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RESEARCH ARTICLE

COEFFICIENT ESTIMATES FOR CERTAIN SUBCLASSES OF ANALYTIC AND BI-UNIVALENT FUNCTION ASSOCIATED WITH FRACTIONAL DERIVATIVE OPERATOR

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ABSTRACT

Characterizing and estimating coefficients for subclasses of analytic and bi-univalent functions using fractional derivative operators. In this paper, we propose two novel subclasses within the realm of analytic and bi-univalent functions defined in the open unit disk U. By introducing these subclasses, we derive estimates for the initial two Taylor-Maclaurin coefficients, specifically $|a_2|$ and $|a_3|$, pertaining to functions falling within these newly defined categories. This research paper sheds light on the behavior of functions in these subclasses and contributes to the broader understanding of their analytic and geometric properties.

Keywords: Analytic functions, Bi-univalent functions, Fractional derivative operators, Taylor-Maclaurin coefficients, Subclasses, Estimates

INTRODUCTION

The study of analytic and bi-univalent functions, particularly those associated with fractional derivative operators, represents a significant area of research within complex analysis. These functions play a crucial role in various mathematical and scientific applications, ranging from theoretical studies to practical implementations in fields like physics and engineering. By investigating coefficient estimates for specific subclasses of these functions, researchers aim to deepen our understanding of their properties and uncover new insights into their behavior. Such endeavors not only contribute to the advancement of pure mathematics but also have implications for diverse areas of applied sciences.

1.1 Contributions

The novel contributions of this paper are:

1. We propose two novel subclasses within the domain of analytic and bi-univalent functions, offering a refined framework for their analysis and characterization.

2. By establishing estimates for the first two Taylor-Maclaurin coefficients, we provide insights into the geometric and analytic properties of functions within these subclasses, advancing the understanding of their behaviour

METHODOLOGY

2.1 Taking a Function Class

By A, we denote the class of all functions of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

analytic in the open unit disk

$$U = \{z : z \in \mathbb{C}, and \mid z \mid < 1\}$$

where, $\mathbb{C}$, as usual, is the set of complex numbers. We also denote by S a subclass of all functions in A that are univalent in U. Some important and well-investigated subclasses of the class of univalent function S include (e.g.) the class of $S^*(\alpha)$ starlike functions of order α in U and the class K(α) of convex functions of order α in U. By definition, we get:

$$S^*(\alpha) := \{ f \in S : \Re\left(\frac{z(f'(z)}{f(z)}\right) > \alpha,\ 0 \leq \alpha < 1, z \in U\}$$

and

$$K(\alpha) := \{ f \in S : f'(0) \neq 0, \Re\left(1 + \frac{z(f''(z)}{f'(z)}\right) > \alpha,\ 0 \leq \alpha < 1, z \in U\}$$

If f and g are analytic functions in U, then we say that f is subordinate to g and write f(z) $\prec$ g(z) if there exists a Schwarz function φ , which (by definition) is analytic in U with φ (0) = 0 and | φ (z)| < 1 for all $z \in U$ such that f(z) = g(φ (z)), $z \in U$. Furthermore, if the function g is univalent in U, then we have the following equivalence:

$$f(z) \prec g(z)(\ z \in U\ ) \Leftrightarrow f(0) = g(0))\ and\ f(U) \subset g(U)$$

By using the method of differential subordination, Obradovic [21] gave some criteria of univalence, expressed by the formula $\Re\{f'(z)\} > 0$, for the linear combinations

$$\alpha\left(1 + \frac{z(f''(z)}{f'(z)}\right) + (1-\alpha)\frac{1}{f'(z)}$$

In [24], Silverman investigated an expression for the quotients of the analytic representations of convex and starlike functions. More precisely, for $0 < b \leq 1$, author consider a class of functions

$$G_b = \left\{ f \in A : \left| \frac{1 + zf''(z)/f'(z)}{zf'(z)/f(z)} - 1 \right| < b, z \in U \right\}$$

and proved that $G_b \subset S^*(2/1 + \sqrt{1+8b})$

For any $f \in S$, the Koebe one-quarter theorem [11] implies that the image of U under f contains a disk of radius 1/4. Thus, every univalent function $f \in S$ possesses the inverse function f^{-1} defined as follows:

$$f^{-1}(f(z)) = z, z \in U$$

and

$$f(f^{-1}(w)) = w, \quad |w| < r_0(f), \quad r_0(f) \le \frac{1}{4}$$

In fact, the inverse function $g = f^{-1}$ is given by the formula:

$$g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^2 - 5a_2 a_3 + a_4)w^4 + ...$$

2.2 Defining new subclass

A function $f \in A$ is called bi-univalent in U if f and f^{-1} are univalent in U. By σ we denote the class of bi-univalent functions in U given by (1). The familiar Koebe function is not an element of σ because it univalently maps the unit disk U onto the entire complex plane minus a slit along the line from $-\frac{1}{4}$ to $-\infty$. Hence, the image domain does not contain the unit disk U.

In 1985, Louis de Branges [3] proved the celebrated Bieberbach conjecture, which states that, for each $f(z) \in S$ given by the Taylor–Maclaurin series expansion (1), the following coefficient inequality is true:

$$|a_n| \le n \quad (n \in N - \{1\})$$

where, N is the set of positive integers. The class of analytic bi-univalent functions was first introduced and studied by Lewin [15] who proved that $|a_2| < 1.51$. Later, Brannan and Clunie [4] improved Lewin's result to $|a_2| \le \sqrt{2}$. Brannan and Taha [6] and Taha [29] considered certain subclasses of bi-univalent functions similar to the familiar subclasses of univalent functions formed by strongly starlike, starlike, and convex functions. They introduced bistarlike functions and bi-convex functions and established nonsharp estimates for the first two Taylor–Maclaurin coefficients $|a_2|$ and $|a_3|$. For the subsequent historical survey of functions from the class σ, see the work by Srivastava, et al. [26] (see also [5, 6]). In fact, the recent pioneering work by Srivastava, et al. [26] cited above has apparently revived the study of analytic and bi-univalent functions in recent years. It was followed by the works by Frasin and Aouf [12], Xu, et al. [31, 32], Hayami [14], and other researchers (see, e.g., [1, 2, 7–10, 13, 16–19, 22, 23, 25, 27, 28, 30]).

In the present work, we deduce estimates for the initial coefficients $|a_2|$ and $|a_3|$ of two new subclass of the class of bi-univalent functions σ

2.3. Coefficient Estimation

In this section, it is assumed that φ is an analytic function with positive real part in the unit disk U such that $\phi(0)=1, \phi'(0)=0, and\ \phi(U)$ is symmetric with respect to the real axis. This function has a Taylor series of the form:

$$\phi(z)=1+B_1z+B_2z^2+B_3z^3+\ldots,\quad B_1>0$$

Suppose that u(z) and v(z) are analytic in the unit disk U with

$$u(0)=v(0)=0, |u(z)|<1,\ \ |v(z)|<1$$

and that

$$v(z)=b_1(z)+\sum_{n=2} b_nz^n,\ v(z)=c_1z+\sum_{n=2} c_nz^n,\quad z\in U \tag{2}$$

It is well known that (see [20, p. 172])

$$|b_1|\le 1,\ \ |b_2|\le 1-|b_1|^2,\ \ |c_1|\le 1,\ \ |c_2|\le 1-|c_1|^2 \tag{3}$$

As a result of simple calculations, we conclude that

$$\phi(u(z))=1+B_1b_1z+(B_1b_2+B_2b_1^2)z^2+\ldots, z\in U \tag{4}$$

and

$$\phi(v(w))=1+B_1c_1w+(B_1c_2+B_2c_1^2)w^2+\ldots, w\in U \tag{5}$$

Definition 1:

We say that a function $f\in\sigma$ belongs to the class $H_\sigma^\lambda(\phi), \lambda\ge 1$, if the following subordinations is true:

$$\lambda\left(\frac{1+zf''(z)}{f'(z)}\right)+(1-\lambda)\frac{1}{f'(z)}\prec\phi(z),\quad \lambda\ge 1,\ \ z\in U$$

and

$$\lambda\left(\frac{1+wg''(w)}{g'(w)}\right)+(1-\lambda)\frac{1}{g'(w)}\prec\phi(w),\quad \lambda\ge 1,\ \ w\in U$$

where $g(w):=f^{-1}(w)$

Theorem 1:

If f given by (1) is in the class $H_\sigma^\lambda(\phi),\ \lambda\ge 1$, then

$$|a_2|\le\frac{B_1\sqrt{B_1}}{\sqrt{4(2\lambda-1)^2B_1+|(\lambda+1)B_1^2-4(2\lambda-1)^2B_2|}}$$

and

$$|a_3|\le\begin{cases}\dfrac{B_1}{\lambda+1}+ & \text{if } |B_2|\le B_1,\\[2mm] \dfrac{4(2\lambda-1)^2B_1|B_2|+B_1\left|(\lambda+1)B_1^2-4(2\lambda-1)^2B_2\right|}{(\lambda+1)[4(2\lambda-1)^2B_1+|(\lambda+1)B_1^2-4(2\lambda-1)^2B_2|]}, & \text{if } |B_2|>B_1.\end{cases}$$

Proof:

Let $f \in H_{\sigma}^{\lambda}(\phi), \lambda \geq 1$, Then there are analytic functions $u, v : U \to U$ given by (2) such that:

$$\lambda\left(1+\frac{zf''(z)}{f'(z)}\right)+(1-\lambda)\frac{1}{f'(z)}=\phi(z(z)), \lambda \geq 1 \tag{6}$$

and

$$\lambda\left(1+\frac{ug''(w)}{g'(w)}\right)+(1-\lambda)\frac{1}{g'(w)}=\phi(v(w)), \lambda \geq 1 \tag{7}$$

where $g(w) := f^{-1}(w)$. Since

$$= 1 + 2(2\lambda-1)a_2 z + [(9\lambda-3)a_3 + 4(1-2\lambda)a_2^2]z^2 + \cdots$$

$$= 1 + 2(2\lambda-1)a_2 z + [(9\lambda-3)a_3 + 4(1-2\lambda)a_2^2]z^2 + \cdots$$

and

$$\lambda\left(1+\frac{wg''(w)}{g'(w)}\right)+(1-\lambda)\frac{1}{g'(w)} = 1 - 2(2\lambda-1)a_2 w + \left[(10\lambda-2)a_2^2 - (9\lambda-3)a_3\right]w^2 + \cdots.$$

it follows from (4), (5), (6), and (7) that

$$2(2\lambda-1)a_2 = B_1 b_1 \tag{8}$$

$$(9\lambda-3)a_3 + 4(1-2\lambda)a_2^2 = B_1 b_2 + B_2 b_1^2 \tag{9}$$

$$-2(2\lambda-1)a_2 = B_1 c_1 \tag{10}$$

$$(10\lambda-2)a_2^2 - (9\lambda-3)a_3 = B_1 c_2 + B_2 c_1^2 \tag{11}$$

In view of (8) and (10), we get

$$b_1 = -c_1 \tag{12}$$

$$8(2\lambda-1)^2 a_2^2 = B_1^2(b_1^2 + c_1^2) \tag{13}$$

Adding (11) and (9), as a result of subsequent computations performed by using (13), we find

$$[2(1+\lambda)B_1^2 - 8(2\lambda-1)^2 B_2]a_2^2 = B_1^3(b_2 + c_2) \tag{14}$$

Relations (12), (14), together with (3) imply that

$$\left| (1+\lambda)B_1^2 - 4(2\lambda-1)^2 B_2 \right| \left|a_2^2\right| \leq B_1^3\left(1 - \left|b_1^2\right|\right) \tag{15}$$

From (8) and (15), we obtain

$$|a_2| \leq \frac{B_1\sqrt{B_1}}{\sqrt{4(2\lambda-1)^2 B_1 + |(1+\lambda)B_1^2 - 4(2\lambda-1)^2 B_2|}}$$

Further, from (11) and (9), we get

$$(1+\lambda)(9\lambda-3)a_3 = (5\lambda-1)B_1 b_2 + 2(2\lambda-1)B_1 c_2 + (9\lambda-3)B_2 b_1^2$$

Thus by virtue of (3), we find

$$(1+\lambda)|a_3| \leq B_1 + [|B_2| - B_1]\left|b_1^2\right|$$

Note that

$$\left|b_1^2\right| = \frac{4(2\lambda-1)^2}{B_1^2}\left|a_2^2\right| \leq \frac{4(2\lambda-1)^2 B_1}{4(2\lambda-1)^2 B_1 + |(1+\lambda)B_1^2 - 4(2\lambda-1)^2 B_2|}$$

This enables us to conclude that

$$|a_3| \leq \begin{cases} \dfrac{B_1}{\lambda+1} + & \text{if } |B_2| \leq B_1, \\ \dfrac{4(2\lambda-1)^2 B_1 |B_2| + B_1\left|(\lambda+1)B_1^2 - 4(2\lambda-1)^2 B_2\right|}{(\lambda+1)[4(2\lambda-1)^2 B_1 + |(\lambda+1)B_1^2 - 4(2\lambda-1)^2 B_2|]}, & \text{if } |B_2| > B_1. \end{cases}$$

Theorem 1 is proved.
If we set

$$\lambda = 1, \phi(z) = \frac{1+z}{1-z} = 1 + 2z + 2z^2 + ..., \ z \in U$$

in Definition 1 of the class of bi-univalent functions $H_\sigma^\lambda(\phi)$, then we determine the class of bi-convex functions $H_\sigma(\phi)$ given by Definition 2:

Definition 2:

A function $f \in \sigma$ is said to be in the class $H_\sigma(\phi)$, if the following conditions are true:

$$\left(\frac{1+zf''(z)}{f'(z)}\right) \prec \phi(z), \qquad z \in U$$

and

$$\left(\frac{1+zf''(w)}{f'(w)}\right) \prec \phi(w), \qquad w \in U$$

where $g(w) := f^{-1}(w)$

2.4. Corollaries

By using the parameter setting of Definition 2 in Theorem 1, we get the following corollary:

Corollary 1:

Let a function $f \in H_{\sigma}^{\lambda}(\phi)$ be given by (1). Then

$$|a_2| \geq 1 \text{ and } |a_3| \geq 1$$

This is a special case of Theorem 4.1 (with β = 0) presented in the work by Brannan and Taha [6]

Definition 3:

A function $f \in \sigma$ is said to be in the class $K_{\sigma}(\phi)$ if and only if

$$\frac{1+\frac{zf''(z)}{f'(z)}}{\frac{zf'(z)}{f(z)}} \prec \phi(z), \frac{1+\frac{wg''(w)}{g'(w)}}{\frac{wg'(w)}{g(w)}} \prec \phi(w)$$

where $g(w) := f^{-1}(w)$

Theorem 2:

If f given by (1) belongs to the class $K_{\sigma}(\phi)$, then

$$|a_2| \leq \frac{B_1\sqrt{B_1}}{\sqrt{B_1+|B_2|}}$$

and

$$|a_3| \leq \begin{cases} \frac{B_1}{4}, & \text{if } B_1 \leq \frac{1}{4}, \\ \left[1-\frac{1}{4B_1}\right]\frac{B_1^3}{B_1+|B_2|}+\frac{B_1}{4}, & \text{if } B_1 > \frac{1}{4}. \end{cases}$$

Proof

Let $f \in K_{\sigma}^{\lambda}(\phi)$. Then there are analytic functions $u, v : U \to U$ given by (2) such that

$$\frac{1+\frac{zf''(z)}{f'(z)}}{\frac{zf'(z)}{f(z)}} = \phi(u(z)) \tag{16}$$

and

$$\frac{1+\frac{wg''(w)}{g'(w)}}{\frac{wg'(w)}{g(w)}}=\phi\big(v(w)\big) \tag{17}$$

where $g(w):=f^{-1}(w)$. Since

$$\frac{1+\frac{zf''(z)}{f'(z)}}{\frac{zf'(z)}{f(z)}}=1+a_2z+4(a_3-a_2^2)z^2+\cdots$$

and

$$\frac{1+\frac{wg''(w)}{g'(w)}}{\frac{wg'(w)}{g(w)}}=1-a_2w-4(a_3-a_2^2)w^2+\cdots,$$

it follows from (4), (5), (16), and (17) that

$$a_2=B_1b_1 \tag{18}$$

$$4(a_3-a_2^2)=B_1b_2+B_2b_1^2 \tag{19}$$

$$-a_2=B_1c_1 \tag{20}$$

$$-4(a_3-a_2^2)=B_1c_2+B_2c_1^2 \tag{21}$$

From (18) and (20), we get

$$b_1=-c_1 \tag{22}$$

and

$$2a_2^2=B_1^2(b_1^2+c_1^2) \tag{23}$$

If we add (19) and (21), then, as a result of subsequent computations with the help of (23), we find

$$2B_2a_2^2=-B_1^3(b_2+c_2) \tag{24}$$

Relations (22) and (24), together with (3), give the inequality

$$|B_2|\left|a_2^2\right|\le B_1^3\left(1-\left|b_1^2\right|\right) \tag{25}$$

It follows from (18) and (25) that

$$|a_2|\le\frac{B_1\sqrt{B_1}}{\sqrt{B_1+|B_2|}}$$

Further, by using (19) and (21), we find

$$8a_3 = 8a_2^2 + B_1(b_2 - c_2) \qquad (26)$$

It follows from (3), (18), (22) and (26), that

$$\begin{aligned} |a_3| &\le a_2^2 + \frac{B_1}{4}\left(1 - |b_1^2|\right) \\ &= \left(1 - \frac{1}{4B_1}\right) a_2^2 + \frac{B_1}{4} \\ &\le \begin{cases} \dfrac{B_1}{4}, & \text{if } B_1 \le \dfrac{1}{4} \\ \left[1 - \dfrac{1}{4B_1}\right] \dfrac{B_1^3}{B_1 + |B_2|} + \dfrac{B_1}{4}, & \text{if } B_1 > \dfrac{1}{4} \end{cases} \end{aligned}$$

Theorem 2 is proved.

CONCLUSION

In conclusion, this paper introduces two novel subclasses of analytic and bi-univalent functions within the framework of fractional derivative operators, providing insights into their coefficient estimates. By characterizing and estimating coefficients for these subclasses, we gain a deeper understanding of their behavior and properties in the open unit disk U. The derived estimates for the initial Taylor-Maclaurin coefficients contribute to our knowledge of these specialized functions and their geometric properties. Additionally, the established theorems and corollaries offer essential bounds and subordination principles for bi-univalent and bi-convex functions, further enriching the field of function theory. Future research endeavors may explore broader classes of functions or delve into the implications of these findings across various mathematical contexts, thereby advancing our comprehension of com.

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