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RESEARCH ARTICLE

MAGNETOHYDRODYNAMIC BIOCONVECTION FLOW OF WALTER'S – B NANOFLUID OVER AN EXPONENTIALLY STRETCHING SURFACE

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ABSTRACT

The combination of nanofluids and bioconvection has a wide range of applications in biological systems, agriculture, pharmaceuticals, biotechnology and industrial applications. This paper presents a numerical analysis of the Magnetohydrodynamic bioconvection flow of a two-dimensional, steady, incompressible Walter's – B nanofluid flow in the presence of gyrotactic microorganisms over an exponentially stretching surface. The Buongiorno model together with the Boussinesq approximation is adopted and takes into account the effects of Brownian motion and thermophoresis in formulating the fluid flow governing equations. The governing equations are non-dimensionalised using appropriate similarity transformations. The resulting first-order ordinary differential equations are numerically solved using the Shooting Technique together with the fourth-order Runge-Kutta method. Simulation of the model and investigation of the effects of pertinent parameters on the fluids' temperature, velocity, nanomaterial concentration and dimensionless motile microorganism density is carried out using MATLAB bvp4c. It is observed that the velocity field is enhanced with the Weissenberg number, Brownian motion parameter and Grashof number but reduced with the Buoyancy ratio parameter and Thermophoretic parameter. The temperature profiles increased with Brinkman number and Hartmann number nevertheless, the Prandtl number has the opposite effect. The nanomaterial concentration and motile microorganisms' density profiles enhanced subject to Buoyancy ratio parameter, however, the Bioconvection Peclet number and Bioconvection Lewis parameter have bi-influence toward motile microorganisms' density profile.

Keywords: Magnetohydrodynamic (MHD), Bioconvection, Walter's – B Nanofluid, Nanomaterials, Gyrotactic Microorganism, Boussinesq approximation, Brownian motion Thermophoresis diffusion.

INTRODUCTION

In industrial and mechanical engineering, the concept of boundary layer flow of non-Newtonian materials is vital in the modelling of numerous manufacturing processes such as cooling systems, fabrication of adhesive tapes, solar energy accumulators, metallic plates, chemical processes of non–Newtonian materials, the aerodynamics of plastic sheets, underground nuclear and non-nuclear waste disposal and

application of coatings on hard surfaces among others. Enhancing heat transfer in a base fluid is of great significance in all these systems with regard to energy conservation[1,2]. From this perspective, it is done by enhancing higher thermal conductivity in nanomaterials suspended in nanofluids. One of the advantages of nanofluids is a significant decrease in pumping power for heat exchangers [3,4].
Walter's – B nanofluid is a non-Newtonian type of viscoelastic fluid that portrays both the properties of viscosity and elasticity. Nandeppanavar et al. [5] studied heat transfer in a Walter's – B fluid across an impermeable stretched sheet with an uneven heat source/sink and elastic deformation and observed that the energy equation includes work done by deformation increases the fluid's temperature. Aziz et al. [6] studied the free convection flow of a boundary layer past a horizontally flat plate embedded in a porous medium occupied by nanofluid containing gyrotactic microorganisms and established that the transfer rates of mass, heat and motile microorganisms are significantly influenced by the bioconvection parameters. Tham et al. [7] altered the Aziz et al. [8] work to study the convection flow of nanofluid past a solid sphere in the presence of gyrotactic microorganisms embedded in a porous medium and the results were the same as the aforementioned study.

Bioconvection is a phenomenon that involves shallow suspensions of random but typically upward swim of micro-organisms that are denser than water. As described by Mutuku and Makinde [9] density stratification and spontaneous pattern formation are caused by the presence of nanoparticles, buoyancy forces, and simultaneous interaction of the denser self-propelled microorganisms. In a similar vein, microbes are also taken into account as a means of convection in materials that imitate the motion of nanomaterials in nanofluids. Adding gyrotactic microorganisms into nanofluids tends to improve nanofluids' stability. The benefit of including motile microorganisms in the suspensions has aided the development of microfluidics devices like bacteria-powered micro mixers and bio micro-systems like enzyme biosensors, particularly in microvolume. More precisely, the advanced applications of bioconvection include building insulation, cooling systems for electronic devices, and geothermal nuclear waste disposal. Kuznetsov [10] examined the suspension of nanofluid bioconvection containing gyrotactic microorganisms and nanoparticles and observed that depending on whether the basic distribution of nanoparticles is bottom or top-heavy, the existence of nanoparticles may either lower or raise the critical Rayleigh number. However, the gyrotactic microorganisms' effect is always unstable. Mutuku and Makinde [11] worked on the hydromagnetic bioconvection of nanofluid due to gyrotactic microorganisms over a permeable vertical plate and discovered that the bioconvection parameters and the buoyancy parameter control the convective process. Khan and Makinde [12] altered the work done by Mutuku and Makinde [12] to investigate the Magnetohydrodynamic bioconvection of nanofluid due to gyrotactic microorganisms past a convective heat stretching sheet. Sk et al. [13] studied the effect of multiple slips on the bioconvection flow of nanofluid containing nanoparticles and gyrotactic microorganisms. Siddiqa et al. [14] studied the gyrotactic bioconvection flow of a nanofluid through a vertically wavy surface.

Makinde and Animasaun [11] extended the work done by Khan and Makinde [9] to study bioconvection in the Magnetohydrodynamic flow of nanofluid with nonlinear thermal radiation and chemical reaction in quartic autocatalysis past a revolution paraboloid's upper surface. Uddin et al. [15] revised the Sk et al. [16] work to inspect the slip flow of Bioconvection nanofluid with uses in nano-biofuel cells over a curvy surface. Jeptoo [8] amended Mutuku and Makinde [12] work by taking into account the Magnetohydrodynamic Bioconvection of nanofluid due to Gyrotactic Microorganisms across a convectively heated vertical plate and observed that the thickness of the thermal boundary layer diminishes as the magnetic field strength increases. Ahmad et al. [1] explored the nanofluid flow of bioconvection due to gyrotactic microorganisms through a porous medium. Shi et al. [14] studied the Magneto-cross bioconvection flow of nanofluid involving gyrotactic microorganisms with the sense of activation energy. Hamid et al. [17] extended the Shi et al. [14] work by numerically studying the Magneto-cross bioconvection flow of nanofluid involving gyrotactic microorganisms with an effective Prandtl number approach and the results were the same as the previous study. Chu et al. [18] studied heat transport and nanomaterial bio-convective flow of Walter's – B fluid containing gyrotactic microorganisms. Waqas et al. [19] conducted the Bioconvection transport of magnetized Walter's – B nanofluid across a cylindrical disk coupled to nonlinear radiative heat transfer. Hayat et al. [7] studied thermal diffusion's effects on the bioconvection of Walter's – B with nanomaterial and gyrotactic microorganisms. Alqarni [2] extended the work done by Hayat et al. [7] to thermo bioconvection of Walter's – B nanofluid containing motile microorganisms past a Riga plate. ur Rahman et al. [19] explored the Irreversibility Assessment of the impact of activation energy and mixed convection on MHD bioconvective flow of nanofluid.

This study intends to numerically investigate the bioconvection induced by the magnetohydrodynamic flow of Walter's – B nanofluid in the presence of nanomaterials and gyrotactic microorganisms over an exponentially stretching surface due to its significance in cooling system, polymer processing, bioenergy, biofuel cell, biological and biotechnological systems, drug delivery and preparation of biodiesel fuel, biofuel, and biofertilizers. Here, nanomaterials are moving as Brownian motion and Thermophoresis with respect to the flow of the base fluid. The model is formulated, transformed to their dimensionless form using similarity transformation, and numerically analysed. The graphs depicting the effect of pertinent parameters on the temperature, velocity and concentration are discussed[20].

MODELLING

Here, we intend to examine the two-dimensional, steady, incompressible flow of water-based electrically conducting nanofluid (Walter's – B) containing nanomaterials and gyrotactic microorganisms towards an exponentially stretching surface (ESS). The microorganisms' volume fraction is so minimal that is, they have no impact on the viscosity and inertia of the suspension of fluid's microorganisms. A transverse magnetic field with a uniform strength B_0 is applied to the flow. Here, the surface is stretching exponentially in the $x-$ direction and the presence of nanomaterials does not affect the swimming direction or velocity of microorganisms.

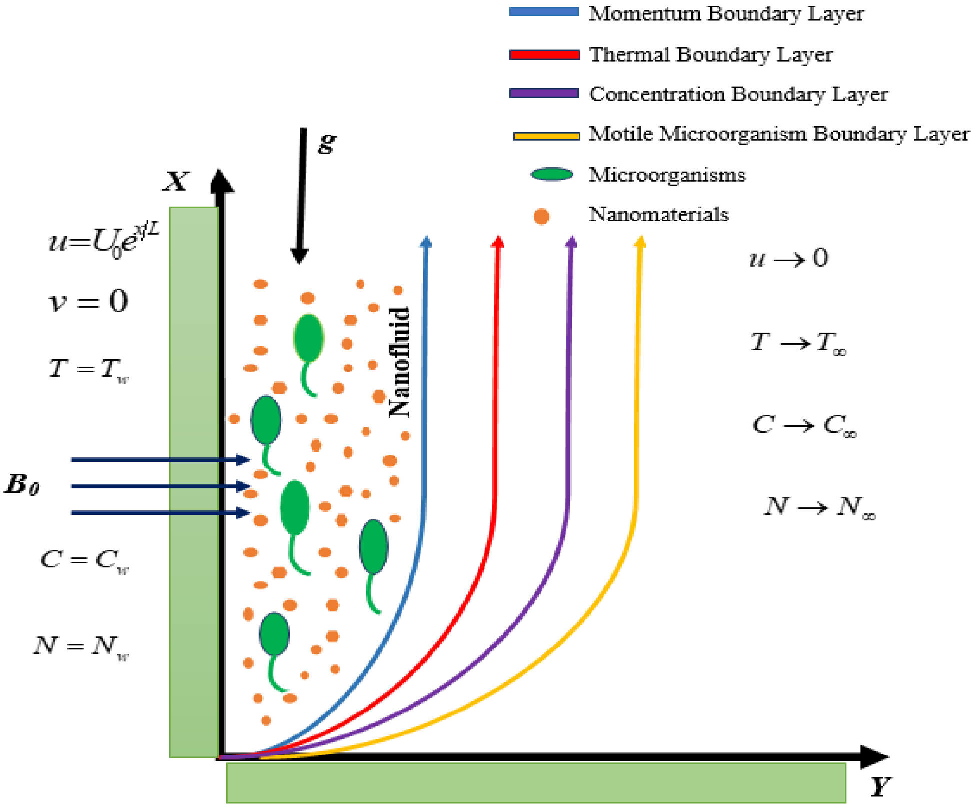

Figure 1: Schematic diagram of the flow

Applying the Buongiorno model together with Boussinesq approximation to correct the dimensionless units and taking into account the effects of Brownian motion and thermophoresis due to nanomaterials, the approximations of boundary layers continuity equation, momentum equation, energy equation, nanomaterial concentration equation and conservation of microorganism's equation are respectively.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon_f\frac{\partial^2 u}{\partial y^2}-\frac{k_0}{\rho_f}\left(u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B_0^2 u}{\rho_f}+$$
$$\frac{1}{\rho_f}\left[(1-C_\infty)\rho_f\beta g(T-T_\infty)-(\rho_n-\rho_f)g(C-C_\infty)-(\rho_m-\rho_f)g\gamma(N-N_\infty)\right] \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left\{D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\}+\frac{\sigma B_0^2 u^2}{(\rho C_p)_f} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}+\frac{bW_c}{(C_w-C_\infty)}\left[\frac{\partial}{\partial y}\left(N\frac{\partial C}{\partial y}\right)\right]=D_m\frac{\partial^2 N}{\partial y^2} \tag{5}$$

Subject to the boundary conditions

$$u=U_w(x)=U_0e^{x/L},\quad v=0,\quad T=T_w=T_\infty+T_0e^{x/2L},\quad C=C_w=C_\infty+C_0e^{x/2L},$$
$$N=N_w=N_\infty+N_0e^{x/2L}\ \text{at}\ y=0 \tag{6}$$

$$u\to 0,\ T\to T_\infty,\ C\to C_\infty,\ N\to N_\infty\ \text{as}\ y\to\infty$$

Where, u and v are the velocity components in the x and y directions respectively, μ is the fluid's dynamic viscosity, υ_f is the fluid's kinematic viscosity, ρ_f is the density of the base fluid, k_0 is the Walter's – B fluid parameter, σ is the electrical conductivity of the fluid, B_0 is the magnetic field strength, α is the base fluid's thermal diffusivity, k is the thermal conductivity, g is the gravitational acceleration, β is the volume expansion coefficient of the fluid, $(\rho C_p)_f$ is the effective heat capacity of the base fluid, $(\rho C_p)_n$ is the effective heat capacity of the nanomaterials, τ is the ratio of effective heat capacity of the nanomaterials to that of the base fluid, ρ_m is the microorganism' density, ρ_n is the density of nanomaterial, D_B is the diffusion coefficient of Brownian motion, D_T is the coefficient of thermophoretic diffusion, D_m is the microorganisms' diffusivity, γ is the average volume of microorganisms, b is the chemotaxis constant, W_c is the maximum cell swimming speed, T is the local temperature, C is the concentration of nanomaterials, N is the concentration of microorganism, T_w, C_w, N_w are temperature, nanomaterial concentration and density of motile microorganisms at the surface, T_∞, C_∞, N_∞ are ambient values of temperature, nanomaterial concentration and density of motile microorganisms, T_0, C_0, N_0 are reference values of temperature, nanomaterial concentration and density of motile microorganisms and L is the reference length.

The base fluid's thermal diffusivity, the ratio of effective heat capacities and the fluid's kinematic viscosity are defined as;

$$\alpha = \frac{k}{\left(\rho C_p\right)_f}, \quad \tau = \frac{\left(\rho C_p\right)_n}{\left(\rho C_p\right)_f}, \quad \upsilon_f = \frac{\mu}{\rho_f} \tag{7}$$

Setting similarity variables and quantities

$$\lambda = y e^{\frac{x}{2L}} \sqrt{\frac{U_0}{2L\upsilon_f}},\ \psi = \sqrt{2LU_0\upsilon_f} f(\lambda) e^{\frac{x}{2L}},\ \Omega(\lambda) = \frac{(T-T_\infty)e^{\frac{-x}{2L}}}{T_0},\ \ \mho(\lambda) = \frac{(C-C_\infty)e^{\frac{-x}{2L}}}{C_0},$$

$$\emptyset(\lambda) = \frac{(N-N_\infty)e^{\frac{-x}{2L}}}{N_0}, u = U_0 e^{\frac{x}{L}} f' \text{ and } v = -\sqrt{\frac{U_0\upsilon_f}{2L}} e^{\frac{x}{2L}}(f+\lambda f'), Br = \frac{\mu U_W^2}{k(T_w-T_\infty)}, We = \frac{k_0 U_w}{L\mu},$$

$$Ha = \frac{L\sigma B_0^2}{U_w \rho_f},\quad Gr = \frac{L\beta g(1-C_\infty)(T_w-T_\infty)}{U_W^2},\quad Nr = \frac{(\rho_n-\rho_f)(C_w-C_\infty)}{\beta\rho_f(1-C_\infty)(T_w-T_\infty)}, Pr = \frac{\upsilon_f}{\alpha},\quad Sc = \frac{\upsilon_f}{D_B},$$

$$Rb = \frac{\gamma(\rho_m-\rho_f)(N_w-N_\infty)}{\beta\rho_f(1-C_\infty)(T_w-T_\infty)},\quad Nb = \frac{\tau D_B(C_w-C_\infty)}{\alpha},\quad Pe = \frac{bW_c}{D_m},\ Nt = \frac{\tau D_T(T_w-T_\infty)}{\alpha T_\infty}, Lb = \frac{\upsilon_f}{D_m},$$

$$R = \frac{N_\infty}{(N_w-N_\infty)}. \tag{8}$$

Where, λ is the similarity variable, f is the dimensionless stream function, Ω is the dimensionless temperature, $\mho$ is the dimensionless nanomaterial concentration, $\emptyset$ is the dimensionless motile microorganism concentration and ψ is the stream function defined as

$$u = \frac{\partial\psi}{\partial y}, \quad \text{and} \quad v = -\frac{\partial\psi}{\partial x} \tag{9}$$

Substituting equation (8) into equations (1) - (6), we obtain the dimensionless equations as follows;

$$\begin{aligned} & f''' + ff'' - 2(f')^2 - \frac{We}{2}\left(6ff''' - ff'''' - 3(f'')^2\right) - 2Haf' + 2Gr\left(\Omega - Nr\mho - Rb\emptyset\right) = 0. \\ & \Omega'' + Nb\mho'\Omega' + Nt(\Omega')^2 - Pr\left(f'\Omega - f\Omega'\right) + 2HaBr\left(f'\right)^2 = 0. \\ & \mho'' + \frac{Nt}{Nb}\Omega'' + Sc\left(f\mho' - \mho f'\right) = 0. \\ & \emptyset'' + Lb\left(f\emptyset' - f'\emptyset\right) - Pe\left(\mho''\left(R+\emptyset\right) + \mho'\emptyset'\right) = 0. \end{aligned} \tag{10}$$

Subject to the boundary conditions

$$f'(0) = 1, f(0) = 0,\ \Omega(0) = 1, \mho(0) = 1, \emptyset(0) = 1. \tag{11}$$
$$f'(\infty) \to 0,\ \Omega(\infty) \to 0,\ \mho(\infty) \to 0,\ \emptyset(\infty) \to 0. \tag{12}$$

Where Primes denote differentiation with respect to λ and We is the Weissenberg number, Ha is the Hartmann number, Gr is the Grashof number, Nr is the Buoyancy ratio parameter, Rb is the Bio-convection Rayleigh parameter, Nb is the Brownian motion parameter, Nt is the Thermophoretic parameter, Pr is the Prandtl parameter, Br is the Brinkman number, Sc is the Schmidt number, Lb is the

Bioconvection Lewis parameter, Pe is the Bioconvection Peclet parameter, R is the Microorganisms' concentration difference parameter.

NUMERICAL APPROACH

The nonlinear boundary value problems (BVP) for the system of the first-order ordinary differential equations labelled from (13) to (23) are numerically solved using the shooting technique coupled with the Runge-Kutta method (RK4). In particular, the shooting method is used to transform the boundary value problems into a set of initial value problems (IVP) with unknown initial conditions. Later, the Runge-Kutta method (RK4) is employed to numerically solve the resulting initial value problems until the boundary conditions are satisfied.

Setting

$$z_1 = f, z_2 = f', z_3 = f'', z_4 = f''', z_4' = f'''', z_5 = \Omega, z_6 = \Omega', z_6' = \Omega'', z_7 = \mho, z_8 = \mho', z_8' = \mho'', z_9 = \emptyset, z_{10} = \emptyset', z_{10}' = \emptyset''. \quad (13)$$

The boundary conditions (11) and (12) are transformed to initial conditions as follows

$$\begin{aligned} &z_1(0)=0, \quad z_2(0)=1, \quad z_3(0)=s_1, \quad z_4(0)=s_2, \quad z_5(0)=1, \\ &z_6(0)=s_3, \quad z_7(0)=1, \quad z_8(0)=s_4, \quad z_9(0)=1, \quad z_{10}(0)=s_5. \end{aligned} \quad (14)$$

The resulting initial value problems (IVP) for the system of first-order ordinary differential equations are;

$$z_1' = z_2. \quad (15)$$

$$z_2' = z_3. \quad (16)$$

$$z_3' = z_4. \quad (17)$$

$$z_4' = \frac{-2z_4 - 2z_1z_3 + 4z_2^2 + 4Haz_2 - 2Gr\left(z_5 - z_7Nr - z_9Rb\right) + We\left(6z_2z_4 - 3z_3^2\right)}{z_1We} \quad (18)$$

$$z_5' = z_6. \quad (19)$$

$$z_6' = Pr\left(z_2z_5 - z_1z_6\right) - Nbz_8z_6 - Ntz_6^2 - 2HaBrz_2^2. \quad (20)$$

$$z_7' = z_8. \quad (21)$$

$$z_8' = Sc\left(z_7z_2 - z_1z_8\right) - \frac{Nt}{Nb}z_6'. \quad (22)$$

$$z_9' = z_{10}. \quad (23)$$

$$z_{10}' = Pe\left(z_8'\left(R + z_9\right) + z_8z_{10}\right) + Lb\left(z_2z_9 - z_1z_{10}\right). \quad (24)$$

Subject to the initial conditions

$$\begin{aligned} &z_1(0)=0, \quad z_2(0)=1, \quad z_3(0)=s_1, \quad z_4(0)=s_2, \quad z_5(0)=1, \\ &z_6(0)=s_3, \quad z_7(0)=1, \quad z_8(0)=s_4, \quad z_9(0)=1, \quad z_{10}(0)=s_5. \end{aligned} \quad (25)$$

The unknown initial conditions s_1, s_2, s_3, s_4, s_5 are assumed. The system of equations (15) to (24) is solved with the initial guess s_1, s_2, s_3, s_4, s_5 and results are compared with the boundary conditions (12). If variations occur, the process is repeated until the improved values of unknown initial conditions are attained.

RESULTS AND DISCUSSION

Velocity Profiles:

In this section, Figs. 2 – 11 portray the effect of the Weissenberg number (We), Hartmann number (Ha), Grashof number (Gr), Buoyancy ratio parameter (Nr), Bioconvection Rayleigh parameter (Rb), Prandtl number (Pr), Brownian motion parameter (Nb), Thermophoretic parameter (Nt), Brinkman number (Br) and Microorganisms' concentration difference parameter (R) on the nanofluids' velocity profiles. It is observed that at the surface the velocity is maximum but exponentially decreases to zero far from the surface satisfying the ambient conditions. Physically, the Lorentz force produced by the magnetic field tends to slow down the flow of the fluid and hence reduces the fluids' velocity. Fig. 2 depicts the effect of the Weissenberg number on the velocity field. Practically, Weissenberg's number compares the viscous and elastic forces. As expected, the increase of the Weissenberg number leads to an increase in the fluid's velocity. Also, the boundary layer thickness (fluid thickness) increases as the Weissenberg number gets large. The Brinkman number is associated with heat conductivity from a wall to a moving viscous fluid and is usually used in the processing of polymers. Fig. 3 shows that the velocity and the boundary layer thickness seem to increase indicating an increase in the buoyancy force as the Brinkman number gets large. The collision of nanomaterials suspended in nanofluid with the stretching surface leads to an increase in the kinetic energy in the flow. Fig. 8 shows that the fluids' velocity decreases as the Prandtl number increases hence decreasing the boundary layer thickness (fluid thickness). It is also noted that an increase in the Gr, Rb, Nb, and R leads to an increase in both fluids' velocity and the boundary layer thickness as depicted in Figs. 4 – 7. the opposite result is seen with an increase in Ha, Nr, and Nt as shown in Figs. 9 – 11.

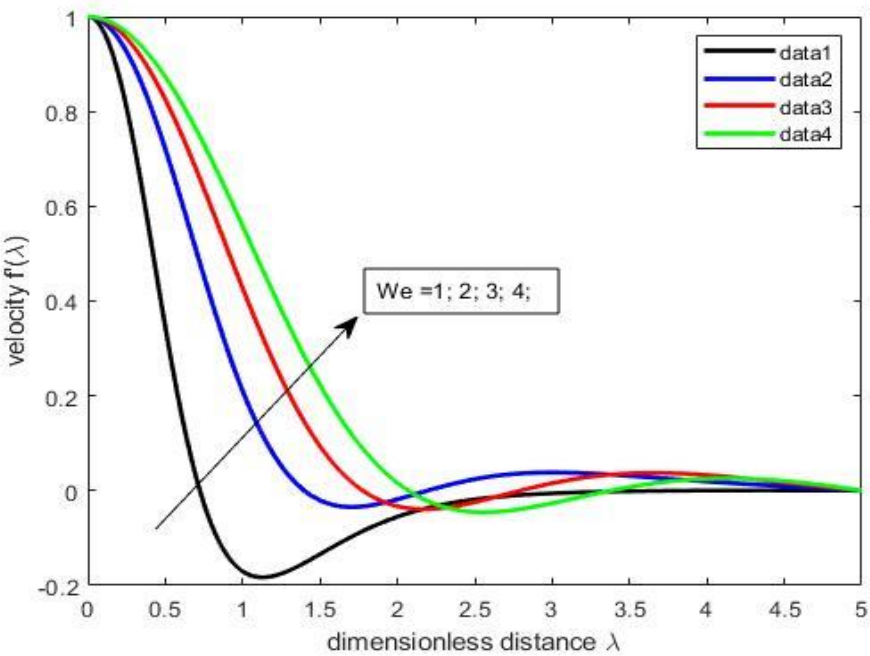

Figure 2: Variation of velocity with Weissenberg number

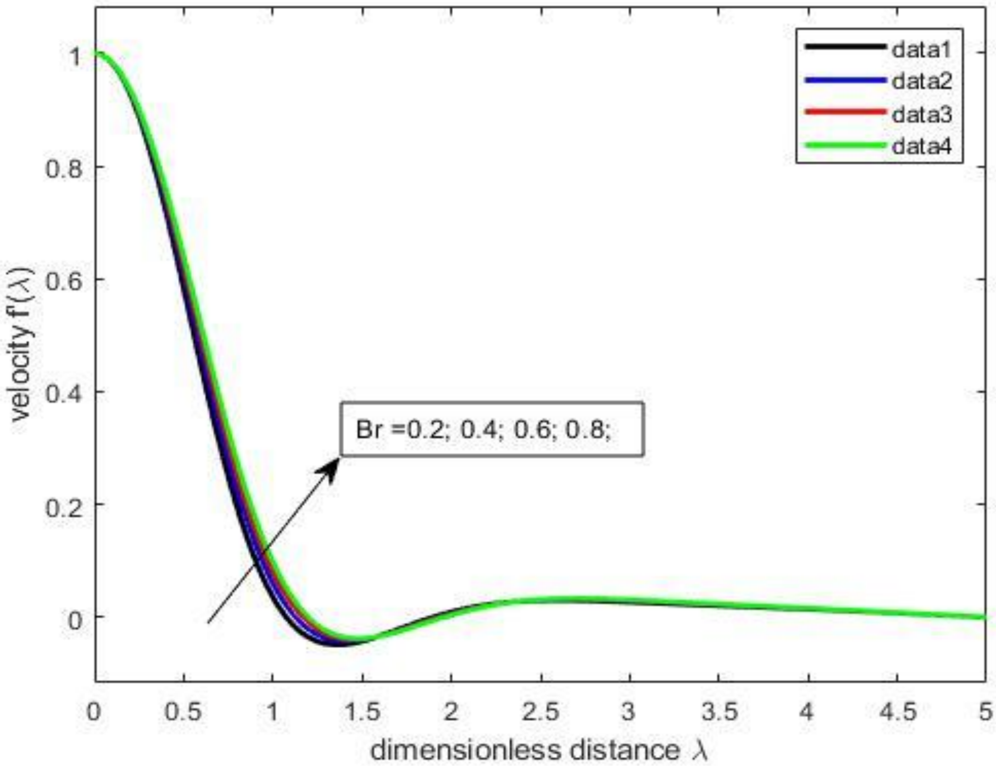

Figure 3: Variation of velocity with Brinkman number

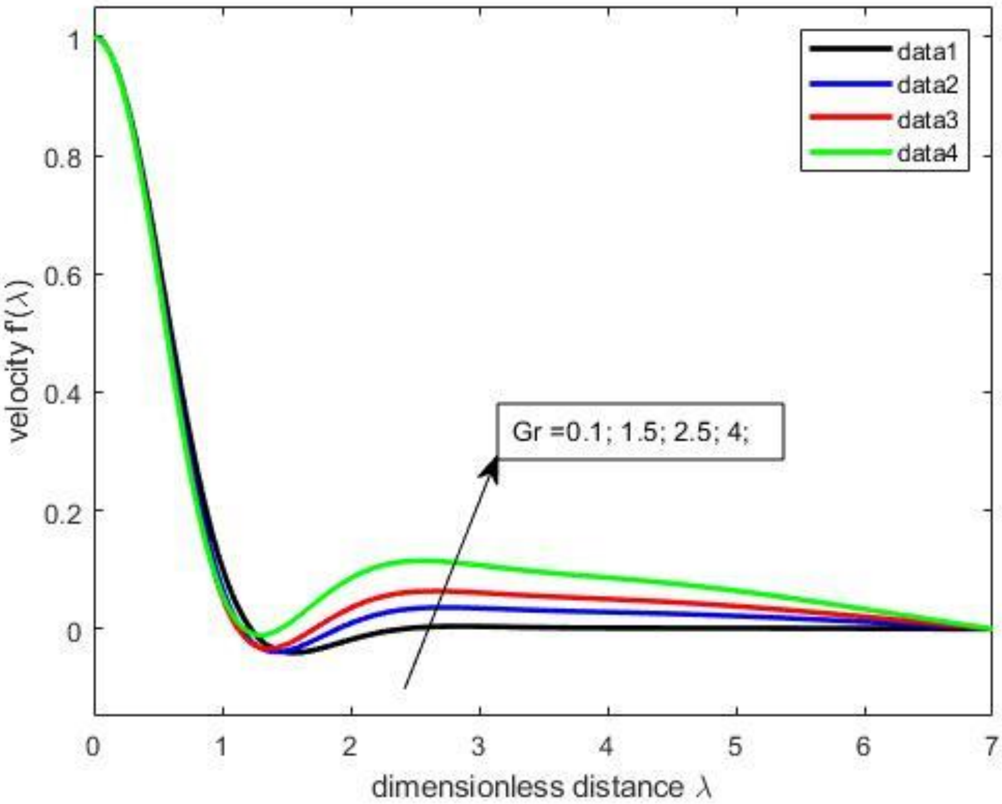

Figure 4: Variation of velocity with Grashof number

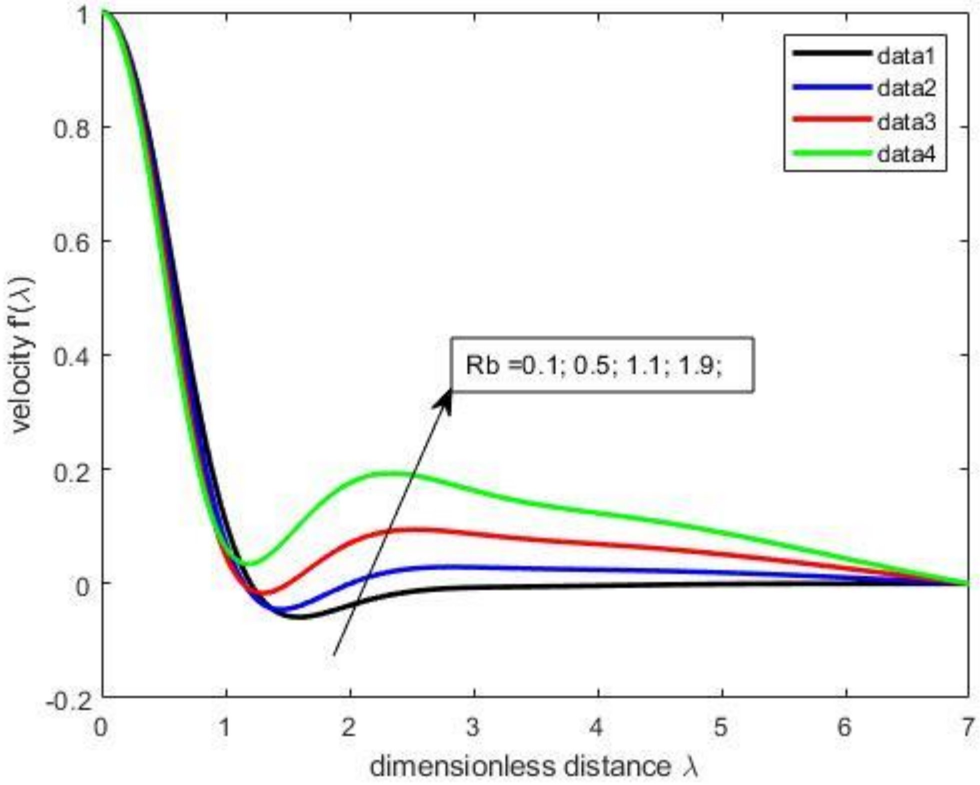

Figure 5: Variation of velocity with Bioconvection Rayleigh parameter

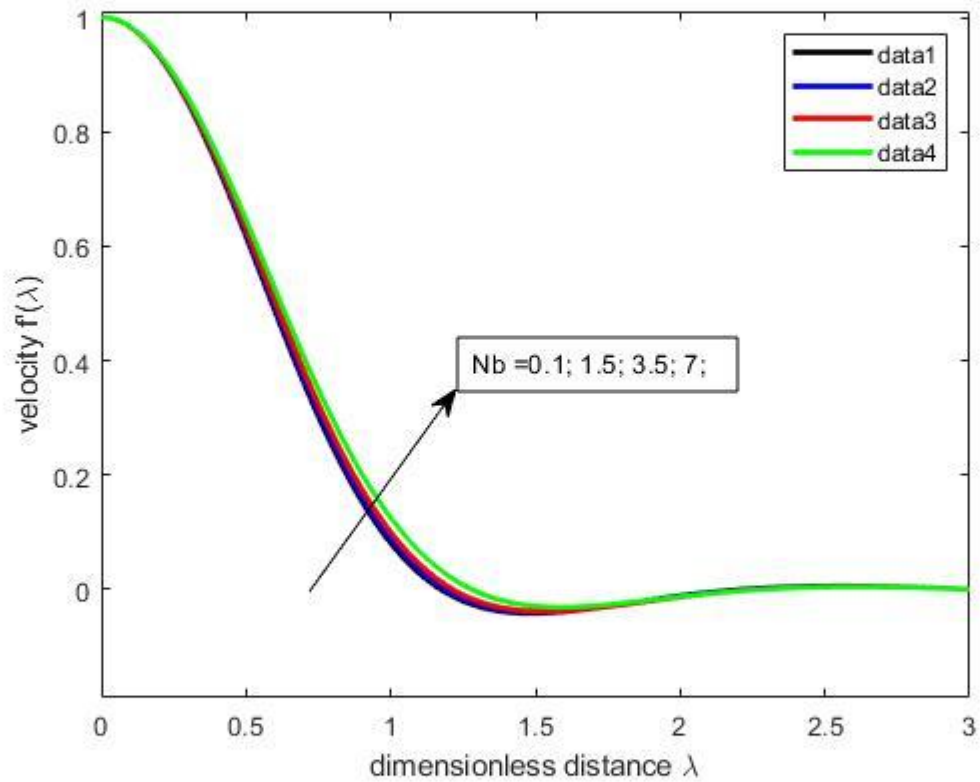

Figure 6: Variation of velocity with Brownian motion parameter

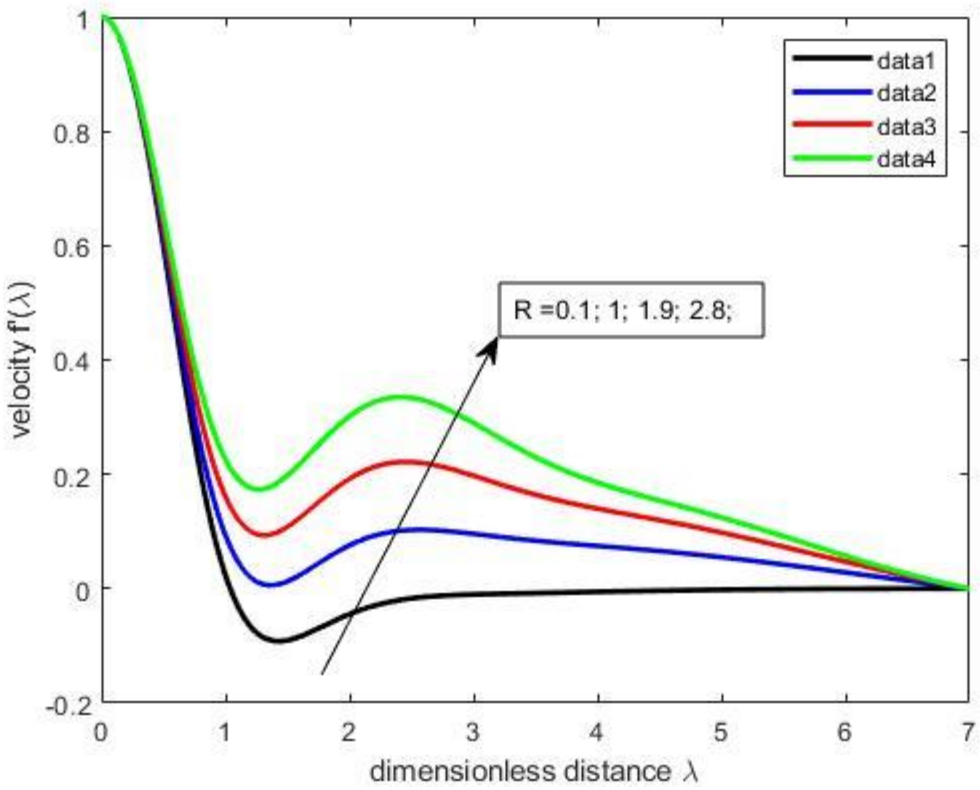

Figure 7: Variation of velocity with Microorganisms' concentration difference parameter

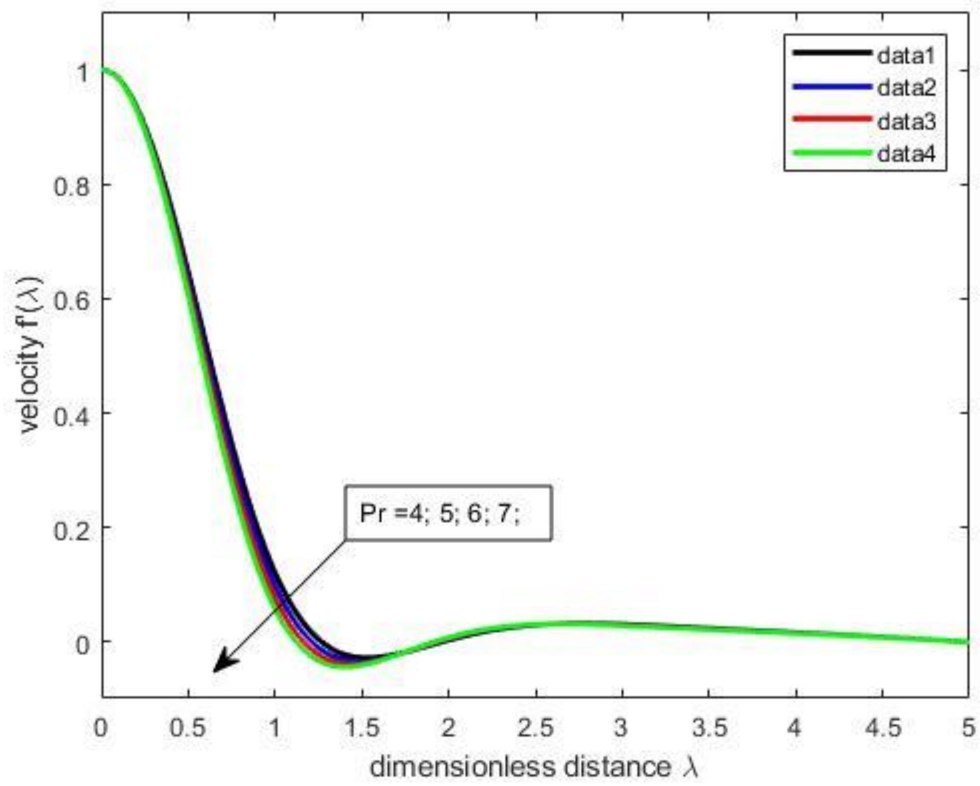

Figure 8: Variation of velocity with Prandtl parameter

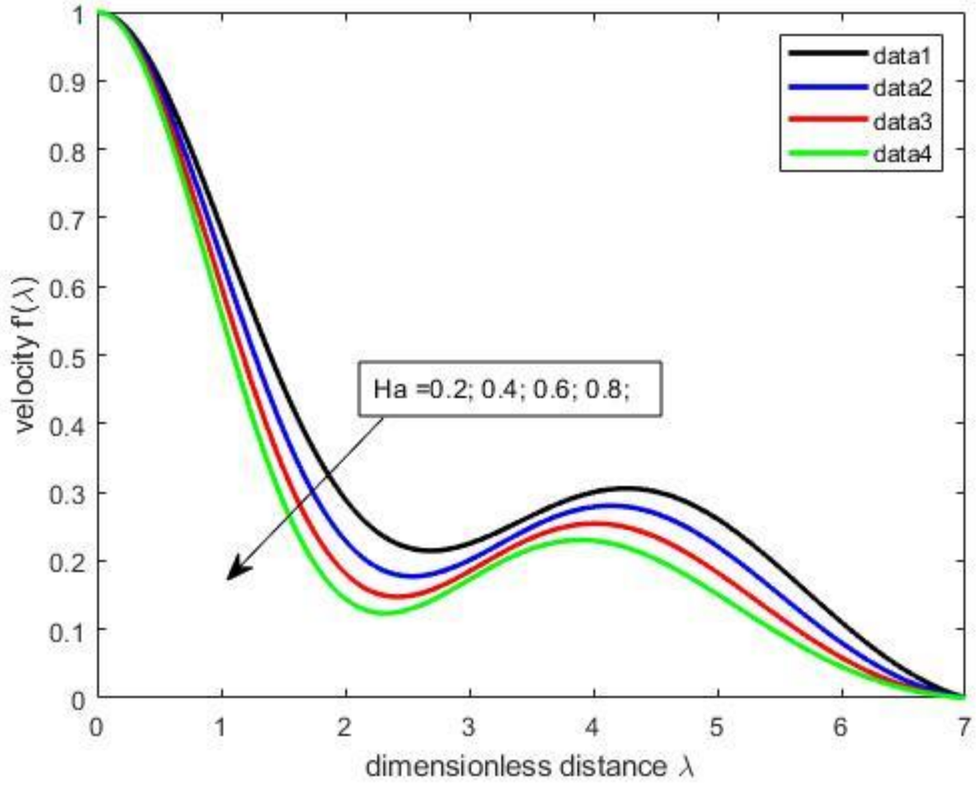

Figure 9: Variation of velocity with Hartmann number

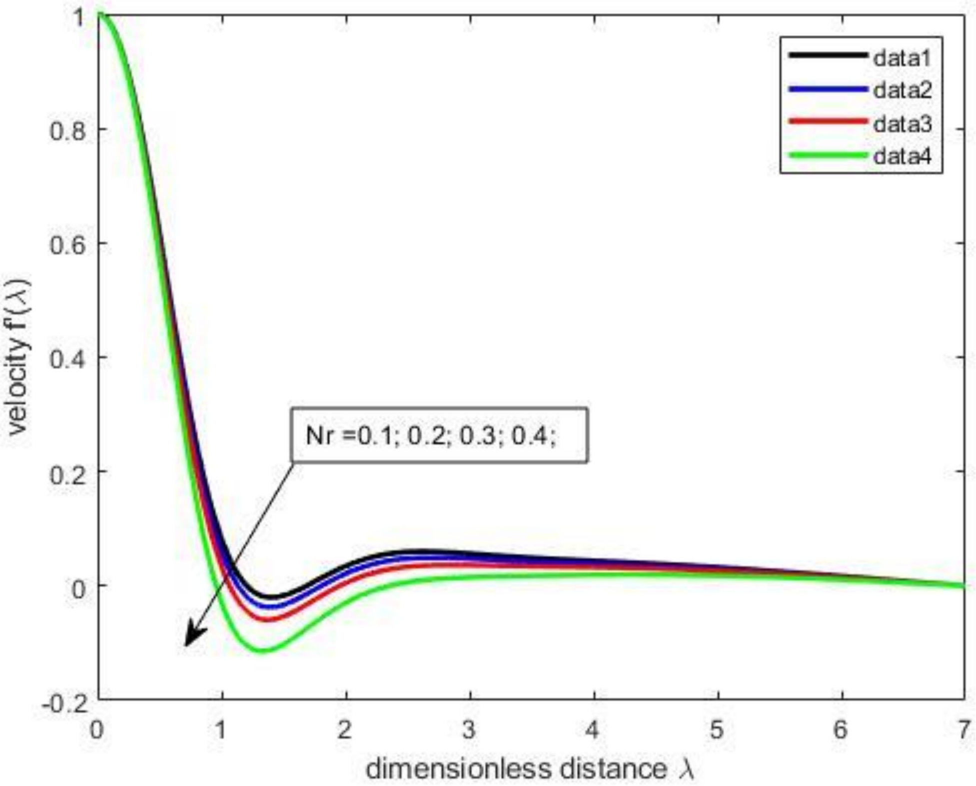

Figure 10: Variation of velocity with Buoyancy ratio parameter

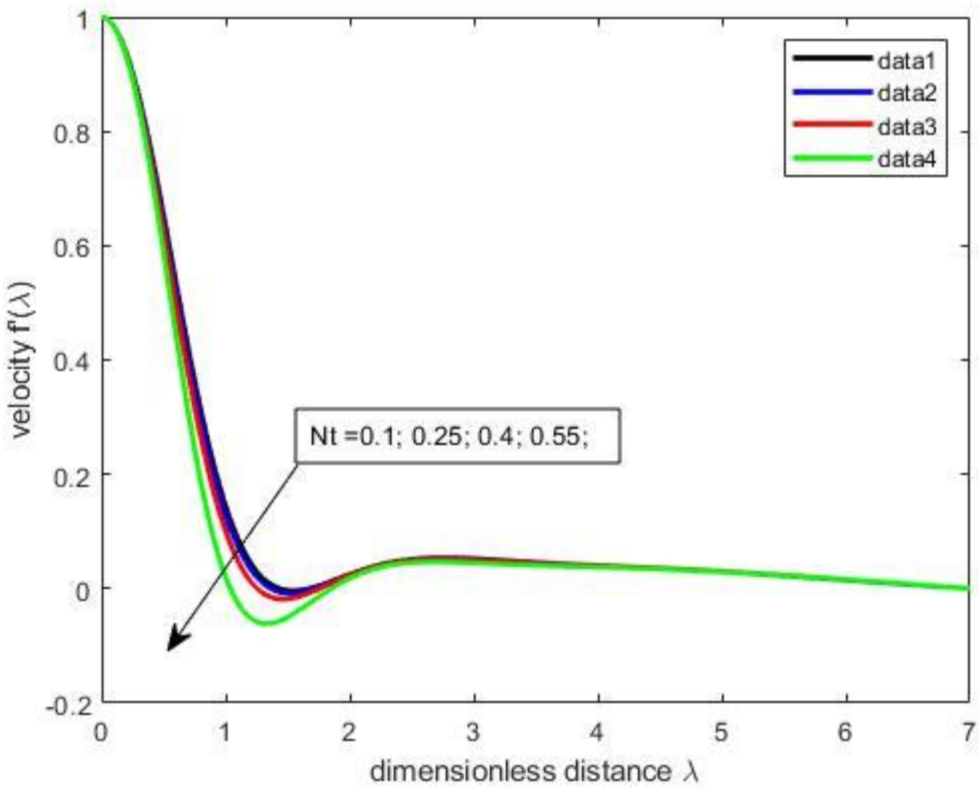

Figure 11: Variation of velocity with Thermophoretic parameter

Temperature Variation:

From the temperature profiles (Figs. 12 – 16), it is remarked that at the surface the temperature is maximum but exponentially decreases to zero far from the surface satisfying the ambient conditions. Figs. 12 and 13 depicted the effect of the Brinkman number (Br) and Hartmann number (Ha) on nanofluids' temperature profile. It is spotted that both temperature and Thermal boundary layer thickness increased for larger values of Brinkman number and Hartmann number. The influence of the Brownian motion parameter (Nb) and Thermophoretic parameter (Nt) on temperature is outlined in Figs. 14 and 15. As anticipated, the temperature is increased for larger values of Brownian motion parameter and Thermophoretic parameter. It is also noted that the thermal boundary layer thickness is enhanced by a higher estimation of the Thermophoretic parameter and Brownian motion parameter. The fact that the occurrence of nanomaterials suspended in the base fluids moves as Brownian motion and thermophoresis increase thermal conductivity hence boosting nanofluids' temperature and thermal boundary layer thickness, thereby enhancing nanofluids' heat transfer. The effect of the Prandtl number (Pr) on nanofluids' temperature profiles is portrayed in Fig. 16. As expected, both nanofluids' temperature and thermal boundary layer thickness decline for increasing values of Prandtl number. Physically, the Prandtl number regulates the relative thickness of the thermal and momentum boundary layers in heat transfer of nanofluid problems. In comparison to velocity (momentum), heat diffuses more quickly when Pr is small. This indicates that the thermal boundary layer is substantially thicker than the velocity boundary layer for metallic liquids.

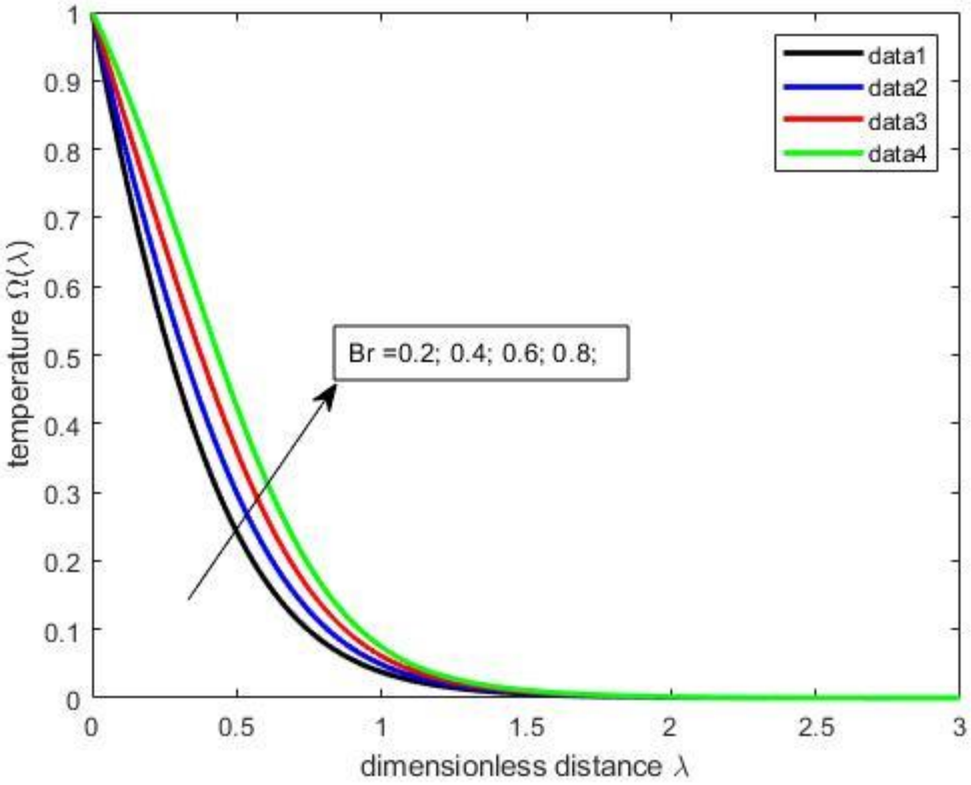

Figure 12: Variation of temperature with Brinkman number

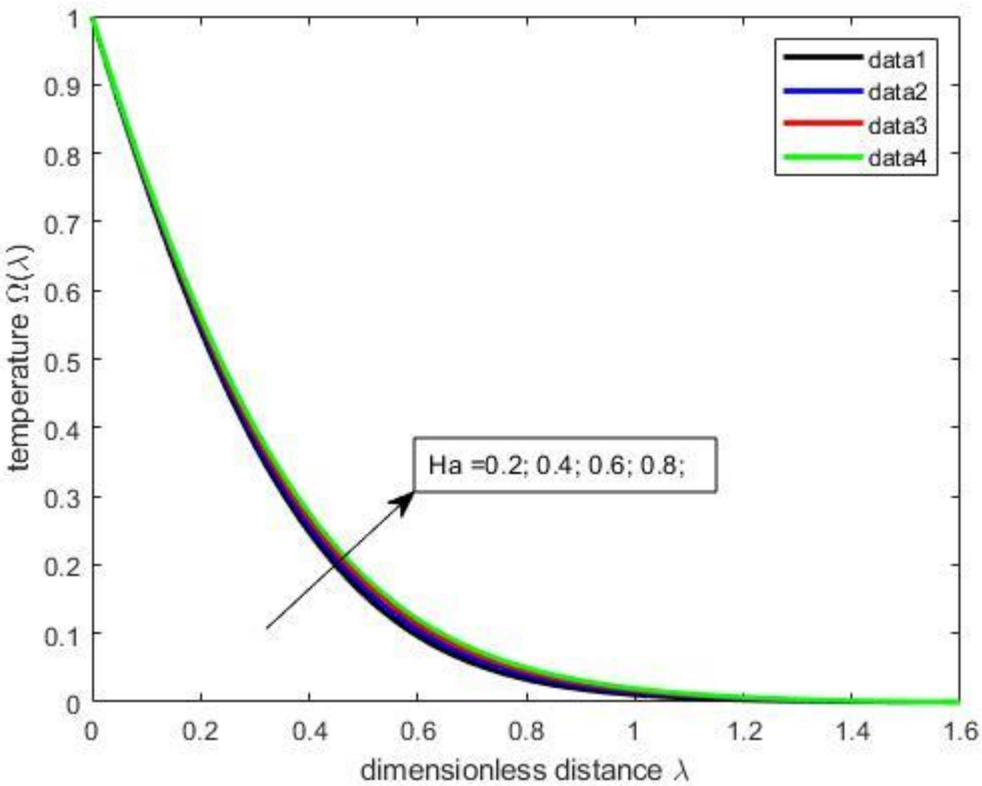

Figure 13: Variation of temperature with Hartmann number

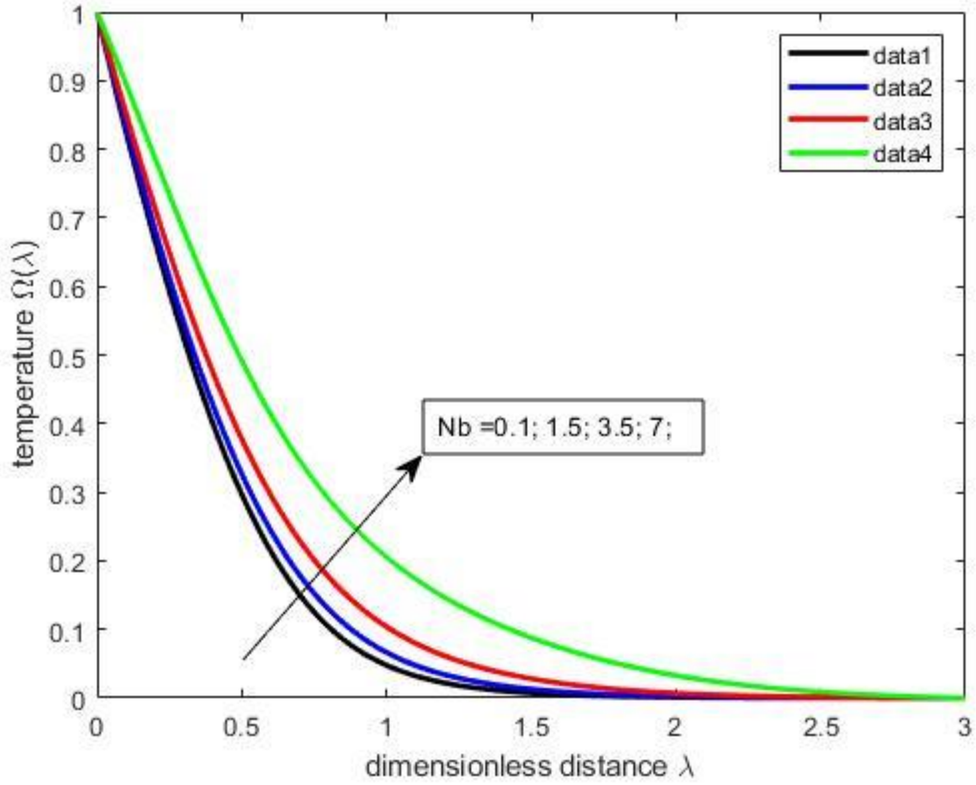

Figure 14: Variation of temperature with Brownian motion parameter

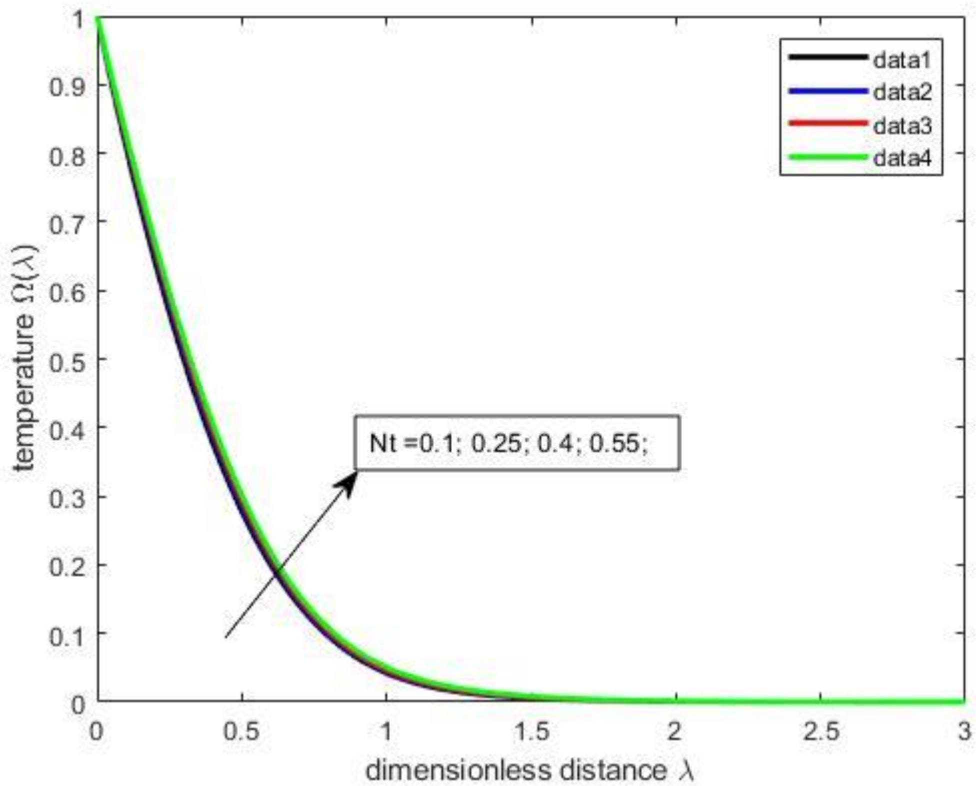

Figure 15: Variation of temperature with Thermophoretic parameter

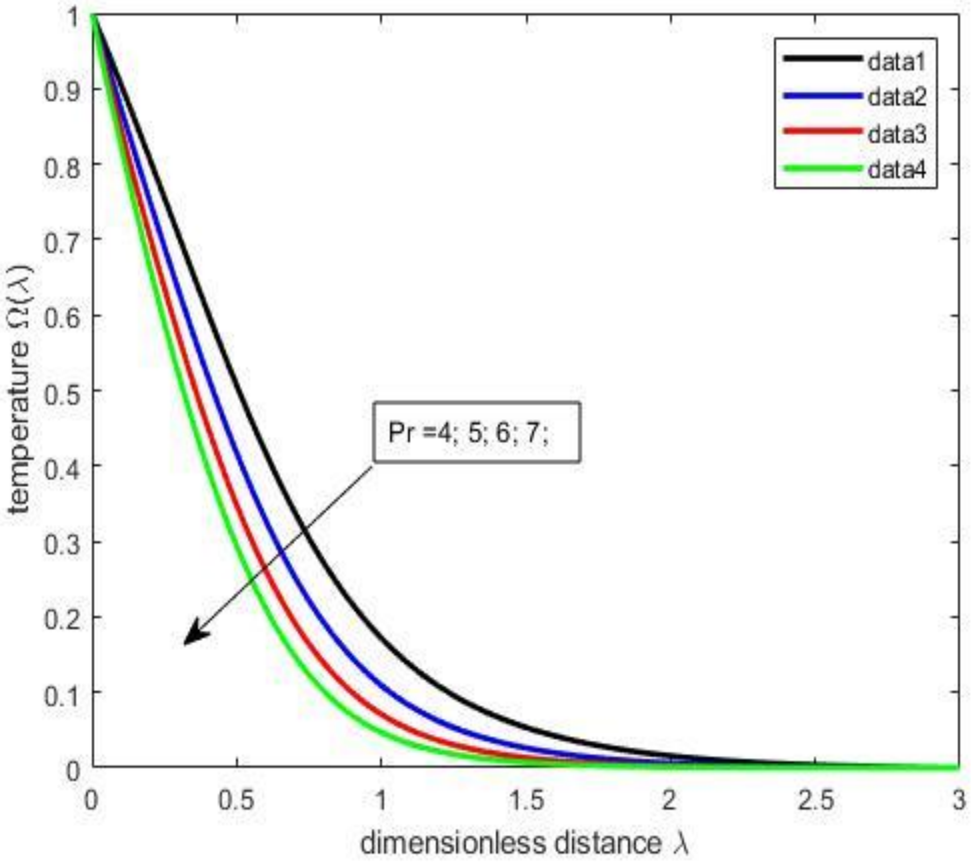

Figure 16: Variation of temperature with Prandtl number

Nanomaterial Concentration Profiles:

Here, Fig. 17 – 25 are sketched to illustrate the nanomaterial concentration subject to Schmidt number (Sc), Hartmann number (Ha), Grashof number (Gr), Buoyancy ratio parameter (Nr), Bioconvection Rayleigh parameter (Rb), Prandtl number (Pr), Brownian motion parameter (Nb) and Thermophoretic parameter (Nt), Brinkman number (Br). The effect of Prandtl number, Buoyancy ratio parameter,

Hartmann number, and Thermophoretic parameter is outlined in Figs. 17 – 20. Clearly, it is observed that increasing the Prandtl number, Buoyancy ratio parameter, Hartmann number, and Thermophoretic parameter boosts up nanomaterial concentration hence upsurging the concentration boundary layer thickness. This is brought by the force produced by the rate of mass transfer at the surface and the rapid flow away from the stretching surface. Also, the influence of the Magnetic field intensity, Bioconvection, and Buoyancy propel the fluids towards the stretchable surface and thereby, as expected increase the nanomaterial concentration. The reverse effect has been observed for the increase of Schmidt number, Grashof number, Bioconvection Rayleigh parameter, Brownian motion parameter, and Brinkman number as spotted in Figs. 21 – 25, thereby, reduced mass diffusivity and Brownian motion of nanomaterials in the boundary layer region.

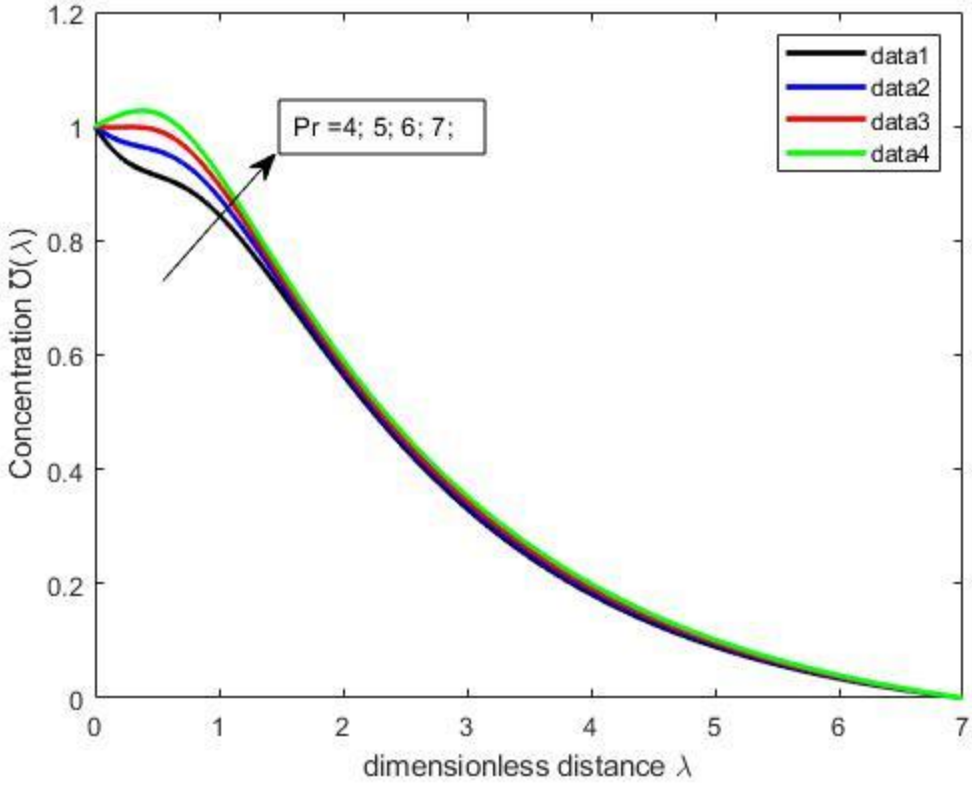

Figure 17: Variation of concentration with Prandtl number

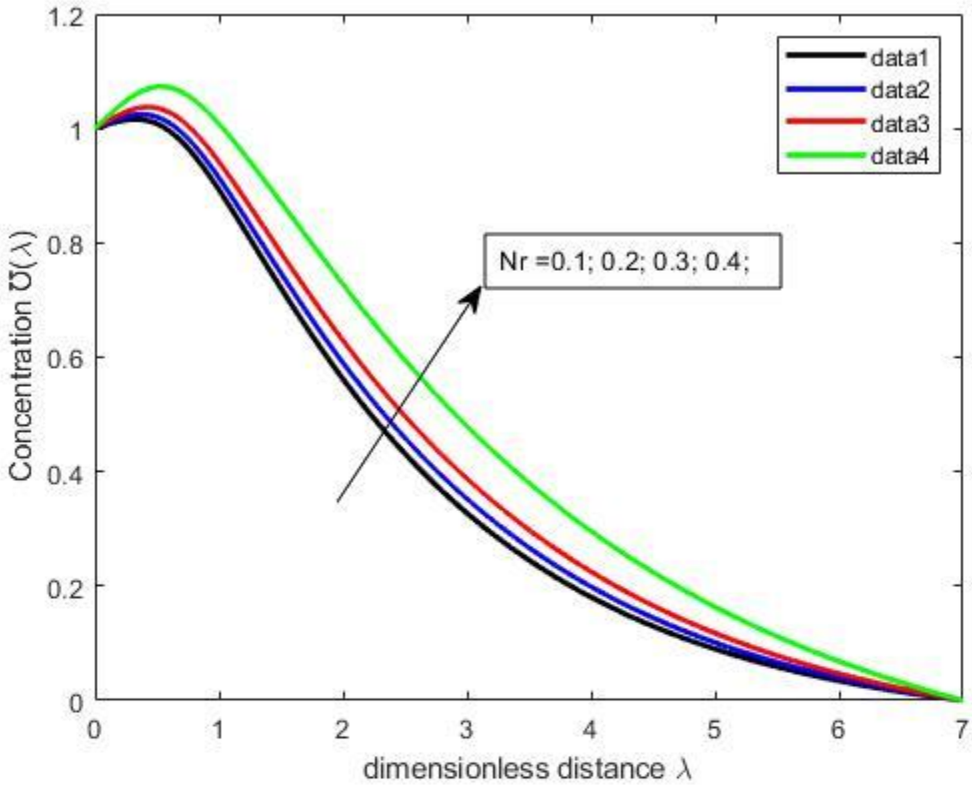

Figure 18: Variation of concentration with Buoyancy ratio parameter

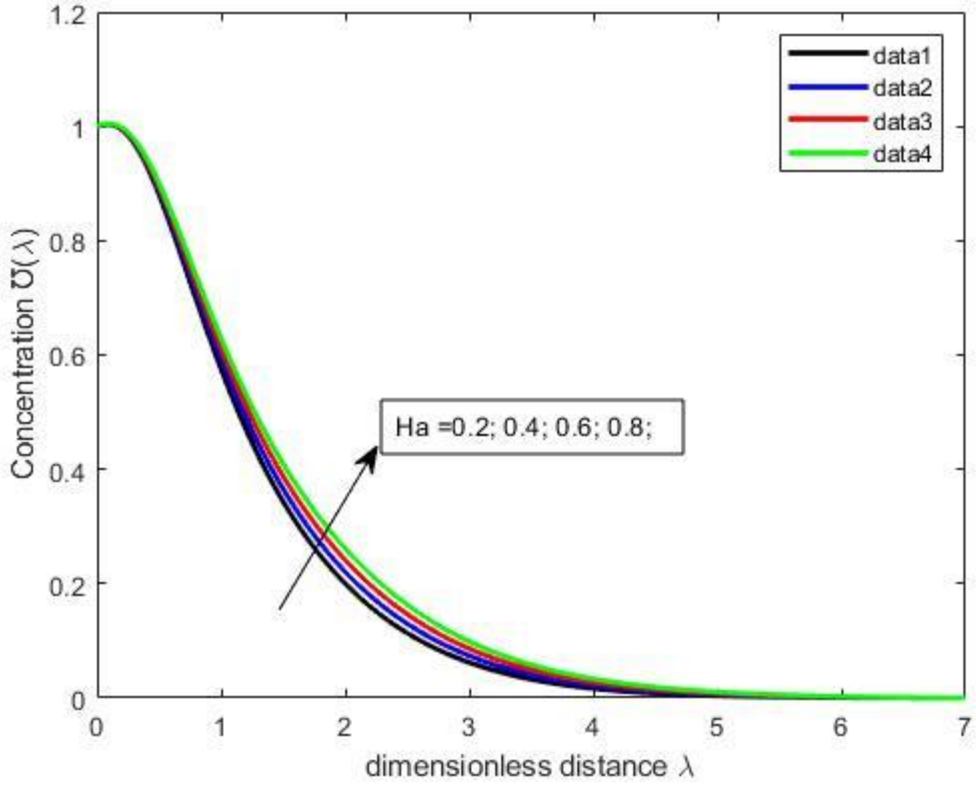

Figure 19: Variation of concentration with Hartmann number

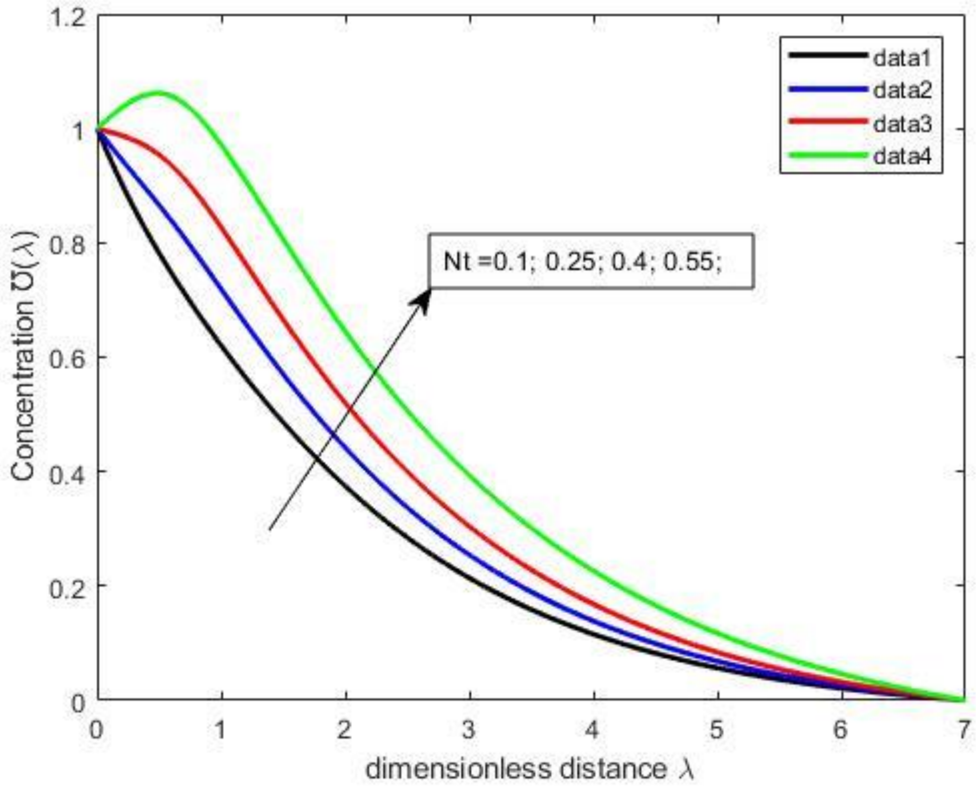

Figure 20: Variation of concentration with Thermophoretic parameter

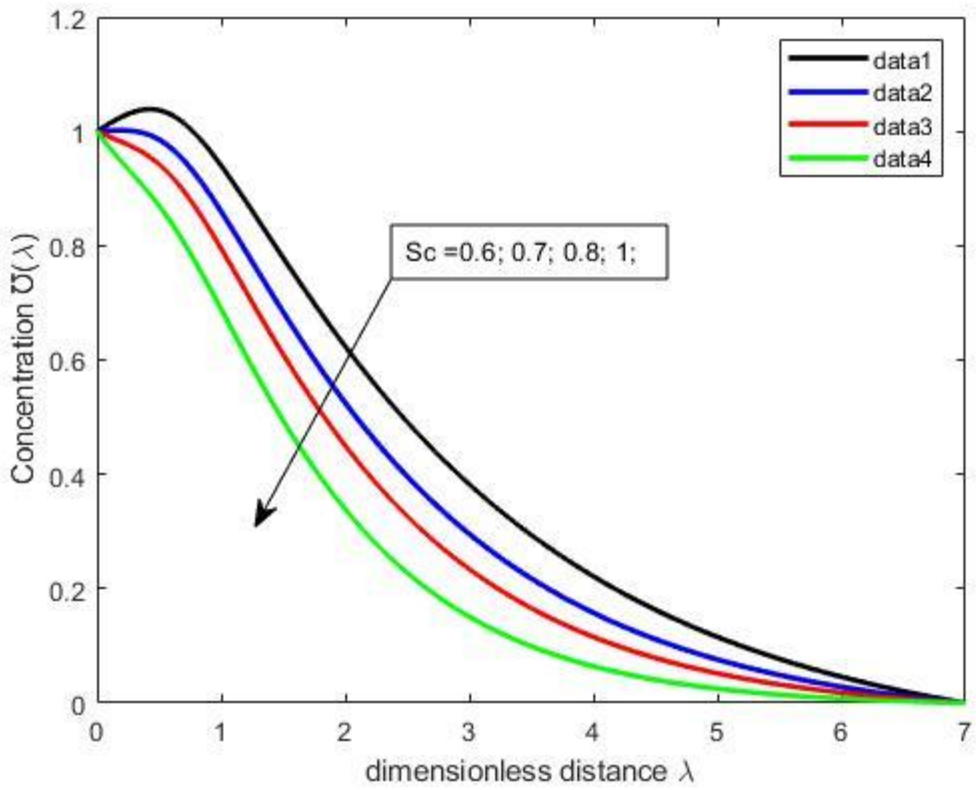

Figure 21: Variation of concentration with Schmidt number

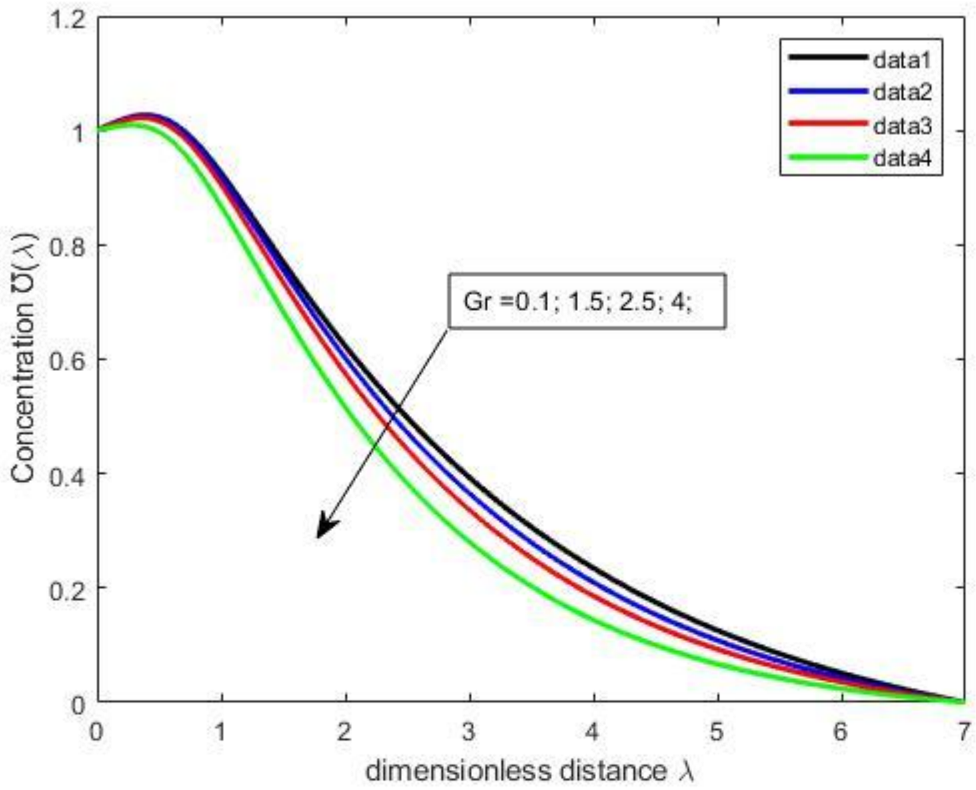

Figure 22: Variation of concentration with Grashof number

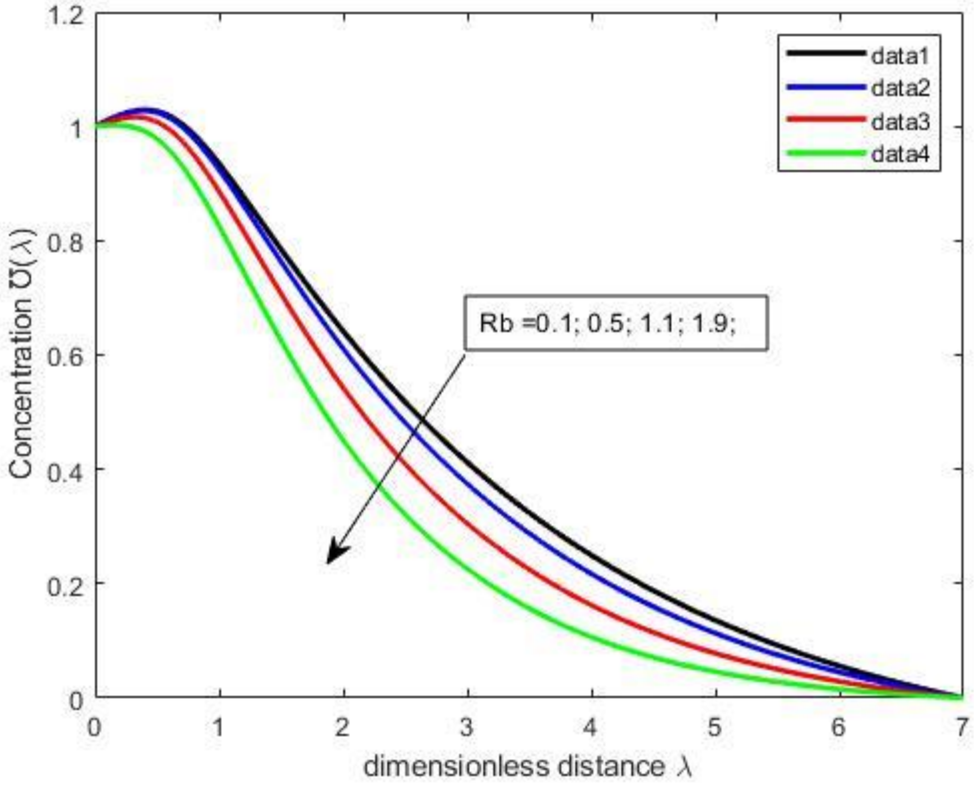

Figure 23: Variation of concentration with Bioconvection Rayleigh number

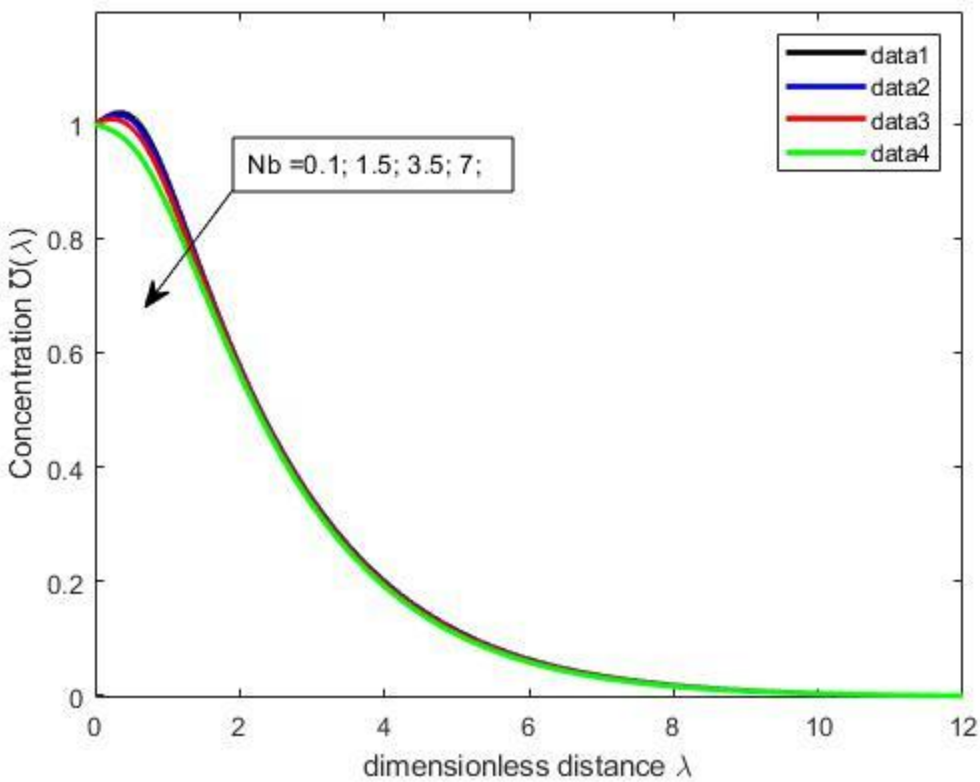

Figure 24: Variation of concentration with Brownian motion parameter

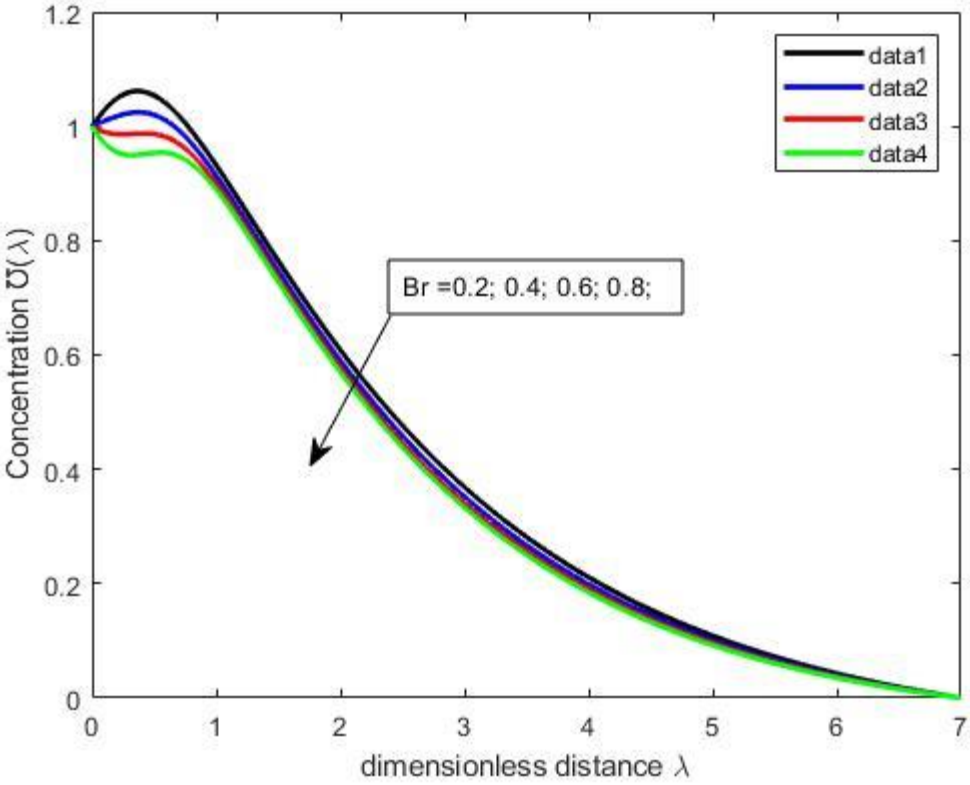

Figure 25: Variation of concentration with Brinkman number

Motile microorganisms' density profiles:

In this part, Figs. 26 – 31 are simulated to examine the influence of the Bioconvection Peclet number (Pe), Bioconvection Rayleigh parameter (Rb), Buoyancy ratio parameter (Nr), Bioconvection Lewis parameter (Lb), Hartmann number (Ha) and Microorganisms' concentration difference parameter (R) on the dimensionless motile microorganisms' density. Here, it is noticed that the dimensionless motile microorganisms' density is strongly affected by Rb and R. So, it is observed that an increase Rb and R decreases in motile microorganisms' density and motile microorganism boundary layer thickness as sketched in Figs. 27 and 31.

Conversely, the motile microorganisms' density increases with an increase in Nr and Ha as spotted in Figs. 28 and 30. The effect of Pe and Lb is outlined in Figs. 26 and 29. Here, it is observed that there is a bi-influence of Pe and Lb toward the dimensionless motile microorganisms' density. Fig. 26 shows that the dimensionless motile microorganisms' density and motile microorganism boundary layer thickness increase from the surface as Pe gets large until the dimensionless quantity $\lambda = 0.7247$, and decreases as it continues to move far away from $\lambda = 0.7247$. Also, Fig. 29 shows that the dimensionless motile microorganisms' density and motile microorganism boundary layer thickness decreases from the surface as Lb gets large until the dimensionless quantity $\lambda = 0.7247$, and increases as it continues to move far away from $\lambda = 0.7247$. As examined previously, the influence of the Magnetic field intensity, Bioconvection, and Buoyancy propel the fluids towards the stretchable surface and thereby, as expected increase the motile microorganisms' density.

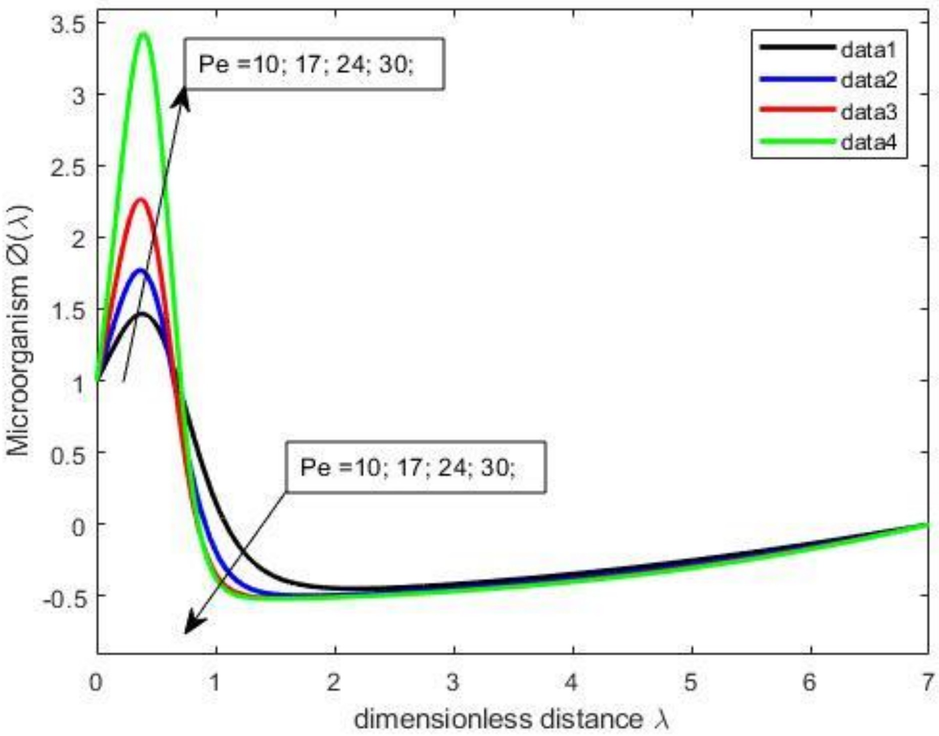

Figure 26: Variation of microorganism conservation with Bioconvection Peclet number

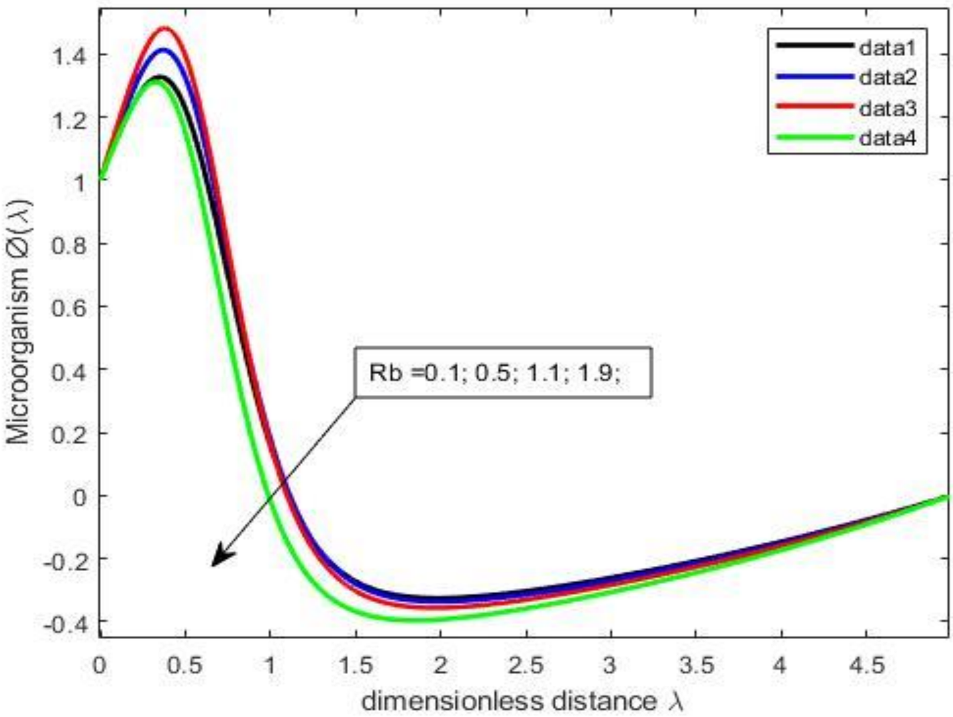

Figure 27: Variation of microorganism conservation with Bioconvection Rayleigh number

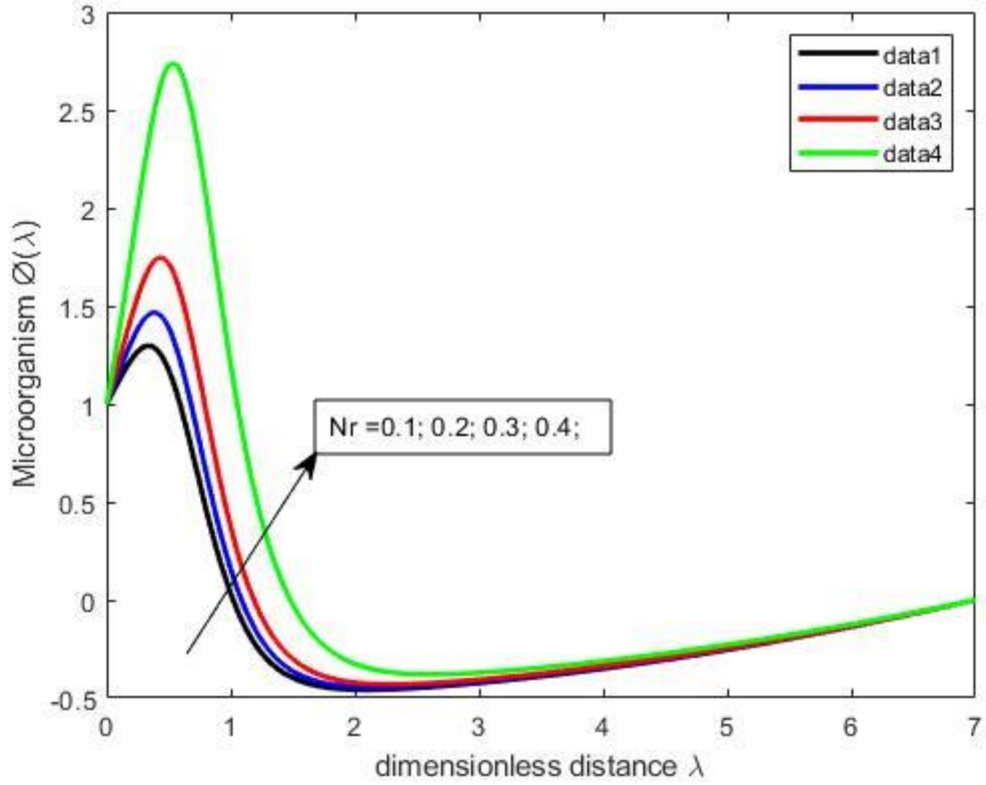

Figure 28: Variation of microorganism conservation with Buoyancy ratio parameter

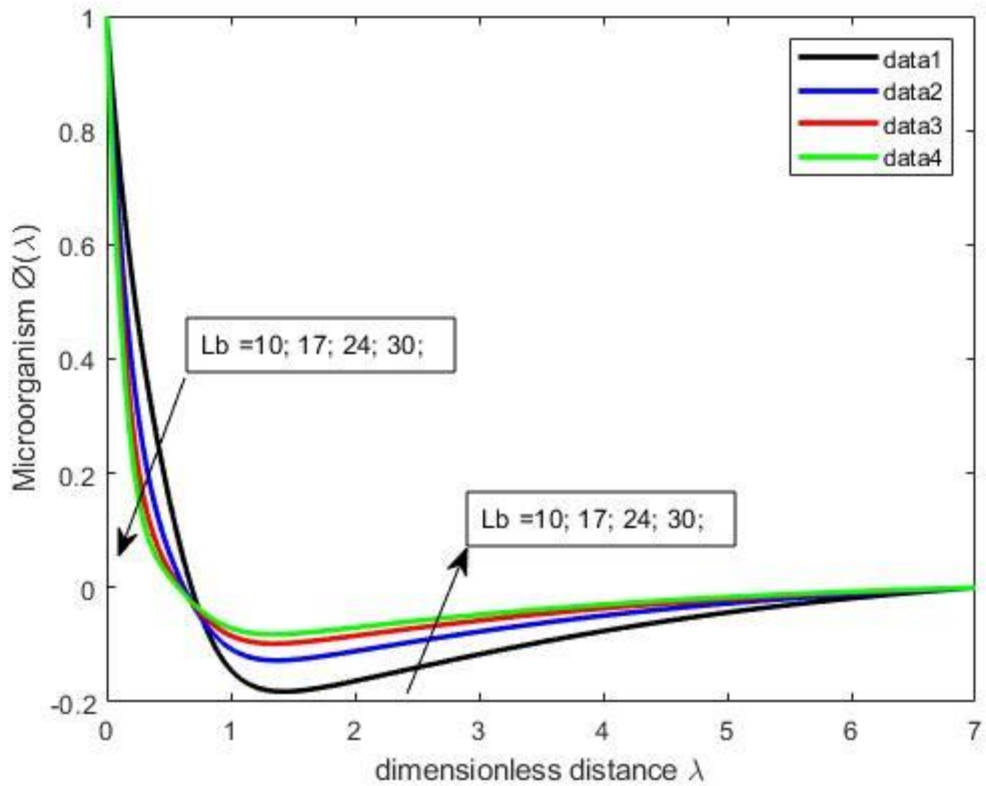

Figure 29: Variation of microorganism conservation with Bioconvection Lewis number

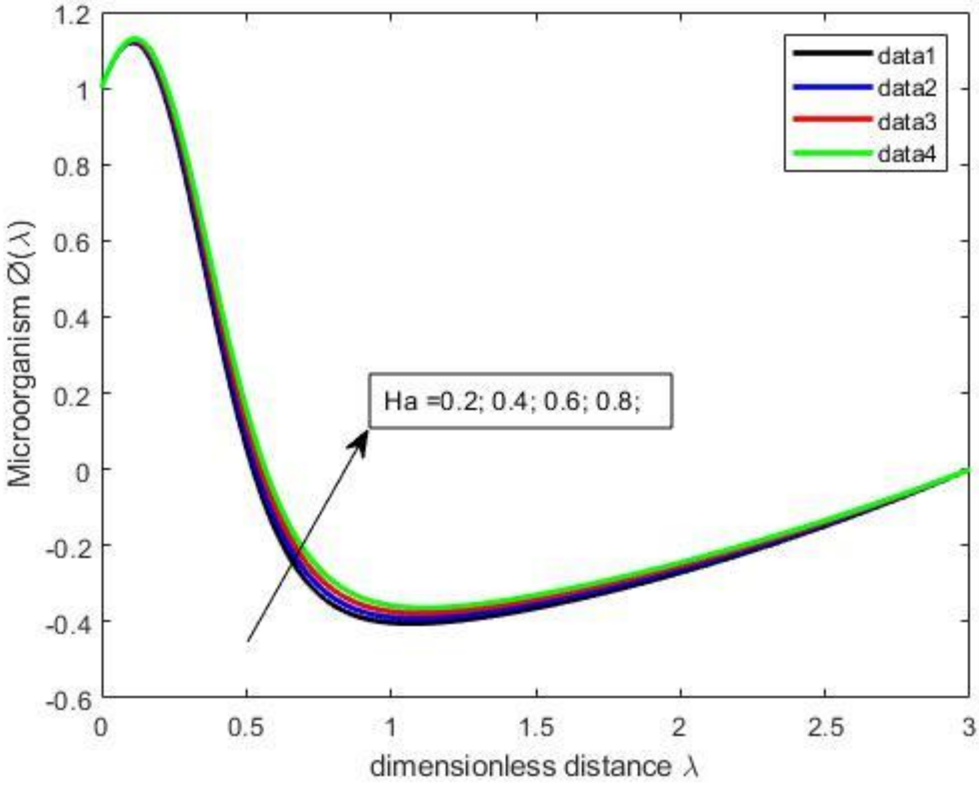

Figure 30: Variation of microorganism conservation with Hartmann number

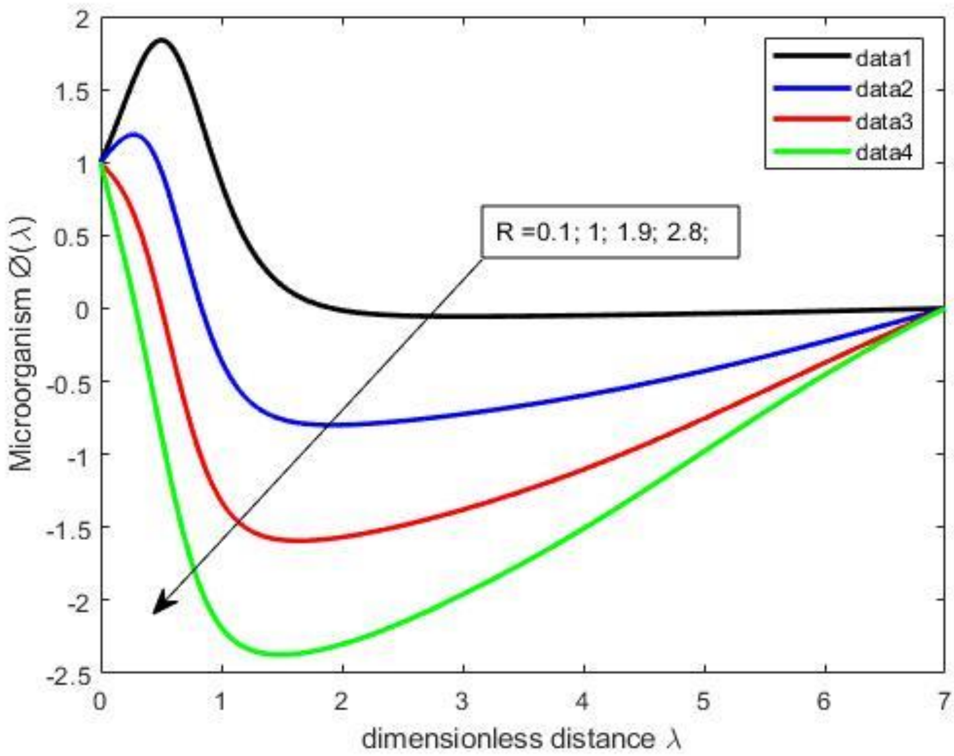

Figure 31: Variation of microorganism conservation with Microorganisms' concentration difference parameter

CONCLUSIONS

The bioconvection induced by the magnetohydrodynamics in an entirely two-dimensional, steady, incompressible flow of Walters – B nanofluid in the presence of nanomaterials and gyrotactic microorganisms over an exponentially stretching surface is numerically explored. Adopting the Buongiorno model together with the Boussinesq approximation and taking into account the effects of Brownian motion and thermophoresis, the governing equations of the flow are formulated. the governing equations are non-dimensionalised using similarity variables. The resulting first-order ordinary differential equations are numerically solved using the Shooting Technique together with the fourth-order Runge-Kutta method. Results are shown in graphs and the summary of the findings is illustrated as follows;

- Fluid velocity and momentum boundary layer thickness increase for increased We, Br, Gr, Rb, Nb and R however, the converse is observed for increased Ha, Nr, Pr and Nt.
- Fluid temperature and thermal boundary layer thickness increase for increased Br, Ha, Nb, and Nt whereas, the reverse effect is spotted for increased Pr.
- Both nanomaterial concentration and concentration boundary layer thickness upsurge for increased Pr, Nr, Ha, and Nt while, the opposite is noticed for increased Sc, Gr, Rb, Nb, and Br.
- Both dimensionless motile microorganisms' density and motile microorganism boundary layer thickness increase for increased Rb, Pe, Lb and R nevertheless, the opposite is spotted for increased Nr, Pe, Ha and Lb.

REFERENCES

1. Ahmad S, Ashraf M, Ali K. Bioconvection due to gyrotactic microbes in a nanofluid flow through a porous medium. Heliyon. 2020 Dec 1;6(12).

2. Alqarni MS. Thermo-bioconvection flow of Walter's B nanofluid over a Riga plate involving swimming motile microorganisms. AIMS Mathematics. 2022 Jan 1;7(9):16231-48.

3. Aziz A, Khan WA, Pop I. Free convection boundary layer flow past a horizontal flat plate embedded in porous medium filled by nanofluid containing gyrotactic microorganisms. International Journal of Thermal Sciences. 2012 Jun 1;56:48-57.

4. Choi SU, Eastman JA. Enhancing thermal conductivity of fluids with nanoparticles.Argonne National Lab. (ANL), Argonne, IL (United States); 1995 Oct 1.

5. Chu YM, ur Rahman M, Khan MI, Kadry S, Rehman WU, Abdelmalek Z. Heat transport and bio-convective nanomaterial flow of Walter's-B fluid containing gyrotactic microorganisms. Ain Shams Engineering Journal. 2021 Sep 1;12(3):3071-9.

6. Hamid A, Khan MI, Kumar RN, Gowda RP, Prasannakumara BC. Numerical study of bio-convection flow of magneto-cross nanofluid containing gyrotactic microorganisms with effective prandtl number approach.

7. Hayat T, Alsaedi A, Ahmad B. Thermo diffusion and diffusion thermo impacts on bioconvection Walter-B nanomaterial involving gyrotactic microorganisms. Alexandria Engineering Journal. 2021 Dec 1;60(6):5537-45.

8. Jeptoo CA. MHD Nanofluid Bioconvection Due to Gyrotactic microorganisms Past a convectively heated vertical plate. IOSR Journal of Mathematics. 2019;15:50-71.

9. Khan WA, Makinde OD. MHD nanofluid bioconvection due to gyrotactic microorganisms over a convectively heat stretching sheet. International Journal of Thermal Sciences. 2014 Jul 1;81:118-24.

10. KuznetsovAV.The onset of nanofluid bioconvection in a suspension containing bothnanoparticles and gyrotactic microorganisms. International Communications in Heat and Mass Transfer. 2010 Dec 1;37(10):1421-5.

11. Makinde OD, Animasaun IL. Bioconvection in MHD nanofluid flow with nonlinear thermal radiation and quartic autocatalysis chemical reaction past an upper surface of a paraboloid of revolution. International Journal of Thermal Sciences. 2016 Nov 1;109:159-71.

12. Mutuku WN, Makinde OD. Hydromagnetic bioconvection of nanofluid over a permeable vertical plate due to gyrotactic microorganisms. Computers & Fluids. 2014 May 22;95:88-97.

13. Nandeppanavar MM, Abel MS, Tawade J. Heat transfer in a Walter's liquid B fluid over an impermeable stretching sheet with non-uniform heat source/sink and elastic deformation. Communications in Nonlinear Science and Numerical Simulation. 2010 Jul 1;15(7):1791-802.

14. Shi QH, Hamid A, Khan MI, Kumar RN, Gowda RP, Prasannakumara BC, Shah NA, Khan SU, Chung JD. Numerical study of bio-convection flow of magneto-cross nanofluid containing gyrotactic microorganisms with activation energy. Scientific Reports. 2021 Aug 6;11(1):16030.

15. Siddiqa S, Sulaiman M, Hossain MA, Islam S, Gorla RS. Gyrotactic bioconvection flow of a nanofluid past a vertical wavy surface. International Journal of Thermal Sciences.2016 Oct 1;108:244-50.

16. Sk MT, Das K, Kundu PK. Multiple slip effects on bioconvection of nanofluid flow containing gyrotactic microorganisms and nanoparticles. Journal of Molecular Liquids. 2016 Aug 1;220:518-26.

17. Tham L, Nazar R, Pop I. Mixed convection flow over a solid sphere embedded in a porous medium filled by a nanofluid containing gyrotactic microorganisms. International Journal of Heat and Mass Transfer. 2013 Jul 1;62:647-60.

18. Uddin MJ, Khan WA, Qureshi SR, Bég OA. Bioconvection nanofluid slip flow past a wavy surface with applications in nano-biofuel cells. Chinese Journal of Physics. 2017 Oct 1;55(5):2048-63.

19. Rahman M, Haq F, Darab PC, Sallah M, Abdelmohsen SA, Fadhl BM, Makhdoum BM. Mixed convection and activation energy impacts on MHD bioconvective flow of nanofluid with irreversibility assessment. Heliyon. 2023 Jun 1;9(6).

20. Waqas H, Alghamdi M, Muhammad T, Khan MA. Bioconvection transport of magnetized Walter's B nanofluid across a cylindrical disk with nonlinear radiative heat transfer. Case Studies in Thermal Engineering. 2021 Aug 1;26:101097.