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RESEARCH ARTICLE

K-SUPER CONTRA HARMONIC MEAN LABELING OF GRAPHS

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ABSTRACT

In Let $f:V(G)\rightarrow\{k,k+1,k+2,...,k+p+q-1\}$ be an injective function. Then the induced edge labeling $f^*(e=uv)$ defined by $f^*(e)=\left\lceil\frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3}\right\rceil or \left\lfloor\frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3}\right\rfloor$, then f is called k-Super heronian Mean labeling, if $\{f(V(G))\}\cup\{f(e)/e\in E(G)\}=\{k,k+1,k+2,...,k+p+q-1\}$.
A graph which admits k-super heronian mean labeling is called a k-super heronian mean graph. In this paper we introduce and study k-super heronian mean labeling graph. Here k denoted as any positive integer greater than or equal to 1.

Keywords: k-Super Lehmer-3 mean Labeling, *k*-Super Lehmer-3 mean graph, triangular snake, double triangular snake, Alternative triangular snake, quadrilateral snake, double quadrilateral snake, Alternative quadrilateral snake.

INTRODUCTION

A graph considered here are finite, undirected and simple. Let $G(V, E)$ be a graph with p vertices and q edges. For standard terminology and notations, we follow[3]. For detailed survey of graph labeling we refer to Gallian [1].The concept of Super Lehmer-3 Mean Labeling was introduced and studied by[4] and also studied [2]. In this paper we introduce the concept of *k*-Super Lehmer-3 Mean Labeling and we investigate the *k*-Super Lehmer-3 meanness of triangular snake, Double triangular snake, Alternative triangular snake, quadrilateral snake, Double quadrilateral snake and Alternative quadrilateral snake.

MAIN RESULTS

Definition 2.1:

Let $f:V(G)\to\{1,2,...,p+q\}$ be an injective function. Then the induced edge labeling $f^*(e=uv)$ defined by

$$f^*(e)=\left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rceil or \left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rfloor$$

,then f is called Super lehmer 3-mean labeling, if $\{f(V(G))\}\cup\{f(e)/e\in E(G)\}=\{1,2,\ldots,p+q\}$. A graph which admits super lehmer 3-mean labeling is called a super lehmer 3-mean graph.

Definition 2.2:

Let $f:V(G)\to\{k,k+1,k+2,...,k+p+q-1\}$ be an injective function. Then the induced edge labeling

$f^*(e=uv)$ defined by $f^*(e)=\left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rceil or \left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rfloor$,

then f is called *k*-Super lehmer 3-mean labeling,

if $\{f(V(G))\}\cup\{f(e)/e\in E(G)\}=\{k,k+1,k+2,...,k+p+q-1\}$.

A graph which admits *k*-super lehmer 3-mean labeling is called a *k*-super lehmer 3-mean graph.

Theorem 2.3:

The triangular snake $T_n(n\geq 2)$ is a *k*-Super Lehmer-3 Mean graph for any *k*.

Proof

Let $\{v_i,1\leq i\leq n,u_i,1\leq i\leq n-1\}$ be the vertices and $\{e_i,1\leq i\leq n,a_i,1\leq i\leq 2(n-1)\}$ be the edges.

First we label the vertices as follows:

For $1\leq i\leq n,\ f(v_i)=k+5(i-1)$

For $1\leq i\leq n-1,\ f(u_i)=k+5i-3$

Then the induced edge labels are:

For $1\leq i\leq n-1,\ f^*(e_i)=k+5i-2$

For
$$1\leq i\leq 2(n-1),\ f^*(a_i)=\begin{cases}\frac{2k+5i-3}{2} & i \text{ is odd}\\ \frac{2k+5i-2}{2} & i \text{ is even}\end{cases}$$

Therefore, the edge labels are all distinct. Hence the triangular snake $T_n(n\geq 2)$ is a *k*-super lehmer-3 mean graph for any *k*.

Theorem 2.4:

The double triangular snake $D(T_n)(n \geq 2)$ is a *k*-Super Lehmer-3 Mean graph for any *k*.

Proof

Let $\{v_i, 1 \leq i \leq n, u_i, w_i\, 1 \leq i \leq n-1\}$ be the vertices and $\{e_i, 1 \leq i \leq n, a_i, b_i\, 1 \leq i \leq 2(n-1)\}$ be the edges.

First we label the vertices as follows:

For $1 \leq i \leq n,\ f(v_i) = k + 8(i-1)$

For $1 \leq i \leq n-1,\ f(u_i) = k + 8i - 6, \qquad f(w_i) = k + 8i - 2$

Then the induced edge labels are:

For $1 \leq i \leq n-1,\ f^*(e_i) = k + 8i - 4$

For $1 \leq i \leq 2(n-1)$,

$$f^*(a_i) = \begin{cases} k + 4i - 3 & i \text{ is odd} \\ k + 4i - 5 & i \text{ is even} \end{cases}$$

$$f^*(b_i) = \begin{cases} k + 4i + 1 & i \text{ is odd} \\ k + 4i - 1 & i \text{ is even} \end{cases}$$

Therefore, the edge labels are all distinct. Hence the double triangular snake $D(T_n)(n \geq 2)$ is a *k*-super lehmer-3 mean graph for any *k*.

Theorem 2.5:

The alternative triangular snake $A(T_n)(n \geq 2)$ is a *k*-super lehmer-3 mean graph for any *k*.

Proof

Let $\left\{v_i, 1 \leq i \leq n, u_i,\ 1 \leq i \leq \frac{n}{2}\right\}$ be the vertices and $\{e_i, 1 \leq i \leq n-1,\ a_i, 1 \leq i \leq n\}$ be the edges.

First we label the vertices as follows:

For $1 \leq i \leq n,\ f(v_i) = k + 2(i-1)$

For $1 \leq i \leq \frac{n}{2},\ f(u_i) = k + 2n + 3(i-1)$

Then the induced edge labels are:

For $1 \leq i \leq n-1,\ f^*(e_i) = k + 2i - 1$

For $1 \leq i \leq n$,

$$f^*(a_i) = \begin{cases} \dfrac{2k + 4n + 3i - 5}{2} & i \text{ is odd} \\ \dfrac{2k + 4n + 3i - 4}{2} & i \text{ is even} \end{cases}$$

$$f^*(b_i) = \begin{cases} k+4i+1 & i \text{ is odd} \\ k+4i-1 & i \text{ is even} \end{cases}$$

Therefore, the edge labels are all distinct. Hence the alternative triangular snake $A(T_n)(n \geq 2)$ is a *k*-super lehmer-3 mean graph for any *k*.

Theorem 2.6:

The quadrilateral snake $Q_n(n \geq 2)$ is a *k*-Super Lehmer-3 Mean graph for any *k*.

Proof

Let $\{v_i, 1 \leq i \leq n, u_i, w_i \; 1 \leq i \leq n-1\}$ be the vertices and $\{a_i, b_i, c_i, 1 \leq i \leq n-2, \; e_i, 1 \leq i \leq n-1\}$ be the edges.

First we label the vertices as follows:

For $1 \leq i \leq n, f(v_i) = k+7(i-1)$

For $1 \leq i \leq n-1, f(u_i) = k+7i-5 \qquad f(w_i) = k+7i-2$

Then the induced edge labels are:

For $1 \leq i \leq n-2, f^*(a_i) = k+7i-6$

$f^*(b_i) = k+7i-1 \qquad f^*(c_i) = k+7i-3$

For $1 \leq i \leq n$, $f^*(e_i) = k+7i-4$

Therefore, the edge labels are all distinct. Hence the quadrilateral graph $Q_n(n \geq 2)$ is a *k*-super lehmer-3 mean graph for any *k*.

Theorem 2.7:

The Alternative quadrilateral snake $A(Q_n)(n \geq 2)$ is a *k*-Super Lehmer-3 Mean graph for any *k*.

Proof

Let $\{v_i, w_i, 1 \leq i \leq n\}$ be the vertices and $\left\{a_i, 1 \leq i \leq n, \; e_i, 1 \leq i \leq n-1, b_i, 1 \leq i \leq \frac{n}{2}\right\}$ be the edges.

First we label the vertices as follows:

For $1 \leq i \leq n, f(v_i) = k+2(i-1)$

For $1 \leq i \leq n$, $$f(u_i) = \begin{cases} \dfrac{2k+4n+5(i-1)}{2} & i \text{ is odd} \\ \dfrac{2k+4n+5i-6}{2} & i \text{ is even} \end{cases}$$

Then the induced edge labels are:

For $1 \leq i \leq n-1, f^*(e_i) = k+2i-6$

For $1 \leq i \leq n$, $$f^*(a_i) = \begin{cases} \dfrac{2k+4n+5i-7}{2} & i \text{ is odd} \\ \dfrac{2k+4n+5i-4}{2} & i \text{ is even} \end{cases}$$

For $1 \leq i \leq \frac{n}{2}, \; f^*(b_i) = k+2n+5i-2$

Therefore, the edge labels are all distinct. Hence the Alternative quadrilateral snake $A(Q_n)(n \geq 2)$ is a *k*-Super Lehmer-3 Mean graph for any *k*.

Theorem 2.8:

The Double quadrilateral snake $D(Q_n)(n \geq 2)$ is a *k*-super lehmer-3 mean graph for any *k*.

Proof

Let $\{v_i, 1 \leq i \leq n, u_i, w_i, 1 \leq i \leq 2(n-1)\}$ be the vertices and $\{e_i, a_i, c_i\, 1 \leq i \leq n-1, b_i, d_i, 1 \leq i \leq 2(n-1)\}$ be the edges.

First we label the vertices as follows:

For $1 \leq i \leq n,\ f(v_i) = k + 12(i-1)$

For $1 \leq i \leq 2(n-1),\quad f(u_i) = \begin{cases} k+6i-4 & i \text{ is odd} \\ k+6i-8 & i \text{ is even} \end{cases}$

$$f(w_i) = \begin{cases} k+6i+2 & i \text{ is odd} \\ k+6i-2 & i \text{ is even} \end{cases}$$

Then the induced edge labels are:

For $1 \leq i \leq n-1,\ f^*(e_i) = k + 12i - 6 \quad f^*(c_i) = k + 12i - 3$

For $1 \leq i \leq 2(n-1),\quad f^*(b_i) = \begin{cases} k+6i-5 & i \text{ is odd} \\ k+6i-7 & i \text{ is even} \end{cases}$

$$f^*(d_i) = \begin{cases} k+6i-2 & i \text{ is odd} \\ k+6i+2 & i \text{ is even} \end{cases}$$

Therefore, the edge labels are all distinct. Hence the Double quadrilateral snake $D(Q_n)(n \geq 2)$ is a *k*-super lehmer-3 mean graph for any *k*.

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