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APPLICATION OF NON-LINEAR EVOLUTION STOCHASTIC EQUATIONS WITH ASYMPTOTIC NULL CONTROLLABILITY ANALYSIS

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ABSTRACT

This paper investigated system of stochastic differential equations with prominence on disparities of drift parameters. These problems were solved analytical by adopting the Ito's method of solution and three different investment solutions were obtained consequently. The necessary conditions were achieved which govern various drift parameters in assessing financial markets. Therefore, the impressions on each solution of investors in financial markets were analyzed graphically. Secondly, stock price data of Transco, LTD were analyzed which covariance matrix were considered and analysis were logically extended to stochastic vector differential equation where control measures were incorporated that would help in predicting different stock price processes, and the result obtained by exploring the properties of the fundamental matrix solution where asymptotic null controllability results were obtained by the singularity of the controllability matrix a function of the drift. Finally, the effects of the significant parameters of stochastic variables were successfully discussed.

Keywords: Stock Prices, Financial markets, Controllability , investors and volatility

INTRODUCTION

Generally, investments are ventures linked with risk which cannot be over emphasized. The human lives and day-to-day activities, are associated with risk; thus, risk is a determinant to effectively manage investment portfolios, because it is instrumental to the ascertainment of fluctuation or variations of returns on the stock and portfolio, which furnishes the investor a mathematical framework for investment decisions[1]. Bonds, stocks, property, etc, are all prototypes of the risk associated to securities. Nevertheless, because of the risk involved in the management of investment portfolios, insurance companies deemed it pertinent for lives, properties, etc, to be insured. In point of fact, insurance companies share third party in the management and control of their financial results. Risk transfer or risk sharing is the methodology employed by insurance firm on financial outcomes of its coverage duty in a number of ways with risk transfer agreement, risk among numerous insurance firms globally. therefore, in a situation of astronomical loses from financial situation as insurance company will not encounter risk, particularly, reinsurance means the division and distribution of risk. In general, risk is an established factor as long as humans are concerned, since we secure risky or riskless assets properly.

However, a finer way to model these factors is as the trajectory or path of a diffusion process defined on many basic or fundamental probability space, possessing the geometric Brownian motion, used as the standard reference model [2]. Modelling financial concepts cannot be exaggerated because of its numerous applications in science and technology. For instance, [3]analyzed the maximization of the exponential utility and the minimization

of the ruin probability and the results obtained displayed the same or kind of investments scheme or approach for zero interest rate [4] studied an optimal reinsurance and investment problem for insurer with jump diffusion risk process. [5,6] examined the risk reserved for an insurer and a reinsurer to follow Brownian motion with drift and applied optimal probability of survival problem under proportional reinsurance and power utility preference. Similarly, [7] studied the excess loss of reinsurance and investment in a financial market and obtained optimal strategies. [8]employed a problem of optimal reinsurance investment for an insurer having jump diffusion risk model when the asset price was control by a CEV model. [9]studied strategies of optimal reinsurance and investment for exponential utility maximization under different capital markets. [10]considered investment problem having multiple risky assets. [11]examined an optimal portfolio selection model for risky assets established on asymptotic power law behaviour where security prices follow a Weibull distribution. Therefore, so many scholars have written extensively on stock market prices such as [11-22], etc.

Nevertheless, Controllability is a qualitative property of dynamical system and is of particular importance in control theory. The concept of controllability has played an important role in the deterministic system theory. It is well known that controllability of deterministic equations is widely used in the analysis and design of a control system. Any control system is said to be controllable if every state corresponding to a process can be affected or controlled in respective time by some control signals. Roughly speaking, controllability generally mean that, it is possible to switch the dynamical system from any feasible past trajectory in the system behavior to any feasible future trajectory using a set of admissible controls after some finite time; that is, there are systems which are completely controllable. If the system is not completely controllable then one can try to prove different kinds of controllability such as approximate, null, local null and local approximate null controllability, etc.Controllability is an important property of a control system, and the controllability property plays a crucial role in many control problems such as, [23,24,25] Studied the stability and controllability analysis of stock market prices ;they developed stochastic vector differential equation with control measures. Results showed stock prices to be stable and asymptotic null controllability results were obtained. [26]worked on controllable kinematic reduction for mechanical system concepts, computational tools and example. They focused on the class of simple mechanical control systems with constraints and model them as connection control systems. They concluded that a number of interesting reduction and controllability conditions can be characterized in terms of a certain vector-valued quadratic form. [27]worked on relative controllability of non-linear systems with delays in state and control. Results showed that sufficient conditions were developed for the euclidean controllability of perturbed non-linear systems with time varying multiple delays in control with the perturbed function having implicit derivative with delays depending on both state and control variable using Dabo’s fixed point’s theorem.[28]

Also worked on Euclidean null controllability of linear systems with delays in state and control. Result showed that sufficient conditions were developed for the Euclidean controllability of linear systems with delays in state and in control. [29]studied the controllability of a class of under-actuated mechanical system with symmetry using exploit, the invariance of the controlled nonlinear dynamics to the group action (symmetry) to drive a set of reduced dynamics for the system. In their research they developed results based on geometric mechanics to study the controllability of a class of controlled under-actuated left invariant mechanical system on lie groups. [30]worked on the controllability of non-homonymic mechanical system with constrained controls. Results showed that the controllability condition obtained have a clear physical meaning. [31]also investigated the controllability of mechanical systems with allowance for the drive dynamics using the dynamics of control drives of mechanical systems. It was discovered that the control drives must tolerate sufficiently fast changes in the control output that is control forces.In the work of[32] studied controllability of mechanical systems in the class of controls bounded together with their derivatives. They applied controllability of the nonlinear dynamic and mechanical systems. They concluded that manipulation robot to be controllable, it is required that the control forces dominate over any other generalized forces such as weight or environmental resistance.

Earlier studies have therefore investigated similar problems but did not consider the disparities of drift parameters as well as controllability analysis. In particular, some studies, for instance [19],[20] and [24], etc. To the best of our knowedge this is the first study that has assessed disparities of drift parameters with control measures and its impacts in financial markets. Therefore , this paper extends the work of [22] in this dynamic area of mathematical finance.

MATHEMATICAL FORMULATION

A Stochastic Differential Equation is a differential equation with stochastic term. Therefore assume that $(\Omega, \mathrm{F}, \wp)$ is a probability space with filteration $\{f_t\}_t \geq 0$ and $W(t) = (W_1(t), W_2(t), ..., W_m(t))^T, t \geq 0$ an m-dimensional Brownian motion on the given probability space. We have SDE in coefficient functions of f and g *as* follows

$$dX(t) = f(t, X(t))dt + g(t, X(t))dZ(t),\ 0 \leq t \leq T,$$
$$X(0) = x_0,$$

where $T > 0,\ x_0$ is an n-dimensional random variable and coefficient functions are in the form $f : [0, \mathrm{T}] \times \mathbb{R}^n$ *and* $g : [0, T] \times \mathbb{R}^n \rightarrow \times \mathbb{R}^{n \times n}$. SDE can also be written in the form of integral as follows:

$$X(t) = x_0 + \int_0^t f(S, X(S))dS + \int_0^t g(S, X(S))dZ(S)$$

Where dX, dZ are terms known as stochastic differentials.The $\mathbb{R}^n$ is a valued stochastic process $X(t)$.

Theorem 1.1: let $T > 0,$ be a given final time and assume that the coefficient functions $f : [0, \mathrm{T}] \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ *and* $g : [0, T] \times \mathbb{R}^n \rightarrow \times \mathbb{R}^{n \times n}$ are continuous. Moreover, $\exists$ finite constant numbers λ *and* β such that $\forall t \in [0, \mathrm{T}]$ and for all $x, y \in \mathbb{R}^n$, the drift and diffusion term satisfly

$$\| f(t,x) - f(t,y) \| + \| \mathrm{g}(t,x) - g(t,y) \| \leq \lambda \| x - y \|,$$
$$\| f(t,x) \| + \| \mathrm{g}(t,x) \| - g(t, \mathrm{x}) \| \leq \beta (1 + \| x \|).$$

Suppose also that x_0 is any $\mathbb{R}^n$ -valued random variable such that $E(\| x_0 \|^2) < \infty.$ then the above SDE has a unique solution X in the interval $[0, T].$ Moreover, it satisfies $E\left(\sup_{0 \leq t \leq T} \| X(t) \|^2\right) < \infty$. the proof of the theorem 1.1 is seen in [23].

Theorem 1.2:(Ito's lemma). Let $f(S, t)$ be a twice continuous differential function on $[0, \infty) \times \mathrm{A}$ and let S_t denotes an Ito's process

$$dS_t = a_t dt + b_t dz(t), t \geq 0$$,

Applying Taylor series expansion of F gives:

$$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}\left(dS_t\right)^2 + \text{higer order terms } \left(h.\text{o}, \text{t}\right)$$,

So, ignoring h.o.t and substituting for dS_t we obtain

$$dF_t = \frac{\partial F}{\partial S_t}\left(a_t dt + bdz(t)\right) + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}\left(a_t dt + bdz(t)\right)^2$$

$$= \frac{\partial F}{\partial S_t}\left(a_t dt + bdz(t)\right) + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2} b_t^2 dt,$$

$$= \left(\frac{\partial F}{\partial S_t} a_t + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2} b_t^2\right) dt + \frac{\partial F}{\partial S_t} b_t dz(t)$$

More so, given the variable $S(\text{t})$ denotes stock price, then following GBM implies (5) and hence, the function $F\left(S,t\right)$,Ito's lemma gives:

$$dF = \left(\mu S \frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 F}{\partial S^2}\right) dt + \sigma S \frac{\partial F}{\partial S} dz(t)$$

Nevertheless, the stochastic analysis on the variations stock drift and it influences in financial markets is considered. The volatility dynamics and other drift coefficients of stock prices was taken to be constant throughout the trading days. The initial stock price which is assumed to follow different trend series was categorized the entire origin of stock dynamics is found in a complete probability space $\left(\Omega, \text{F}, \wp\right)$ with a finite time investment horizon $T > 0$. Therefore, we have the following system of stochastic differential equations below;

$$dX_t = -\beta\mu X_t dt + \sigma X_t dW_t^1 \quad (0.1)$$

$$dX_\phi = K \tanh X_\phi dt + \sigma X_\phi dW_t^2 \quad (0.2)$$

$$dX_\varpi = \left(-\beta\alpha + K \tanh\right) X_\varpi dt + \sigma X_\varpi dW_t^3 \quad (0.3)$$

where μ is an expected rate of returns on stock, σ is the volatility of the stock , dt is the relative change in the price during the period of time and $W_t^1 = W_t^2 = W_t^3$ is a Wiener process, β, K are constants and $\tanh$ is periodic events

METHOD OF SOLUTION

The propose model (1.1) - (1.3) consist of a system of variable coefficient system of stochastic differential equations whose solutions are not trivial. we solve equations independently as follows using Ito's theorem 1.2:

From (1.1) let $f(X_t, t) = \ln X_t$

Taking the partial derivative yields

$$\frac{\partial f}{\partial X_t} = \frac{1}{X_t}, \quad \frac{\partial^2 f}{\partial X_t^2} = \frac{-1}{X_t^2}, \quad \frac{\partial f}{\partial t} = 0 \tag{0.4}$$

According to Ito's gives:

$$df(X_t, t) = \sigma X_t \frac{\partial f}{\partial X_t} dW_t^1 + \left(-\beta\mu X_t \frac{\partial f}{\partial X_t} + \frac{1}{2}\sigma^2 X_t^2 \frac{\partial^2 f}{\partial X_t^2} + \frac{\partial f}{\partial t} \right) dt \tag{0.5}$$

Subtitling (1.4) into (1.5) gives

$$df(X_t, t) = \sigma X_t \frac{1}{X_t} dW_t^1 + \left(-\beta\mu X_t \frac{1}{X_t} + \frac{1}{2}\sigma^2 X_t^2 \left(-\frac{1}{X_t^2} \right) + 0 \right) dt \tag{0.6}$$

$$= \sigma \frac{X_t}{X_t} dW_t^1 + \left(-\beta\mu X_t \frac{X_t}{X_t} - \frac{1}{2X_t^2}\sigma^2 X_t^2 \right) dt = \sigma dW_t^1 + \left(-\beta\mu - \frac{1}{2}\sigma^2 \right) dt = \left(-\beta\mu - \frac{1}{2}\sigma^2 \right) dt + \sigma dW_t^1$$

Integrating the above expression

$$\begin{aligned} \int_0^t d\ln X_t &= \int_0^t df(X_t u, u) = \int_0^t \left(-\beta\mu - \frac{1}{2}\sigma^2 \right) du + \int_0^t \sigma dW_t^1 \\ \ln X_t - \ln X_0 &= \left[-\beta\mu u - \frac{1}{2}\sigma_u^2 \right]_0^t + \left[\sigma W_u^1 \right]_0^t = \ln\left[\frac{X_t}{X_0} \right] = \left[-\beta\mu - \frac{1}{2}\sigma^2 \right] t + \sigma W_t^1 \end{aligned} \tag{0.7}$$

Taking ln of the both sides gives

$$X_t = X_0 e \left(-\beta\mu - \frac{1}{2}\sigma^2 \right) t + \sigma W_t^1 \tag{0.8}$$

Where is a Brownian Motion

From (1.2) let $f\left(X_{\phi},t\right)=\ln X_{\phi}$

Taking the partial derivative yields

$$\frac{\partial f}{\partial X_{\phi}}=\frac{1}{X_{\phi}},\ \ \frac{\partial^2 f}{\partial X_{\phi}^2}=\frac{-1}{X_{\phi}^2},\ \ \frac{\partial f}{\partial t}=0 \qquad (1.9)$$

According to Ito's gives:

$$df\left(X_{\phi},t\right)=\sigma X_{\phi}\frac{\partial f}{\partial X_{\phi}}dW_t^2+\left(K\tanh X_{\phi}\frac{\partial f}{\partial X_{\phi}}+\frac{1}{2}\sigma^2 X_{\phi}^2\frac{\partial^2 f}{\partial X_t^2}+\frac{\partial f}{\partial t}\right)dt \qquad (1.10)$$

Substituting (1.9) into (1.10) gives

$$df\left(X_{\phi},t\right)=\sigma X_{\phi}\frac{1}{X_{\phi}}dW_t^2+\left(K\tanh X_{\phi}\frac{1}{X_{\phi}}+\frac{1}{2}\sigma^2 X_{\phi}^2\left(-\frac{1}{X_{\phi}^2}\right)+0\right)dt \qquad (1.11)$$

$$=\sigma\frac{X_{\phi}}{X_{\phi}}dW_t^2+\left(K\tanh X_{\phi}\frac{X_{\phi}}{X_{\phi}}-\frac{1}{2X_{\phi}^2}\sigma^2 X_{\phi}^2\right)dt=\sigma dW_t^2+\left(K\tanh-\frac{1}{2}\sigma^2\right)dt=\left(K\tanh-\frac{1}{2}\sigma^2\right)dt+\sigma dW_t^2$$

Integrating the above expression

$$\int_0^t d\ln X_{\phi}=\int_0^t df\left(X_{\phi}u,u\right)=\int_0^t\left(K\tanh-\frac{1}{2}\sigma^2\right)du+\int_0^t\sigma dW_t^2 \qquad (1.12)$$

$$\ln X_{\phi}-\ln X_0=\left[K\tanh u-\frac{1}{2}\sigma_u^2\right]_0^t+\left[\sigma W_u^2\right]_0^t=\ln\left[\frac{X_{\phi}}{X_0}\right]=\left[K\tanh-\frac{1}{2}\sigma^2\right]t+\sigma W_t^2$$

Taking ln of the both sides gives

$$X_{\phi}=X_0 e\left(k\tanh-\frac{1}{2}\sigma^2\right)t+\sigma W_t^2 \qquad (1.13)$$

Where is a Brownian Motion

From (1.3) let $f\left(X_{\varpi},t\right)=\ln X_{\varpi}$

Taking the partial derivative yields

$$\frac{\partial f}{\partial X_{\varpi}}=\frac{1}{X_{\varpi}},\ \frac{\partial^2 f}{\partial X_{\varpi}^2}=\frac{-1}{X_{\varpi}^2},\ \frac{\partial f}{\partial t}=0 \qquad (1.14)$$

According to Ito's gives:

$$df\left(X_{\varpi},t\right)=\sigma X_{\varpi}\frac{\partial f}{\partial X_{\varpi}}dW_t^3+\left((-\beta\alpha+K\tanh)X_{\varpi}\frac{\partial f}{\partial X_{\varpi}}+\frac{1}{2}\sigma^2X_{\varpi}^2\frac{\partial^2 f}{\partial X_{\varpi}^2}+\frac{\partial f}{\partial t}\right)dt \qquad (1.15)$$

Substituting (1.14) into (1.15) gives

$$df\left(X_{\varpi},t\right)=\sigma X_{\varpi}\frac{1}{X_{\varpi}}dW_t^3+\left((-\beta\alpha+K\tanh)X_{\varpi}\frac{1}{X_{\varpi}}+\frac{1}{2}\sigma^2X_{\varpi}^2\left(-\frac{1}{X_{\varpi}^2}\right)+0\right)dt \qquad (1.16)$$

$$=\sigma\frac{X_{\varpi}}{X_{\varpi}}dW_t^3+\left((-\beta\alpha+K\tanh)X_{\varpi}\frac{X_{\varpi}}{X_{\varpi}}-\frac{1}{2X_{\phi}^2}\sigma^2X_{\varpi}^2\right)dt=\sigma dW_t^3+\left((-\beta\alpha+K\tanh)-\frac{1}{2}\sigma^2\right)dt$$

$$=\left((-\beta\alpha+K\tanh)-\frac{1}{2}\sigma^2\right)dt+\sigma dW_t^3$$

Integrating the above expression

$$\int_0^t d\ln X_{\varpi}=\int_0^t df\left(X_{\varpi}u,u\right)=\int_0^t\left((-\beta\alpha+K\tanh)-\frac{1}{2}\sigma^2\right)du+\int_0^t\sigma dW_t^3 \qquad (1.17)$$

$$\ln X_{\varpi}-\ln X_0=\left[(-\beta\alpha+K\tanh)u-\frac{1}{2}\sigma_{\varpi}^2\right]_0^t+\left[\sigma W_u^3\right]_0^t=\ln\left[\frac{X_{\phi}}{X_0}\right]=\left[(-\beta\alpha+K\tanh)-\frac{1}{2}\sigma^2\right]t+\sigma W_t^3$$

Taking ln of the both sides gives

$$X_{\varpi}=X_0e\left(\left(-\beta\alpha+k\tanh-\frac{1}{2}\sigma^2\right)t+\sigma W_t^3\right) \qquad (1.18)$$

Where is a Brownian Motion

Nevertheless, a generalized equation for the vector valued Stochastic Differential Equation (SDE) can now be put in the form

$$dx(t)=A(t)x(t)dt+\sum_{i=1}^{n}B_i(t,x(t))\,dw_i(t),\quad x(0)=x_0 \qquad (1.19)$$

Where $A(t)\in\mathbb{R}^{n\times n}, B_i(t)\in\mathbb{R}^{n\times n}, w_i(t)\in\mathbb{R}^n$ is an n-dimensional Brownian motion, $x(t)\in\mathbb{R}^n$ and $x(t)$ for equation (1.19) is normally distributed because the Brownian motion is just multiplied by time-dependent factors. Let $X(t)\in\mathbb{R}^{n\times n}$ be fundamental matrix of the homogenous stochastic differential equation (1.19). It is assumed that x(t) is a continuously differentiable function in t,

Controllability Analysis of Stock prices of Transco, LTD.

In this Section, we study controllability of the system (1.19) when some control measure are introduced into the system as given in equation (1.19) by following the methods of [23]. The control equation of system will be given by

$dx(t) = A(t)x(t)dt + C(t)u(t)dt + \Sigma B_i(t)\xi$

where $C \in \mathbb{R}^{n\times m}$, $u \in \mathbb{R}^{m}$, $x(t_0) = x_0$, (1.20)
the matrices A, C, B_i are continuous in their arguments and $u \in [-h, 0] \rightarrow \mathbb{R}^{m}$, $t \in \mathbb{R}_+$ for h > 0

Where $C(t)$ represents time dependent control measures

In order to effectively study the impact of control measures on the share price movements, we therefore define the following $X(t) = X(t,t_0)$, *then* $X(t,t_0) = X(t)X^{-1}(t_0)$ to have:

$$\int_0^t X(t,s)C(s)ds \qquad (1.21)$$

Define $Y(s) = X(t,s)C(s)$ and the controllability matrix

$$W(t) = \int_0^t Y(s)Y^T(s)ds \qquad (1.22)$$

Where T denotes the transpose of the matrices of each share prices , following [15]. We assume that the following limits exists : $\lim_{t\to\infty} W = W$, $\lim_{t\to\infty} X(t)X^{-1}(t_0) = \bar{X}$, $\lim_{t\to\infty} X(t) = X \neq 0.$

Theorem 5: equation (1.20) is null controllable if and only if W is non-singular. The proof of the controllability is seen in [15] and [23] etc.

RESULTS AND DISCUSSION

This Section presents the graphical results for whose solutions are in (1.8), (1.13) , and (1.18) respectively. Hence the following parameter values were used in the simulation study:
$X_0 = 52.25,\ \beta = 25.6\ ,\sigma = 0.03,\ \mu = 0.88,\ \mathrm{t} = 1,\ \mathrm{W}_t^1 = \mathrm{W}_t^2 = \mathrm{W}_t^3 = 1, \alpha = 0.95, K = 30.7$ and $h = 0.75$

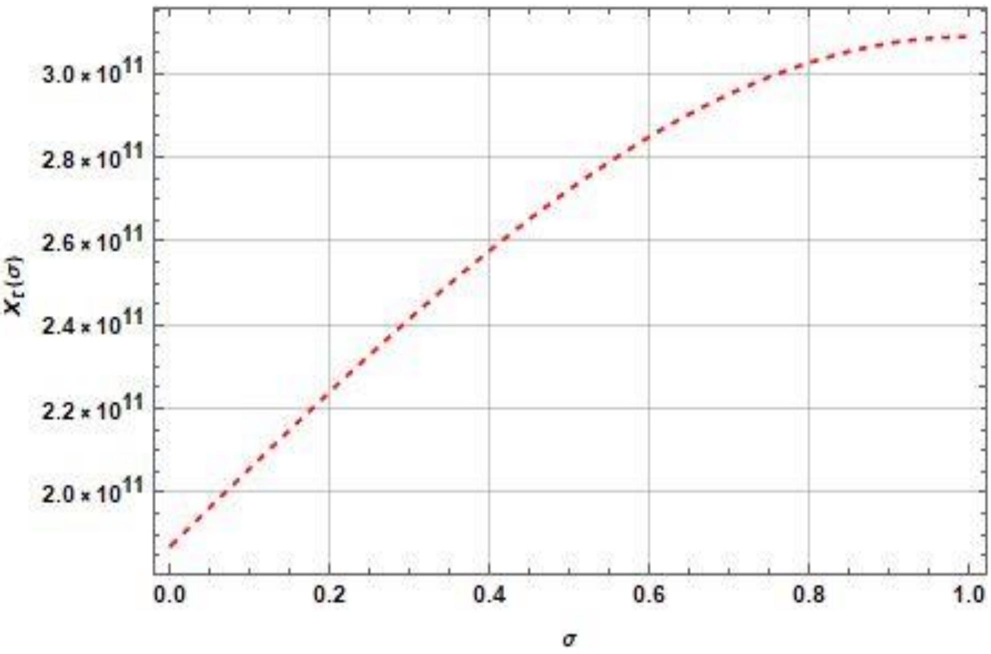

Figure `1: The effect of negative drift coefficient on financial market against volatility

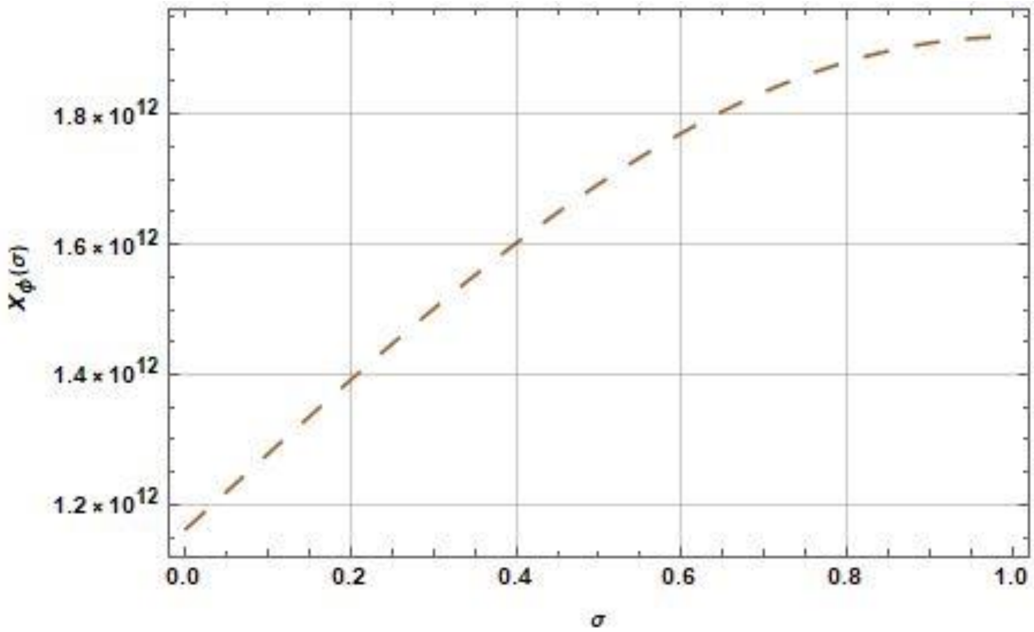

Figure 2: The effect of periodic drift coefficient on financial market against volatility

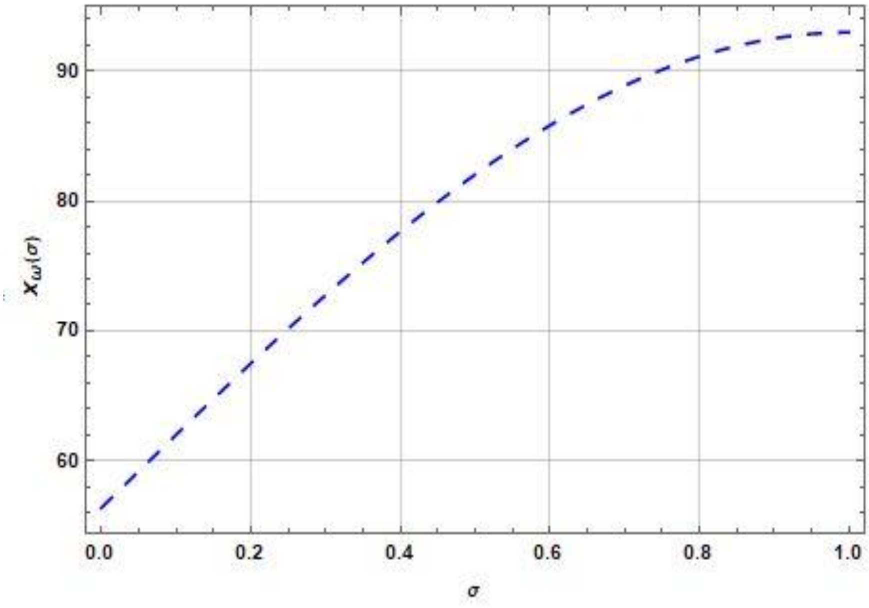

Figure 3: The effect of constant terms with periodic drift coefficient function on financial market against volatility

Figures 1,2 and 3 describes a market that is growing in value but is also highly volatile. This means that the value of the market is increasing over time, but there is also a lot of risk associated with investing in this market. Volatility is a measure of how much the markets value is changing over time and high volatility means that there is a lot of uncertainty in the market.

Finally, volatility of plots is an indicator of the overall health of the market. High volatility often indicates that investors are uncertain about the future and are adjusting their portfolios accordingly.

Illustrations of Covariance and Asymptotic Null Controllability Results for Transco,LTD.

To illustrate the stock price data of Transco LTD extracted from Nigeria Stock Exchange (NSE)
The stock price data of Transco.LTD were transformed to 3x3 matrix which later produced covariance matrix we have the following:

$$A(\text{t}) = \begin{pmatrix} 4.043 & 3.79 & 3.323 \\ 3.743 & 3.96 & 2.723 \\ 3.533 & 3.73 & 2.953 \end{pmatrix}, Cov(A) \begin{pmatrix} 0.0657 & 0.0047 & 0.0534 \\ 0.0047 & 0.0142 & -0.0221 \\ 0.0534 & -0.0221 & 0.0916 \end{pmatrix},$$

In the covariance matrix, each element of the matrix is the covariance between two stocks measures how closely their prices tend to move together. The negative covariance as seen means that the prices tend to move in opposite directions; while positive covariance means that the prices of the two stocks tend to move in the same directions.

with eigen values $\lambda_1 = 0.1644$, $\lambda_2 = 10.627$ and $\lambda_3 = 0.1645$

Which was solved following the method of [15] given as:

In this context, the eigenvalues are a measure of the importance of each stock in terms of how it contributes to the overall risk and returns of Transco investments. The second eigenvalues is larger which tells more important the stock is in terms of the overall risk and return of the investments. In general stocks with large eigenvalues are more volatile and have a greater impact on the investments

$$X(t,s)=\begin{pmatrix} -0.8527e^{0.1644t} & 1.09304e^{10.627t} & -0.8527e^{0.165t} \\ 0.06005e^{0.1644t} & 1.02208e^{10.627t} & 1.0000e^{0.1645t} \\ 1.0000e^{0.1644t} & 1.0000e^{10.627t} & 1.0000e^{0.1645t} \end{pmatrix}, \quad C(t)=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

We show the nonsigularity of the controllability matrix Transco using theorem 5 seen in [15] as follows. let $Y(s)=X(t,s)C(s)$ and $X(t,s)$ respectively.

$$Y(s)=X(t,s)C(s)=\begin{pmatrix} -0.8527e^{0.1644t} & 1.09304e^{10.627t} & -0.8527e^{0.165t} \\ 0.06005e^{0.1644t} & 1.02208e^{10.627t} & 1.0000e^{0.1645t} \\ 1.0000e^{0.1644t} & 1.0000e^{10.627t} & 1.0000e^{0.1645t} \end{pmatrix}$$

$$/W/=\int Y(s)Y^{T}(s)ds=\begin{pmatrix} -2.6491 & 1.0148 & -0.6124 \\ 1.0148 & 1.0519 & 1.1422 \\ -0.6124 & 1.1422 & 3.0000 \end{pmatrix}=-9.8073$$

In all, the determinants of the controllability matrices arenonsigular and therefore the stock price of Transco are asymptotically null controllable. Since the stock prices showed asymptotically null controllable, it means the financial market is stable in the long run. This has several benefits for the market. First, it gives investors confidence that their investments will be worth something in future. Secondly, it helps to reduce volatility in the market, since investors know that prices will eventually stabilize .Third ,it makes it easier for the market to recover from shocks and discruptions. Asymptotic null controllability is therefore a desirable property for financial markets

CONCLUSION

The stochastic differential equations are well known predominant mathematical tools used for the prediction of stock market variables. Therefore, we considered system of stochastic differential equations with disparities of drift parameters in the model. These problems were solved analytical by adopting the Ito's lemma method of solution and three different solutions were obtained accurately. From the analysis of the graphical solutions we deduce that ; shows a market that is growing in value but is also highly volatile, negative covariance which means that the prices of two stock tend to move in opposite direction, positive covariance informs investors that the prices of two stocks tends to move in the same direction, the second eigenvalues:10.627 is larger which tells more important the stock is in terms of the overall risk and return of the Transco investments. More so, incorporating some control measure on the stochastic vector equation, asymptotic null controllability results were obtained by the singularity of the controllability matrix a function of the drift as it affect Transco, LTD investments.

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