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SUITABILITY OF COINTEGRATION TESTS ON DATA STRUCTURE OF DIFFERENT ORDERS

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ABSTRACT

When selecting a method to evaluate theories about the relationship between two variables that have a unit root or, it is necessary to consider the potential existence of cointegration. If the relationship exists between the two variables, it should be able to forecast one variable based on the other, which is why cointegration is significant for time series data including many variables. Using the three approaches, this research investigates the cointegration processes and integration level. Determine whether the time series is stationary and if there is a seasonal effect before looking at cointegration in a combination of variables. A time series plot is used to monitor patterns and the time series data's behaviors. Applying the log transformation and differencing approach will make the data stationar. The data was then subjected to the Augmented Dickey Fuller (ADF) test, which verifies whether or not a unit-root exists by following a unit-root procedure. In the event that the series lacks a unit root process, the data may be considered stationary. The analysis techniques used in the research include the Granger Causality Test, Johansen test, Phillips-Ouliarisco integration test, Engle–Granger two-step method, and simple correlation and regression analysis. R statistical software was used for all of the analyses on a time series data set containing these variables. In conclusion, the results of the three tests indicate cointegration, with the Phillips–Ouliaris test being the most effective whether the sample size is small, medium, or big, respectively, for both normal and gamma distributions. Engle–Granger and Johansen tests are then optimal. Additionally, it was noted that as correlation confidence levels rose, so did the strength of the determination of the cointegration across the correlation.

Keywords: Unit root, Cointegration, Simulation, Integrating order

INTRODUCTION

Cointegration is important in time series data that involve more than one variable due to the fact that if relationship between two variables holds, it is possible to predict one from another, that is, for example, if markets move together in the long-run, this hypothesis will hold (Akeyede et al, 2018)[(1)]. Cointegration is a statistical property associated with a collection of time series variables $(X_1, X_2, \ldots, X_k)$. First, all the series must be integrated with order d, and if a linear combination of this collection is integrated of order less than d, then the collection is said to be co-integrated. Formally, if (X, Y, Z) are each integrated with order d, and there exist coefficients a, b, c such that $aX + bY + cZ$ is integrated with order less than d, then $X,\ Y, and\ Z$ are said to be cointegrated (Adeleke *et al;* 2018)[2].

Invariably, if two or more series are individually integrated (in the time series sense) but some linear combination of them has a lower order of integration, then the series are said to be cointegrated. A common example is where the individual series are first-order integrated $I(1)$but some (cointegrating) vector of coefficients exists to form a stationary linear combination of them. For instance, a stock market index and the price of its associated futures contract move through time, each roughly following a random walk. Testing the hypothesis that there is a statistically significant connection between the futures price and the spot price could be done by testing for the existence of a cointegrated combination of the two series (Born and Demetrescu, 2015)[3].

Cointegration is an important property in contemporary time series analysis which often have either deterministic or stochastic trends. Kasa, 1992[4] provided statistical evidence that many US macroeconomic time series like GNP, wages, employment, etc. have stochastic trends. Cointegration has many implications for both financial theory and for portfolio management of the individual investor. Cointegration has also implications on the individual investor, in order to hedge risk, investors diversify their portfolios by investing in assets traded in different categories. If cointegration between variables is present, their indices will behave in a similar way in the long-run and give similar returns (French and Poterba, 1991[5]; Richards, 1995[6]).

METHODOLOGY

This paper examines the cointegration procedures and level of integration using the three methods. Before examining cointegration in combination of variables, it is necessary to identify whether the time series is stationary and whether it has any seasonal effect. Time series plot is used to track trends and the behaviors of the time series data. The stationarity of data can be achieved by applying differencing method and log transformation. Augmented Dickey Fuller (ADF) test was then applied to conform the stationarity of data, this test follows a unit-root process and the test indicates whether unit-root exist or not. If the series does not have a unit root process, the data can be taken as stationary. The paper employs, simple correlation and regression analysis, Engle–Granger two-step method, Johansen test and Phillips–Ouliariscointegration test as well as Granger Causality Test as methods for analyses. All the analyses were carried out for a time series data with these variables using R statistical software.

1.1 Source of Data

Data used for this paper was fully simulated from the most commonly continuous distribution that are generally related to real life situations. The distributions to considered in this paper are the Normal and Gammadistributions. The simulation was carried out for 3 sample sizes. The data was generated from different variables and non-stationarity was imposed on every data generated such that it has to be integrated once, twice or three times before it attains a stationarity status. In every case, the test of non-stationarity (ADF test), was applied to ensure the status on every data generated before the level of cointegration between the variables is checked. This was assessed based on the underline distribution at every sample size.

1.2 Parameter and Sample Size Fixed for Simulation.

Parameter was fixed for every stage of simulation in such a way that the assumption of stationarity in terms of parameters will be violated. Using systematic sampling, the sample sizeconsidered for every case of simulation are 30, 60 and 90 to ensure the performance of different methods of cointegration test from small sample sizes to large sample sizes.

2.3 Method of Analysis

Each of the three tests of cointegration (Engle–Granger two-step method, Johansen test and Phillips–Ouliariscointegration test) was used to analyze the simulated data from normal, exponential, gamma and uniform distribution at different sample sizes. The number of times a test wrongly rejects the two-hypothesis fixed (type I error) was counted and recorded in tables. More so, the number of times a test is accepting true alternative hypothesis values (power) was also counted and recorded. These was repeated for all the test statistics under study on each simulated data and sample size.

2.4 Criteria for Assessment

The test with the lowest type I error and/or highest power was classified as the most robust test to a distribution at a particular sample size. The robustness of the tests was measured based on type I error (proportion of a test in rejection of a fixed cointegration) and power of the test (proportion of a test in rejection of a fixed cointegration) using p-values. The one with lowest type I error and highest power was considered as the robust test. Other Criteria that was used are adjusted R and integrated order.

2.5 Concept of Stationarity and Unit Root

It is important to distinguish between stationary and non-stationary time series, as well as weak and strict stationarity. This is relevant for cointegration analysis between related variables, as we expect some set of similar data to be non-stationary. A time series is considered strictly stationary if the probability distribution of its values does not change over time as shown in the equation below (Brooks, 2008)[7]:

$$f(y_{t1}, y_{t2}, \dots, y_{m}) = f(y_{t1+k}, y_{t2+k}, \dots, y_{tm+k})$$

The concept of strict stationarity implies that all higher-order moments are constant, including mean and variance. However, strict stationary time series are rarely found in practice. Therefore, the study will focus on weakly stationary processes in further analysis. Conditions and assumptions of weak stationary processes are sufficient to be regarded as stationary. A time series is considered weak stationary when mean, variance and autocovariance are constant over time (Enders, 2008)[9].

On the other hand, the properties of non-stationary time series change over time. For this type of time series, mean and variance have different values at different time-points. Its variance will increase as sample size tends to infinity (Harris and Sollis, 2003)[9].

The stationary conditions can simply be shown by using a simple autoregressive (AR) process: $y_t = \mu + \rho y_{t-1} + e_t$
where the current value of variable $y_t$depends on the constant term μ, value of the variable y from last period t-1 and an error term e_t . The interest is in the value of ρ which indicate whether the process is stationary or non-stationary.

There are three possible cases that could occur, or three possible values of ρ, (Brooks, 2008)[7]:

i. $|\rho| < 1$; a shock to the system in current time period t is temporary; it will die away over time and this series is stationary. It has constant mean, variance and autocovariance. A stationary time series will return to its mean value in the long run.
ii. $\rho = 1$; a shock in time period t which will not die away over time, it is permanent and its variance approaches infinity over time. This time series is regarded as non-stationary, better known as the unit root case. The variable y contains a unit root.
iii. $\rho > 1$; a shock in time period t will explode over time and this sort of time series is also non-stationary. There is no mean reversion to its true value over time.

2.6 Augmented Dickey – Fuller Test (ADF Test) For Unit – Root

The ADF test is used to test for unit root, the testing procedure for the ADF test is the same as for the Dickey-Fuller test but it is applied to the model. A random walk a stochastic or random process, that describes a path that consists of a succession of random steps on some mathematical space such as the integers. A random walk with drift and trend is represented as; $Y_t = \alpha + Y_{t-1} + \beta_t + e_t$
where α is a constant, β the coefficient on a time trend and e the lag order of the autoregressive process. Imposing the constraints $\alpha = 0$ and $\beta = 0$ corresponds to modelling a random walk and using the walk with a drift.
The test statistic, value is calculated as follows: $t = \frac{\hat{Y}}{\sigma\hat{Y}}$

where$\hat{Y}$ is the estimated coefficient and $\sigma\hat{Y}$ is the standard error in the coefficient estimate.

The null – hypothesis for an ADF test: Ho: ϒ = 0 Vs H_1 : ϒ < 0

Where Ho: is the null hypothesis (has unit root) and H_1: Does not have unit root. The test statistics value t is compared to the relevant critical value for the Dickey-Fuller test. If the test statistic is less than the critical value, we reject the null hypothesis and conclude that no unit – root is present. The ADF test does not directly test for stationarity but indirectly through the existence (or absence) of a unit – root.
Decision rule:

If t^*> ADF critical value = do not reject null hypothesis, that is, unit root exists.
If t^*< ADF critical value = reject null hypothesis, that is, unit root does not exist. Using the usual 5% threshold, differencing is required if the p – value is greater than 0.05.

2.7 Concept of Cointegration

The concept of cointegration has its roots in the work of Engle and Granger (1987)[10]. Two variables are cointegrated if they share a common stochastic trend in the long-run. The general rule when combining two integrated variables is that their combination will always be integrated at the higher of the two orders of integration. The most common order of integration in time series is either zero or one (Brooks, 2008)[7];

1. If $y_t \sim I(0), and\ x_t \sim I(0) y_t \sim I(0)$, then their combination $ax_t + by_t$will also be $I(0)$.
2. If $y_t \sim I(0), and\ x_t \sim I(0)$, then their combination $ax_t + by_t$will now be $I(1)$, because $I(1)$ is higher order of integration and dominates the lower order of integration $I(0)$,
3. If $y_t \sim I(1), and\ x_t \sim I(1)$, then their combination $ax_t + by_t$ will also be $I(1)$, in the general case.

However, if there exists such linear combination of non-stationary variables $I(1)$ that is stationary, $I(0)$, cointegration between those variables exists. The following regression model includes two I(1) non-stationary variables $y_t$and x_t:$y_t = \mu + \beta x_t + e_t$

If the OLS estimate is such that the linear combination of $y_t$and $x_t$stationary, these two variables are cointegrated. The error term between them is constant over time (stationary):$e_t = y_t - \beta x_t$
In order for two variables to be cointegrated they need to be integrated of the same order. For example, if one variable is $I(0)$ and the other one is $I(1)$, they cannot be cointegrated. The highest order of integration of the two variables will dominate and cointegration will not exist. However, if there is a linear combination of the stock indices that is stationary, cointegration between them exists.

2.8 The Engle-Granger test

The Engle-Granger test is a single-equation method used to determine whether there is a cointegrating relationship between two variables (Engle and Granger, 1987)[11]. The precondition to examine cointegration is that the variables are both non-stationary and integrated of the same order. The Engle-Granger (EG) method can be performed by following the next four step procedure:

Step 1: Perform the ADF test as demonstrated in 3.1.1 to pretest for the order of integration. If the variables are both $I(1)$, cointegration is theoretically possible and we can proceed to step 2. If the variables are of different order, the conclusion is that cointegration is not possible.
Step 2: Estimate the long-run, static relationship or equilibrium by running the OLS regression on the general equation: $y_t = \mu + \beta x_t + e_t)$

This equation can be expanded with a constant term and a time trend, If the variables are cointegrated, an OLS regression will give a "super-consistent" estimator, denoted as $\hat{\beta}$, implying that the coefficient βwill converge faster to its true value than using OLS on stationary variables, $I(0)$. If there is a linear combination of variables $y_t$and $x_t$that is stationary, the variables are said to be cointegrated. This linear combination of the variables can then be presented with the estimated error term; $\hat{e}_t = y_t - \hat{\beta}x_t$

Step 3: Store the residuals $\hat{e}_t$ and examine whether they are stationary or not. Here an ADF test, as explained earlier, is performed on the saved residuals from every regression equation above. The hypotheses for the EG test for cointegration are:

$$H_0: \hat{e}_t - I(1) - \text{non} - \text{stationary residual and nocointegration between variables}$$
$$H_1: \hat{e}_t - I(0) - \text{stationary residual and cointegration between variables}$$

If the null hypothesis is rejected, the variables from the model are cointegrated. The test statistics is the same as the one used for the ADF test, but the critical values are different. Since the Engle-Granger method includes testing on estimated residuals ($\hat{e}_t$) instead of the actual values, the estimation error will change the distribution of the test statistics. Therefore, the critical values used in an Engle-Granger approach will be larger in absolute value, or more negative compared to those used in a DF or ADF test. This means that the magnitude of the test statistics must be much larger in order to reject the null hypothesis, compared to the usual DF critical values. Akeyede et al, (2018)[1] provide appropriate critical values for residual-based cointegration testing, depending on whether and which deterministic terms are included in the model.

Step 4: If cointegration is found between the variables, estimate an error-correction model. However, this will not be part of our analysis, since we are interested only in detecting cointegration.

Johansen Test
The Johansen test is a test for cointegration allows for more than one cointegrating relationship, unlike the Engle–Granger method, this test is subject to asymptotic properties, i.e. large samples. If the sample size is too small, then the results will not be reliable and one should use Auto Regressive Distributed Lags.

Phillips–OuliarisCointegration Test
Phillips (1986)[11] show that residual-based unit root tests applied to the estimated cointegrating residuals do not have the usual Dickey–Fuller distributions under the null hypothesis of no-cointegration. Because of the spurious regression phenomenon under the null hypothesis, the distribution of these tests has asymptotic distributions that depend on;

1. The number of deterministic trend terms and.
2. The number of variables with which co-integration is being tested.

These distributions are known as Phillips–Ouliaris distributions and critical values have been tabulated. In finite samples, a superior alternative to the use of these asymptotic critical value is to generate critical values from simulations.

RESULTS AND DISCURSIONS

The data obtained at every category were analysed to check if the data is stationary or has a unit root using Augmented Dickey Fuller (ADF), and therefore check for cointegration among the variables using Engle Granger method, Johansen test and Phillips–Ouliariscointegrationmethods for analyses.

3.1 Testing for Unit Root/ Stationarity in the Generated data

The stationarity/unit root test was carried out on data whose error terms are generated from normal, exponential, gamma and uniform distributions using Augmented Dickey Fuller (ADF). The statistic tests the null hypothesis that the data series has a unit root with the alternative that the data series is stationary.

Table 1: Results of Unit Root Tests on the Two Generated Data Sets

Variable	Sample Size(T)	First Variable (X)				Second Variable (Y)			
Distribution	Sample Size	Values	Lag Order	P-value	Remark	Values	Lag Order	P-value	Remark
Normal	30	-3.2391	3	0.0989	N/S	-3.128	3	0.1391	NS
	60	-2.8138	3	0.2458	N/S	-2.288	3	0.4584	NS
	90	-4.4813	3	0.01	N/S	-3.138	3	0.0985	NS
Gamma	30	-24.134	9	0.01	N/S	-16.45	9	0.01	NS
	60	-9.7953	9	0.01	N/S	-5.810	9	0.01	NS
	90	-8.352	9	0.01	N/S	-9.179	9	0.01	NS

NS implies Not Stationary

Table 1 shows the unit root test of the set of data simulated under different underlined distributions, normal and gamma distributions at sample sizes of 30, 60 and 90 respectively which small, moderate and large sample sizes. It was observed from the table that most of the p-values from normal distributions except for gamma distribution are greater than 5% and therefore accept the null hypothesis of data generated being have a unit root except those that generated with error term being normal.Therefore, the data series need to be differenced and differenced data are hereby carried out in the following section.

Table 2: Results of Unit Root Tests on the Two Sets of Data (Differenced Data)

Variable	Sample Size (T)	First Variable (X)					Second Variable (Y)		
Distribution	Sample Size	Values	Lag Order	P-value	Remark	Values	Lag Order	P-value	Remark
Normal	30	-16.30	9	0.01	NS	-15.59	9	0.01	NS
	60	-15.04	9	0.01	NS	-10.95	9	0.01	NS
	90	-13.24	9	0.01	NS	-12.26	9	0.01	NS
Gamma	30	-11.21	9	0.01	NS	-18.67	9	0.01	NS
	60	-17.15	9	0.01	NS	-20.36	9	0.01	NS
	90	-15.26	9	0.01	NS	-16.94	9	0.01	NS

NS implies non stationary

Table 2 above shows the ADF test for the differenced generated data at different sample sizes and other category of investigation with the null hypothesis of a unit root against an alternative of a level stationarity. The p-values of all cases of simulated data are less than the 1% level of significance which indicate that, the null hypothesis of having a unit root series should be rejected in favour of alternative of being stationary. Therefore, the differenced data series are considered to be stationary. We therefore proceed to determine the long run relationship between the variables using co-integration technique.

3.2 Cointegration Tests Comparison

Using the Eagle-Granger method, Johansen test and Phillips–Ouliaris cointegrationmethods, a pairwise analysis of two variables with different strength of relationship are carried out using the procedures for testing cointegration. We tested whether a linear combination of a pair variable is stationary. If it is found to be stationary, the two data set are cointegrated.

The performances of three tests of cointegration mentioned in section 2 are studied and compared when error term is distributed normal, gamma and This is carried out from low to high strength of correlation ($r = 0, 0.3, 0.6, , 0.9$) between the pair of variables at different sample sizes.

Table 3: Results of Cointegration Test when Error Term is Normal (T = 30)

Test	Eagle-Granger			Johansen test			Phillips–Ouliaris		
r	Test Value	P-value	Adjusted R-squared	Test Value	P-value	Adjusted R-squared	Test Value	P-value	Adjusted R-squared
0	0.2183	0.186	0.2182	1229.9	0.0141	0.4534	1057.7	0.0232	0.3966
0.3	0.177	0.1938	0.1937	742.53	0.0392	0.2348	665.83	0.0392	0.3899
0.6	0.052	0.0585	0.0585	701.33	0.0094	0.1475	454.5	0.0283	-0.5097
0.9	0.175	0.2381	0.2381	642.53	0.0078	0.1439	353.31	0.0021	0.5867

Table 3 shows the relative performance of Eagle-Granger, Johansen test and Phillips–Ouliaris in determining the cointegration of the pair of the data generated at different levels of correlations between the two variables when the sample size is 30. It was observed that, both Johansen test and Phillips–Ouliaris reject the hypothesis of no cointegration due to their p-values less than 5% while Eagle Granger do not reject the hypothesis. Hence, there is no cointegration based on Eagle Granger, whereas, there is cointegration based on the other two tests. It was also observed that the strength of determining the existence of the cointegration across the cointegration decreases as the levels of the correlation increases with Phillips–Ouliaris as the best and has the best fit as indicated by R^2 at all levels followed by Johnsen test.

Table 4: Results of Cointegration Test when Error Term is Normal (T = 60)

Test	Engle-Granger			Johansen test			Phillips–Ouliaris		
R	Test Statistics	P-value	Adjusted R-squared	Test Statistics	P-value	Adjusted R-squared	Test Statistics	P-value	Adjusted R-squared
0	14.052	0.0034	0.8695	959.04	0.0250	0.5690	669.17	0.0029	0.5726
0.3	12.621	0.0039	0.2818	782.99	0.0320	0.5002	1950.8	0.0006	0.7433
0.6	13.978	0.0039	0.2092	710.22	-0.032	0.5007	906.11	5.06e-5	0.7501
0.9	8.096	0.0041	0.2668	535.49	0.0446	0.4897	950.26	2.2e-5	0.7444

Table 4 presents the results of the three tests in determining the cointegration of the pair of the data generated at different levels of correlations between the two variables when the sample size is 60. The results in table 4.4 shows that all the three tests reject the hypothesis of no cointegration due to their p-values less than 5% in favour of the alternative that there is cointegration. Therefore, there is cointegration based on the three tests. However, the strength of determining the existence of the cointegration across the correlation levels decreases as the levels of the correlation increases by Engle-Granger and Johnsen tests. Phillips–Ouliaris seems to be the best as indicated by p-values and R^2 at all levels followed by Engle-Granger at this category.

Table 5: Results of Cointegration Test when Error Term is Normal (T = 90)

Test	Engle-Granger			Johansen test			Phillips–Ouliaris		
r	Test Statistics	P-value	Adjusted R-squared	Test Statistics	P-value	Adjusted R-squared	Test Statistics	P-value	Adjusted R-squared

0	13.385	0.0022	0.5024	889.83	0.0398	0.4782	744.01	0.994	-0.00101
0.3	5.681	0.0031	0.5777	585.10	0.0394	0.4115	1154.7	2.186e	0.0558
0.6	4.200	0.0033	0.5776	337.14	0.0450	0.4105	629.42s	2.2e	0.6637
0.9	4.015	0.010	0.5774	470.91	0.0514	0.4200	850.27	2.2e-16	0.2197

The relative performance of Engle-Granger, Johansen test and Phillips–Ouliaris in determining the cointegration of the pair of the data generated at different levels of correlations between the two variables when the sample size is 90 are shown table 5. All the three tests reject the null hypothesis of no cointegration and accept the alternative. Hence, they all show that cointegration exists. However, Phillips–Ouliaris has the best performance and is also observed that the strength of determining the existence of the cointegration across the correlation levels, decreases as the levels of the correlation increases.

Table 6: Results of Cointegration Test when Error Term is Gamma (T = 30)

Test	Engle-Granger			Johansen test			Phillips–Ouliaris		
R	Test Statistics	P-value	Adjusted R-squared	Test Statistics	P-value	Adjusted R-squared	Test Statistics	P-value	Adjusted R-squared
0	13.721	0.0058	0.3131	464.08	5.48e-05	0.2356	74.806	1.9e-03	0.2852
0.3	16.483	0.0001	0.3745	494.24	6.92e-07	0.3421	998.74	1.9e-03	0.3411
0.6	895.36	0.0001	0.4518	895.36	1.23 e-08	0.3765	995.93	1.9e-03	0.4341
0.9	895.36	0.0010	0.4918	913.75	1.97e-07	0.4412	1023.3	1.9e-03	0.4264

Table 6 shows the three tests of cointegration when the error term is distributed gamma. The test statistics, p-value and $R^2$of each test are recorded at all levels of correlation of the pair variable. It was observed that Johansen testperformed more than others in determine the cointegration between the pair variable at various correlation levels due to its smallest p-values. This followed by Phillips–Ouliaris**.** The performance of all the three tests improves as correlation level increases based on the p-vale and R^2.

Table 7: Results of Cointegration Test when Error Term is Gamma (T = 60)

Test	Engle-Granger			Johansen test			Phillips–Ouliaris		
r	Test Statistics	P-value	Adjusted R-squared	Test Statistics	P-value	Adjusted R-squared	Test Statistics	P-value	Adjusted R-squared
0	0.033	0.0001	0.1411	869.29	0.0093	0..1097	927.43	1.5e-07	0.02634
0.3	13.945	0.0001	0.2594	883.42	0.0075	0.1175	1365.74	1.5e-10	0.0010
0.6	13.822	0.0001	0.3412	451.98	0.0055	0.4586	829.93	2.2e-16	0.9915
0.9	12.585	0.0001	0.4294	471.18	0.0009	0.4598	66.704	4.4e-07	0.02432

The table 7 reveals the relative performance of the cointegration term when the error term is generated from gamma distribution and sample sizes is 60. The results show that the Phillips–Ouliaris is the best at various correlation levels followed by Engle-Granger. The performance of the three tests improve as correlation level increases.

Table 8: Results of Cointegration Test when Error Term is Gamma (T = 90)

Test	Engle-Granger			Johansen test			Phillips–Ouliaris		
R	Test Statistics	P-value	Adjusted R-squared	Test Statistics	P-value	Adjusted R-squared	Test Statistics	P-value	Adjusted R-squared
0	6.591	0.0001	0.41054	436.017	0.0086	0.4112	503.61	2.2e-16	0.8766
0.3	10.060	0.0001	0.5351	920.71	0.0034	0.6578	1028.84	1.17e-11	0.9440
0.6	6.606	0.0001	0.6192	570.28	0.0092	0.7854	508.192	2.2e-16	0.9763
0.9	8.969	0.0001	0.7918	535.65	0.0008	0.7867	817.442	2.2e-16	0.9894

The table 8 presents the relative performance of the cointegration term when the error term is generated from gamma distribution and sample sizes is 90. The results show that the Phillips–Ouliaris is the best at various correlation levels followed by Engle-Granger. The performances of the three tests improve as correlation level increases.

CONCLUSION

The relative performance of the Engle-Granger, Johansen test and Phillips–Ouliaris in determining the cointegration of the pair of the data generated at different levels of correlations between the two variables when the sample size is 30, 60 and 90 and error term is normal are carried out and presented in tables 3, 4 and 5 respectively. It was observed that, both Johansen test and Phillips–Ouliaris reject the hypothesis of no cointegration due to their p-values less than 5% while Engle-Granger do not reject the hypothesis. Hence, there is no cointegration based on Engle-Granger, whereas, there is cointegration based on the other two tests. It was also observed that the strength of determining the existence of the cointegration across the cointegration decreases as the levels of the correlation increases with Phillips–Ouliaris as the best and has the best fit as indicated by R^2 at all levels followed by Johnsen test. Therefore, there is cointegration based on the three tests. However, the strength of determining the existence of the cointegration across the correlation levels decreases as the levels of the correlation increases by Engle-Granger and Johnsen tests. Phillips–Ouliaris seems to be the best as indicated by p-values and R^2 at all levels followed by Engle-Granger at this category.The results of three tests of cointegration when the error term is distributed gamma are shown in Tables 6, 7 and 8 for sample size of 30, 60 and 90 respectively across various levels of correlation. It was observed that Johansen testperformed more than others in determine the cointegration between the pair variable at various correlation levels due to its smallest p-values. This followed by Phillips–Ouliaris when sample size is 30 while Phillips–Ouliaris is the best determinant at larger sample sizes of 60 and 90**.** The performance of all the three tests improves as correlation level increases based on the p-vale and R^2.
In conclusion, from the three tests, it shows that there is cointegration with Phillips–Ouliaris as the best followed by Engle-Granger and Johansen test when sample size is small, medium and large respectively for both normal and gamma distributions. It was also observed that the strength of determining the existence of the cointegration across the correlation increase as the levels of the correlation confidents increased.

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