

The second step in AK Decomposition Method is that we represent solution as an infinite series given below

$$u = \sum_{m=0}^{\infty} u_m(x,t) \quad (5)$$

The nonlinear operator $Nu = \Psi(u)$is decompose as

$$Nu(x,t) = \sum_{m=0}^{\infty} A_n \quad (6)$$

Where, $A_n$is called Adomian's polynomial. This can be calculated for various classes of nonlinearity according to:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}[\Psi\left(\sum_{i=0}^{n}\lambda^i u_i\right)]_{\lambda=0}$$

By substituting (5) and (6) in (4) we will get

$$\sum_{m=0}^{\infty} M_i[u_n(x,t)] = \frac{\beta}{s}M_i[g(t)] + \beta b + -\frac{\beta}{s}M_i[Ru(t)] - \frac{\beta}{s}M_i\sum_{m=0}^{\infty} A_n \quad (7)$$

Now, if we compare both sides of equation (7), we can get the following recurrence relation:

$$M_i[u_0(t)] = \beta b + \frac{\beta}{s}M_i[g(t)] \quad (8)$$

$$M_i[u_1(t)] = -\frac{\beta}{s}M_i\big(Ru_0(t)\big) - \frac{\beta}{s}M_i[A_0] \quad (9)$$

$$M_i[u_2(x,t)] = -\frac{\beta}{s}M_i\big(Ru_1(t)\big) - \frac{\beta}{s}M_i[A_1] \quad (10)$$

In general the recurrence relation is given by

$$M_i[u_{n+1}(x,t)] = -\frac{\beta}{s}M_i\big(Ru_n(t)\big) - \frac{\beta}{s}M_i[A_n], \quad n \geq 0 \quad (11)$$

Applying inverse AK transform to (8) and (11), the required recursive relation is given below

$$\begin{cases} u_0(t) = M_i^{-1}\left[\beta p(x) + \frac{\beta^2}{s} q(x) + \frac{\beta^2}{s^2} M_i[g(x,t)]\right] = M_i^{-1}[F(s,\beta)] = F(t) \\ \qquad [u_{n+1}(x,t)] = -M_i^{-1}\left[\frac{\beta}{s} M_i\big(Ru_n(t)\big) + \frac{\beta}{s} M_i[A_n]\right], \quad n \geq 0 \end{cases} \tag{12}$$

Where $F(t)$represents the term arising from the source term and the prescribed initial conditions.

**SIR Model**

The most basic model that describes whether or not an epidemic will occur and how it occurs in a population is the SIR epidemic model. It was first developed by Kermack and Mckendrick in 1927 (Britton, 2003; Murray, 2004; Ellner and Guckenheimer, 2006). Modification of this model exist in literature, examples of this can be found in a book by Hethcote (Hethcote, 2000), Dieckmann and Heesterbeek (Dieckmann and Heesterbeek, 2000), Anderson and May (Anderson and May, 1992), and Murray (Murray, 2004).

Consider a disease for which the population can be placed into three compartments:

- ❖ S(t) the susceptible compartment, who can catch the disease;
- ❖ I(t) the infective compartment, who have and transmit the disease;
- ❖ R(t) the removed compartment, who have been isolated, or who have recovered and are immune to the disease, or have died due to the disease during the course of the epidemic.

Then the equations describing the time evolution of numbers in the susceptible, infective and removed compartments are given by

$$\begin{cases} \dfrac{dS}{dt} = -rIS \\ \dfrac{dI}{dt} = rIS - aI \\ \dfrac{dR}{dt} = aI \end{cases}$$

With initial condition $S(0) = S_0, I(0) = I_0, R(0) = R_0$

**Lotka-voltra model**

The Lotka-Volterra model is a pair of differential equations representing the populations of a predator and prey species which interact with each other. The model was independently proposed in 1925 by American statistician Alfred J. Lotka and Italian mathematician Vito Volterra.

**Biological model**

Let's consider two species namely rabbit and fox in a given ecosystem

**Biological assumption**

a) The number of rabbit population increase by their own growth rate
b) The number of rabbit population decrease as they eaten by foxes
c) The number fox decrease in the absence of rabbit
d) The number of fox increase in the presences of rabbit

Let $x =$ the number of rabbit population (prey)

$y =$the fox population (predator)

$\frac{dx}{dt}$ =the rate of change of the number of rabbit with respect to $t$

$\frac{dy}{dt}$ =the rate of change of the number of fox with respect to $t$

$$\begin{cases} \frac{dx}{dt} = ax - bxy \\ \frac{dy}{dt} = -dy + cxy \end{cases}$$

Where $a$ birth rate of rabbit

$b$ Contact rate of rabbit and fox

$c$ Conversion rate

$d$ Death rate of the fox

**a) Numerical Application of SIR model**

Consider SIR model with initial condition $I(0) = 10, S(0) = 15, R(0) = 5$ and parameters$r = 0.3, a = 0.1$. The SIR model can be rewritten as:

$$\begin{cases} \frac{dS}{dt} = -0.3IS \\ \frac{dI}{dt} = 0.3IS - 0.1I \\ \frac{dR}{dt} = 0.1I \end{cases} \tag{13}$$

With initial condition $S(0) = 15, I(0) = 10, R(0) = 5$

Taking the AK transform on both sides of equation (13)

$$M_i\left[\frac{dS}{dt} = -0.3IS\right]$$
$$M_i\left[\frac{dI}{dt} = 0.3IS - 0.1I\right]$$
$$M_i\left[\frac{dR}{dt} = 0.1I\right]$$

$$\begin{cases} \frac{s}{\beta}\bar{S}(s,\beta) - sS(0) = -0.3M_i[IS] \\ \frac{s}{\beta}\bar{I}(s,\beta) - sI(0) = 0.3M_i[IS] - 0.1M_i[I] \\ \frac{s}{\beta}\bar{R}(s,\beta) - sR(0) = 0.1M_i[I] \end{cases}$$

Using the given initial condition we get

$$\begin{cases} \frac{s}{\beta}\bar{S}(s,\beta) - 15s = -0.3M_i[IS] \\ \frac{s}{\beta}\bar{I}(s,\beta) - 10s = 0.3M_i[IS] - 0.1M_i[I] \\ \frac{s}{\beta}\bar{R}(s,\beta) - 5s = 0.1M_i[I] \end{cases}$$

$$\Rightarrow \begin{cases} \bar{S}(s,\beta) = 0.999\beta - 0.3\frac{\beta}{s}M_i[IS] \\ \bar{I}(s,\beta) = 0.001\beta + 0.3\frac{\beta}{s}M_i[IS] - 0.1\frac{\beta}{s}M_i[I] \\ \bar{R}(s,\beta) = 0.1\frac{\beta}{s}M_i[I] \end{cases}$$

By taking the inverse of AK transform, we get

$$\bar{S}(s,\beta) = 15\beta - 0.3\frac{\beta}{s}M_i[IS]$$

$$\bar{I}(s,\beta) = 10\beta + 0.3\frac{\beta}{s}M_i[IS] - 0.1\frac{\beta}{s}M_i[I]$$

$$\bar{R}(s,\beta) = 5s + 0.1\frac{\beta}{s}M_i[I]$$

$$\begin{cases} {M_i}^{-1}[\bar{S}(s,\beta)] = {M_i}^{-1}\left(15\beta - 0.3\frac{\beta}{s}M_i[IS]\right) \\ {M_i}^{-1}[\bar{I}(s,\beta)] = {M_i}^{-1}\left(10\beta + 0.3\frac{\beta}{s}M_i[IS] - 0.1\frac{\beta}{s}M_i[I]\right) \\ {M_i}^{-1}(\bar{R}(s,\beta)) = 5 + {M_i}^{-1}\left[0.1\frac{\beta}{s}M_i[I]\right] \end{cases}$$

Suppose the solutionof the unknown function is given as an infinite sum of the form

$$\begin{cases} \sum_{n=0}^{\infty} S_n(t) = 15 - {M_i}^{-1}\left[\frac{\beta}{s}M_i\left(0.3\sum_{n=0}^{\infty} A_n\right)\right] \\ \sum_{n=0}^{\infty} I_n(t) = 10 + {M_i}^{-1}\left[\frac{\beta}{s}M_i\left(0.3\sum_{n=0}^{\infty} A_n - 0.1\sum_{n=0}^{\infty} I_n\right)\right] \\ \sum_{n=0}^{\infty} R_n(t) = 5 + {M_i}^{-1}\left[\frac{\beta}{s}M_i\left(0.1\sum_{n=0}^{\infty} R_n\right)\right] \end{cases}$$

Where, $A_n$is called Adomian's polynomial of the nonlinear term $I(t)S(t)$.Consequently, we can write

$$S_0 = 15, \qquad I_0 = 10, \qquad R_0 = 5$$

$$S_1 = -{M_i}^{-1}\left[\frac{\beta}{s}M_i\left(0.3\sum_{n=0}^{\infty} A_0\right)\right]$$

$$S_2 = -{M_i}^{-1}\left[\frac{\beta}{s}M_i\left(0.3\sum_{n=0}^{\infty} A_2\right)\right]$$

$\vdots$

Finally, we have the general recurrence relation in the following way

$$S_{n+1} = -{M_i}^{-1}\left[\frac{\beta}{s}M_i(0.3\textstyle\sum_{n=0}^{\infty} A_n)\right], n \geq 0 \tag{14}$$

$$I_1 = {M_i}^{-1}\left[\frac{\beta}{s}M_i\left(0.3\sum_{n=0}^{\infty}A_0 - 0.1\sum_{n=0}^{\infty}I_0\right)\right]$$

$$I_2 = {M_i}^{-1}\left[\frac{\beta}{s}M_i\left(0.3\sum_{n=0}^{\infty}A_1 - 0.1\sum_{n=0}^{\infty}I_1\right)\right]$$

$\vdots$

Finally, we have the general recurrence relation in the following way

$$I_{n+1} = {M_i}^{-1}\left[\frac{\beta}{s}M_i(0.3\textstyle\sum_{n=0}^{\infty}A_n - 0.1\sum_{n=0}^{\infty}I_n)\right], n \geq 0 \qquad (15)$$

and

$$R_1 = {M_i}^{-1}\left[\frac{\beta}{s}M_i\left(0.1\sum_{n=0}^{\infty}R_0\right)\right]$$

$$R_2 = {M_i}^{-1}\left[\frac{\beta}{s}M_i\left(0.1\sum_{n=0}^{\infty}R_1\right)\right]$$

$\vdots$

Finally, we have the general recurrence relation in the following way

$$R_{n+1} = {M_i}^{-1}\left[\frac{\beta}{s}M_i(0.1\textstyle\sum_{n=0}^{\infty}R_n)\right], n \geq 0 \qquad (16)$$

The first few components are thus determined as follows:

$$S_1 = -{M_i}^{-1}\left[\frac{\beta}{s}M_i(0.3A_0)\right]$$

$$= -{M_i}^{-1}\left[\frac{\beta}{s}M_i(0.3S_0I_0)\right]$$

$$= -{M_i}^{-1}\left[\frac{\beta}{s}M_i(0.3 \times 150)\right]$$

$$= -{M_i}^{-1}\left[45\beta \times \frac{\beta}{s}\right] = -45t$$

$$I_1 = {M_i}^{-1}\left[\frac{\beta}{s}M_i(0.3A_0 - 0.1I_0)\right]$$

$$= {M_i}^{-1}\left[\frac{\beta}{s}M_i(44)\right] = 44t$$

$$R_1 = {M_i}^{-1}\left[\frac{\beta}{s}M_i(0.1R_0)\right] = 0.5t$$

$$S_2 = -{M_i}^{-1}\left[\frac{\beta}{s}M_i(0.3A_1)\right]$$

$$S_2 = -{M_i}^{-1}\left[\frac{\beta}{s}M_i(63t)\right]$$

$$= -{M_i}^{-1}\left(63\frac{\beta^3}{s^2}\right) = \frac{63}{2}t^2$$

$$I_2 = {M_i}^{-1}\left[\frac{\beta}{s}M_i(0.3A_1 - 0.1I_1)\right]$$

$$= {M_i}^{-1}\left[\frac{\beta}{s}M_i\left(\frac{293}{5}t\right)\right]$$

$$= {M_i}^{-1}\left(\frac{293}{5}\frac{\beta^3}{s^2}\right) = \frac{293}{10}t^2$$

$$R_2 = {M_i}^{-1}\left[\frac{\beta}{s}M_i(0.1R_1)\right]$$

$$= {M_i}^{-1}\left(0.05\frac{\beta^3}{s^2}\right) = 0.025t^2$$

$$S_3 = -{M_i}^{-1}\left[\frac{\beta}{s}M_i(0.3A_2)\right]$$

$$= -{M_i}^{-1}\left[-\frac{\beta}{s}M_i(367.65t^2)\right]$$

$$= 735.3{M_i}^{-1}\left(\frac{\beta^4}{s^3}\right) = 122.55t^3$$

$$I_3 = {M_i}^{-1}\left[\frac{\beta}{s}M_i(0.3A_2 - 0.1I_2)\right]$$

$$= {M_i}^{-1}\left[-\frac{\beta}{s}M_i(370.58t^2)\right]$$

$$= -741.16{M_i}^{-1}\left(\frac{\beta^4}{s^3}\right) = -123.53t^3$$

$$R_3 = {M_i}^{-1}\left[\frac{\beta}{s}M_i(0.1R_2)\right]$$

$$= {M_i}^{-1}\left[\frac{\beta}{s}M_i(0.0025t^2)\right]$$

$$= 0.005{M_i}^{-1}\left[\frac{\beta^4}{s^3}\right] = \frac{0.005}{6}t^3$$

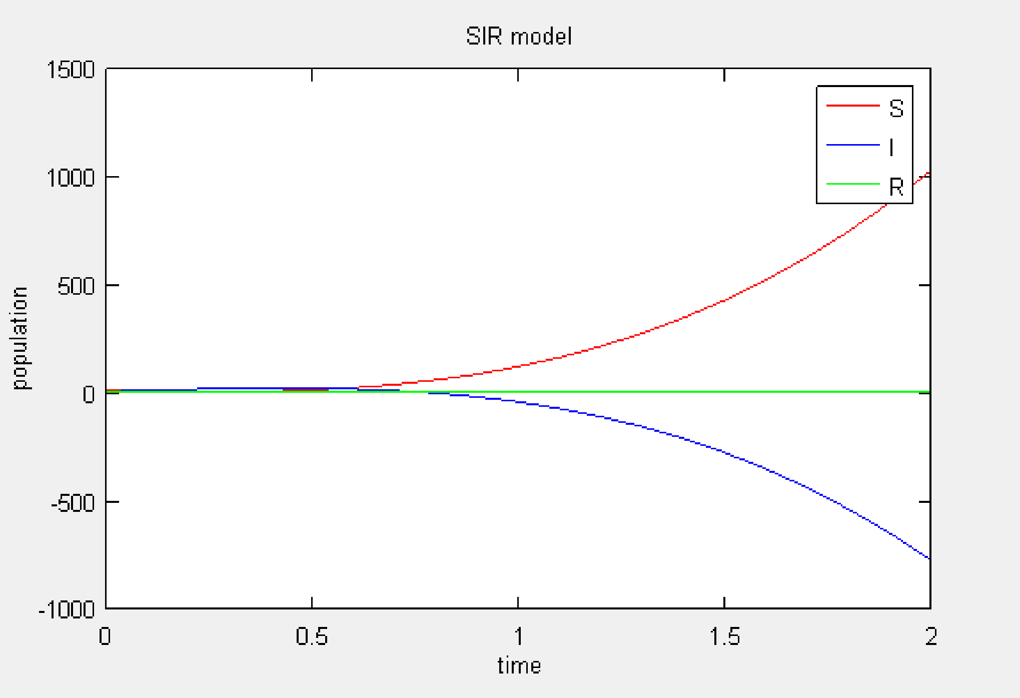
And so on…

Collect all these solutions together we obtained

$$S(t) = 15 - 45t + 31.5t^2 + 122.55t^3 + \cdots$$

$$I(t) = 10 + 44t + 29.3t^2 - 123.53t^3 + \cdots$$

$R(t) = 5 + 0.5t + 0.025t^2 + 0.00083t^3 + \cdots$




From the above graph it can be seen clearly that during the period of epidemic with a constant population as the number of susceptible individual increases so also the number of infectious individual decreases and vice versa while the number of immune individual increases.

**b) Numerical Application of Lotka-voltra model**

Consider the following Lotka-voltra model with $a = 1, b = 0.5, c = 0.25$ and $d = 0.75$

$$\begin{cases} \frac{dx}{dt} = x(1 - 0.5y) \\ \frac{dy}{dt} = y(-0.75 + 0.25x) \end{cases} \quad (17)$$

With initial conditions $x(0) = 2, \quad y(0) = 1$

Taking the AK transform both sides of equation (17), we have

$$\begin{cases} M_i\left[\frac{dx}{dt} = x(1 - 0.5y)\right] \\ M_i\left[\frac{dy}{dt} = y(-0.75 + 0.25x)\right] \end{cases}$$

Using the differentiation property of AK transform we get

$$\begin{cases} \frac{s}{\beta}\bar{x}(s,\beta) - sx(0) = M_i(x(1 - 0.5y)) \\ \frac{s}{\beta}\bar{y}(s,\beta) - sy(0) = M_i(y[-0.75 + 0.25x]) \end{cases}$$

Substituting the given initial condition, leads to

$$\begin{cases} \frac{s}{\beta}\bar{x}(s,\beta) - 2s = M_i(x(1 - 0.5y)) \\ \frac{s}{\beta}\bar{y}(s,\beta) - s = M_i(y[-0.75 + 0.25x]) \end{cases}$$

$$\Rightarrow \begin{cases} \bar{x}(s,\beta) = 2\beta + \frac{\beta}{s} M_i(x(1-0.5y)) \\ \bar{y}(s,\beta) = \beta + \frac{\beta}{s} M_i(y[-0.75+0.25x]) \end{cases}$$

By applying inverse AK transform we get

$$\begin{cases} x(t) = 2 + M_i^{-1}\left[\frac{\beta}{s} M_i(x(1-0.5y))\right] \\ y(t) = 1 + M_i^{-1}\left(\frac{\beta}{s} M_i(y[-0.75+0.25x])\right) \end{cases} \quad (18)$$

Now let an infinite series solution of the unknown function $x(t)$ and $y(t)$ be

$x(t) = \sum_{n=0}^{\infty} x_n(t)$ and $y(t) = \sum_{n=0}^{\infty} y_n(t)$ (19)

By using equation (19), we can write equation (18) in the form

$$\begin{cases} \sum_{n=0}^{\infty} x_n(t) = 2 + M_i^{-1}\left[\frac{\beta}{s} M_i\left(\sum_{n=0}^{\infty} x_n(t) - 0.5\sum_{n=0}^{\infty} A_n\right)\right] \\ \sum_{n=0}^{\infty} y_n(t) = 1 + M_i^{-1}\left[\frac{\beta}{s} M_i\left(-0.75\sum_{n=0}^{\infty} y_n(t) + 0.25\sum_{n=0}^{\infty} A_n\right)\right] \end{cases}$$

$A_n$is called adomian's polynomial of the nonlinear term. Consequently, we get the first three adomian's polynomial components as follows:

$A_0 = x_0 y_0, \quad A_1 = x_1 y_0 + x_0 y_1, \qquad A_3 = x_0 y_2 + x_1 y_1 + x_2 y_0$

And the recurrence relation is given as follow:

$x_0 = 1$

$$x_1 = M_i^{-1}\left[\frac{\beta}{s} M_i(x_0 - 0.5A_0)\right]$$

$$x_2 = M_i^{-1}\left[\frac{\beta}{s} M_i(x_1 - 0.5A_1)\right]$$

Finally, we have the general recurrence relation in the following way

$$x_{m+1} = M_i^{-1}\left[\frac{\beta}{s} M_i(x_m - 0.5A_m)\right] \quad m \geq 0$$

And

$y_0 = 2$

$$y_1 = M_i^{-1}\left[\frac{\beta}{s} M_i(-0.75y_0 + 0.25A_0)\right]$$

$$y_2 = M_i^{-1}\left[\frac{\beta}{s} M_i(-0.75y_1 + 0.25A_1)\right]$$

$$y_{m+1} = M_i^{-1}\left[\frac{\beta}{s}M_i(-0.75y_m + 0.25A_m)\right] \qquad m \geq 0$$

The first few components are thus determined as follows:

$$x_0 = 1, \qquad y_0 = 2$$

$$x_1 = M_i^{-1}\left[\frac{\beta}{s}M_i(x_0 - 0.5A_0)\right]$$

$$= M_i^{-1}\left[\frac{\beta}{s}M_i(1 - 1)\right] = 0$$

$$y_1 = M_i^{-1}\left[\frac{\beta}{s}M_i(-0.75y_0 + 0.25A_0)\right]$$

$$= M_i^{-1}\left[\frac{\beta}{s}M_i(-1.5 + 0.5)\right]$$

$$= -M_i^{-1}\left[\frac{\beta^2}{s}\right] = -t$$

$$x_2 = M_i^{-1}\left[\frac{\beta}{s}M_i(0.5t)\right]$$

$$= M_i^{-1}\left[0.5\frac{\beta^3}{\beta^2}\right] = \frac{t^2}{4}$$

$$y_2 = M_i^{-1}\left[\frac{\beta}{s}M_i(0.75t - 0.25t)\right]$$

$$= M_i^{-1}\left[0.5\frac{\beta^3}{s^2}\right] = \frac{t^2}{4}$$

$$x_3 = M_i^{-1}\left[\frac{\beta}{s}M_i(0.5t^2 - 0.75t^2)\right]$$

$$= -0.25M_i^{-1}\left[\frac{\beta^4}{s^3}\right] = -\frac{0.25t^3}{6}$$

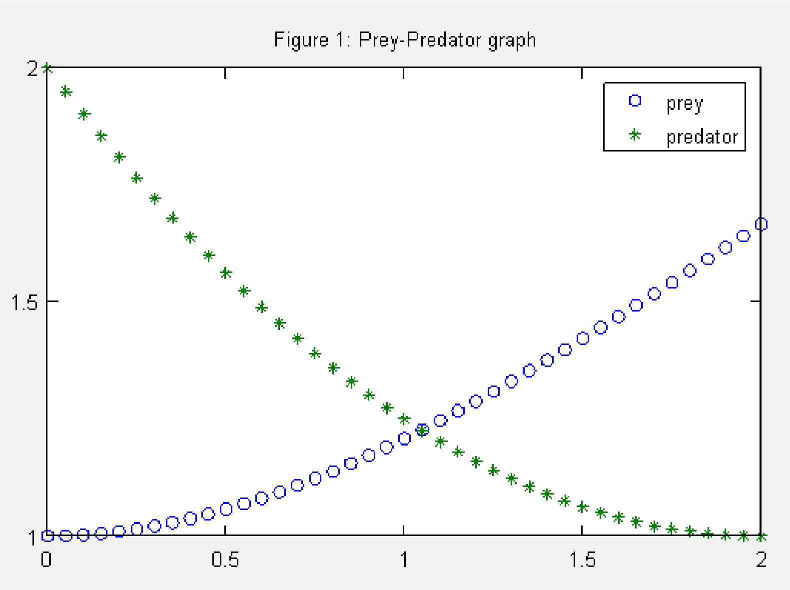
$$y_3 = M_i^{-1}\left[\frac{\beta}{s}M_i(-0.375t^2 - 0.375t^2)\right] = 0$$

Since the approximate solution is given by:

$$x(t) = \sum_{n=0}^{\infty} x_n(t) = x_0 + x_1 + x_2 + x_3 + \cdots = 1 + 0.25t^2 - 0.0417t^3 + \cdots$$

and

$$y(t) = \sum_{n=0}^{\infty} y_n(t) = y_0 + y_1 + y_2 + y_3 + \cdots 2 - t + 0.25t^2 + \cdots$$




From the above graph it can be seen clearly that the number of prey (rabbit) individual increases due to the absence of predator or natural growth rate and also the number of predator individual (fox) decreases due to natural death rate (the absence of prey).

## CONLUSION

New integral transform method called AK has been developed, and also some fundamental theorems of this new method are provided. From this study we show the use of a new method called AK transform for the solution of (SIR) model and Lotka-voltra model. Since both models are nonlinear system of ordinary differential equation, in order to attack the nonlinear part we used Adomain decomposition method. We also used mat lab to analysis the series solution of both models graphically.

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