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RESEARCH ARTICLE

GOLDBACH'S PROBLEMS

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ABSTRACT

The Goldbach-Euler binary problem is formulated as follows: Any even number, starting from 4, can be represented as the sum of two primes. The ternary Goldbach problem is formulated as follows: Every odd number greater than 7 can be represented as the sum of three odd primes, which was finally solved in 2013. [1]-[8]. In 1995, Olivier Ramare proved that any even number is the sum of no more than 6 primes.[9]

Keywords: solving, problems, in, number, theory

INTRODUCTION

Theorem. Difference between any odd number and a prime odd number is equal to any even number and vice versa the difference of any even number and of a prime odd number is equal to any odd number.

Proof.

$$2K+1-p=2N \qquad (01)$$

where

$$K=K_{1,}K_1+1,\dots,K_i=K_1+i-1,\dots\infty$$

$$N=N_{1,}N_1+1,\dots,N_i=N_1+i-1,\dots\infty$$

p is a prime odd number,

j-serial number of a continuous series of natural numbers, starting accordingly with K_1, N_1

K and N are an infinite, continuous series of integers that begin with

K_1, N_1 , p -any prime number(fixed value, some constant).
Thus we have (01). And similarly:

$$2N - p = 2K + 1 \quad (02)$$

the difference of any even and odd numbers andconversely allow to represent any prime odd number.

Corollary1.

If the sum of six primes is any even number, then the sum primes less than six if odd, any odd number, if even any even number with corresponding initial values $N_{1,} K_{1.}$

From the equality of the sum of six primes to any even number it follows:

$$p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + 2 = p_7 + p_8 + p_9 + p_{10} + p_{11} + p_{12} \quad (03)$$

$$p_1 + p_2 + p_3 + p_4 + p_5 + p_6 - p_{12} + 2 = p_7 + p_8 + p_9 + p_{10} + p_{11} \quad (04)$$

$$2N - p_{12} + 2 = p_7 + p_8 + p_9 + p_{10} + p_{11} \quad (05)$$

$$2K + 1 = p_7 + ... + p_{11} \quad (06)$$

$$p_7 + p_8 + p_9 + p_{10} + p_{11} + 2 = p_{12} + p_{13} + p_{14} + p_{15} + p_{16} \quad (07)$$

…

(the index under p is not critical)

$$p_1 + p_2 + p_3 + p_4 + 2 = p_5 + p_6 + p_7 + p_8 \quad (08)$$

$$p_1 + p_2 + p_3 + p_4 - p_8 + 2 = p_5 + p_6 + p_7 \quad (09)$$

$$p_5 + p_6 + p_7 = 2K + 1 \quad (10)$$

where K=3,4,5..., ∞

Weak Goldbach problem.

$$p_1 + p_2 + p_3 + 2 = p_4 + p_5 + p_6 \quad (11)$$

$$p_1 + p_2 + p_3 - p_4 + 2 = p_5 + p_6 \quad (12)$$

$$p_5 + p_6 = 2N \quad (13)$$

where N=2,3,..., ∞

Strong Goldbach problem. Based on the corollary, we solve the following problems.

Twin primes are infinite

Theorem2. Starting from 14, even numbers are the sum of two odd primes not less than two different representations.

$p_1 + p_2 + p_3 + p_4 = p_1 + p_5 = 2N$ (01)

$p_2 + p_3 + p_4 = p_5$ (02)

Assume by analogy with (02):

$p_1 + p_3 + p_4 = p_6$ (03)

add up (02)+(03):

$p_5 + p_6 = p_1 + p_2 + 2(p_3 + p_4)$ (04)

according to Corollary1 :

$2(p_3 + p_4) = p_7 + p_8 = 4N_j$ (05)

where $4N_j$ is a fixed even number.

and

$p_5 + p_6 = p_1 + p_2 + p_7 + p_8$ (06)

and further :

$p_5 + p_6 - p_7 = p_1 + p_2 + 4N_j - p_7$ (07)

$p_1 + p_2 + 2p_3 + 2p_4 = p_1 + p_2 + 4N_j$ (08)

and finally:

$p_3 + p_4 = 2N_j$ (09)

corresponds to Corollary1, which confirms Assumption (03). (03),(04) - inequality in case (09) is not equal to the correspondingcertain even number $2N_j$ with respect to 2N.

However redistribution by replacing simple $p_{1,}\ p_2\ , p_3\ , p_4$ we find an even $2N_j$ for $p_1 \neq p_2$, which means two representations by the sum of two prime for even 2N.

Let's say $p_1 = p_2$, then $2p_5 = 2N$ .Introducing an even through the sum of four simple ones:

$p_5 + p_1 + p_3 + p_4 = 2p_5$ (10)

$p_5 = p_1 + p_3 + p_4$ (11)

$p_6 = p_5 + p_3 + p_4$ (12)

Thus we have $p_5 \neq p_6$, $p_1 \neq p_2$.

The second representation would be absent if there were even numbers that cannot be represented as the sum of two prime numbers.

From this follows:

$p_5 + p_1 = p_6 + p_2 = 2$ N (13)

where N= 7, 8, 9,10.., ∞

As a result, even numbers starting with 16 are the sum of two prime numbers, at least than two presentations. Up to 16 we determine arithmetically -6,8,12 in one presentations. Hence the values of N.

Corollary 2: The number of twins is infinite. Corollary 2 is a special case of the above theorem

Let $p_{1,} p_2$ a pair of twins.
Then according to (13) $p_{5,} p_6$ inevitably next set of twins.
Next, instead of $p_{1,} p_2$, we substitute in (13) $p_{5,} p_6$ we have the next pair, etc. So the process is endless and there is no finite pair of twins!

Corollary 3: A prime number starting at 5 is the arithmetic mean of two simple .

According to Corollary1:

$p_1 + p_2 = 2\ p_3$ (14)

where is one representation of an even number, indicating different values in (02) and (03).

And the second representation of an even number:

$p_3 + p_3 = 2\ p_3$ (15)

and (11) confirms the infinity of primes!

REFERENCES:

1. Terence Tao Google+- Busy day in analytic number theory; – Harald. Helfgott has…
2. Major arcs for Goldbach`s, H.A. Helfgott // arxiv 1305.2897
3. Goldbach Variations// SciAm blogs, Evelyn Lamb, May 15, 2013.
4. Two Proofs Spark a Prime Week for Number Theory // Science 24 May, 2013.

5. Vol. 340 no. 6135 p. 913. Doi:10.1126/Science.340.6135.913

6. Weisstein, Eric W. Goldbach Conjecture(English.)on the site Wolfram MathWorld.

7. Yuri Matiyasevich. Hilbert’s Tenth Problem: What was done and what is to be done.

8. de Polignac A. Recherches nouvelles sur les nombres premiers. 1849 .

9. en.wikipedia.org//wiki/Olivier_Ramar%C3%A9