

## RESEARCH ARTICLE

## On the Choice of Compressor Pressure in the Process of Pneumatic Transport to Increase Energy Saving

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### ABSTRACT

In this paper, we consider a possibility to choose value of pressure of devices for pneumatic transport to increase energy saving. We also introduce an analytical approach for the prognosis of transport of free-flowing materials to estimate velocity of the transport and choosing of the required value of pressure.

**Key words:** Energy saving, pneumatic transport, prognosis of transport

### INTRODUCTION

At present, in many industries, during the processing of various bulk materials, their pneumatic transport through pipelines<sup>[1-4]</sup> is used. Such industries include the production and processing of building materials, agricultural, food, and pharmaceutical products. For example, in the construction industry, cement, plaster, and lime are transported through pneumatic conveying systems. The advantages of pneumatic transport include high productivity, large radii of action, complete absence of losses of transported material in pneumatic lines, high environmental characteristics of plants, and the possibility of building branched systems adapted to complete control automation. The disadvantages of pneumatic transport are the relatively high specific consumption of electricity per unit mass of transported material and the wear of pneumatic lines due to the abrasive effect.

In the framework of this paper, a pipeline in the form of a straight circular cylinder [Figure 1] is considered. Through its left end is a put in free-flowing loose material. The main aim of this work is to choose the pressure value, at which energy saving would increase when pneumatic transport of this material. The accompanying goal of this work is the formation of an analytical technique for predicting the transport of bulk materials to assess the transfer rates and the choice of the desired pressure.

### METHOD OF SOLUTION

To solve our aim, we determine the spatiotemporal distribution of the velocity in the pipeline in question and perform its analysis. The required spatiotemporal distribution of the velocity field was calculated by solving of the Navier–Stokes equation.<sup>[5-7]</sup>

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}, \nabla) \vec{v} = \frac{\vec{F}(z)}{\rho S h} - \frac{1}{\rho} \text{grad}(P) + \nu \Delta(\vec{v}) + \frac{\nu}{3} \text{grad}[\text{div}(\vec{v})] \quad (1)$$

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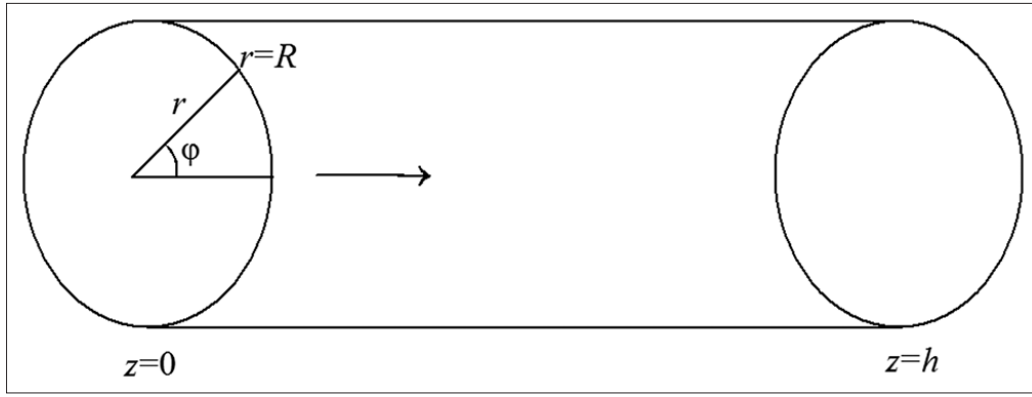


Figure 1: Cylindrical piping

Where  $\vec{v} = \vec{v}(r, z, t)$  is the velocity of transport of the considered free-flowing material,  $\vec{F}(z) = P(z)S\vec{k}$ ,  $S$  is the pipe section area,  $\vec{k}$  is the unit vector along the axis  $Oz$ ,  $P(z) = P_k - (P_k - P_a)z/h$ ,  $P_k$  is the compressor pressure,  $P_a$  is the atmosphere pressure,  $\rho$  is the gas density, and  $\nu$  is the kinematic viscosity. The boundary and initial conditions to these equations could be written in the following form.

$$\vec{v}(0, z, t) \neq \infty, \left. \frac{\partial \vec{v}(r, z, t)}{\partial r} \right|_{r=R} = 0, \vec{v}(r, z, 0) = 0, \vec{v}(r, 0, t) = \vec{v}_{in}, \vec{v}(r, h, t) = \vec{v}_{out}, \vec{v}(r, z, 0) = 0 \quad (2)$$

The projections of the velocity defined by Eq. (1) in the cylindrical coordinate system have the following form,

$$\begin{aligned} \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} &= \nu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_r}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_z}{\partial r} \right) \right] + \frac{\nu}{3} \frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{\partial v_r}{\partial z} \right] \\ \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} &= \frac{S}{\rho} \left[ P_k - (P_k - P_a) \frac{z}{h} \right] + \nu \left( \frac{\partial^2 v_r}{\partial z^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \frac{\nu}{3} \frac{\partial}{\partial z} \left[ \frac{1}{r} \frac{\partial}{\partial r} (r v_z) + \frac{\partial v_z}{\partial z} \right] - S \frac{P_k - P_a}{\rho h} \end{aligned} \quad (3)$$

Then, we transform the Eq. (3) to the following integral form,

$$\begin{aligned} v_r = v_r + \frac{\Theta}{R^4} &\left[ \frac{\nu}{3} \left( r^2 \int_0^t \int_0^z (z-v) \frac{\partial v_r}{\partial r} dv d\tau - r \int_0^t \int_0^z (z-v) v_r dv d\tau - 2 \int_0^t \int_0^r \int_0^z v_r dv du d\tau \right. \right. \\ &+ r^2 \int_0^t \int_0^z v_r dv d\tau \left. \right) - \frac{r^2}{2} \int_0^t \int_0^z (z-v) \frac{\partial v_r}{\partial r} dv d\tau + r \int_0^t \int_0^z (z-v) v_r dv d\tau - \int_0^t \int_0^r \int_0^z (z-v) v_r dv du d\tau \\ &- \frac{r^2}{2} \int_0^t \int_0^z (z-v) \frac{\partial v_z}{\partial r} dv d\tau + \nu \left( r^2 \int_0^t \int_0^z (z-v) \frac{\partial v_r}{\partial r} dv d\tau - r \int_0^t \int_0^z (z-v) v_r dv d\tau + \int_0^t \int_0^r \int_0^z (z-v) \right. \\ &\times v_r dv du d\tau + \int_0^t \int_0^r \int_0^z (z-v) v_z dv du d\tau + r^2 \int_0^t \int_0^z (z-v) \frac{\partial v_z}{\partial r} dz d\tau - r \int_0^t \int_0^z (z-v) v_z dv d\tau \left. \right) \\ &\left. + r \int_0^t v_z d\tau - \int_0^t \int_0^r v_z du d\tau - \int_0^t \int_0^r u^2 \int_0^z (z-v) v_r dv du \right] \end{aligned} \quad (3a)$$

$$\begin{aligned}
 v_z = v_z + \frac{\Theta}{R^4} & \left\{ v \left( \int_0^t \int_0^r u^2 v_r du d\tau + \int_0^t \int_0^r u^2 v_z du d\tau \right) - \int_0^t \int_0^r u^2 \int_0^z (z-v) v_r \frac{\partial v_z}{\partial u} dv du d\tau + r^3 \frac{tS}{3\rho} \right. \\
 & \times z^2 \left[ \frac{P_k}{2} - (P_k - P_a) \frac{z}{6h} \right] - r^3 z^2 t S \frac{P_k - P_a}{6\rho h} + \frac{v}{3} \left( \int_0^t \int_0^r u \int_0^z v v_z dv du d\tau + \int_0^t \int_0^r u^2 \int_0^z \frac{\partial v_r}{\partial u} dv du d\tau \right. \\
 & \left. + \int_0^t \int_0^r u^2 \int_0^z \frac{\partial v_z}{\partial z} dv du d\tau \right) - v v_{\sigma x} t \frac{r^3}{3} - \frac{1}{2} \int_0^t \int_0^r u^2 \int_0^z v v_z^2 dv du d\tau + z \left[ \frac{1}{h} \int_0^r u^2 \int_0^h (h-v) v_z dv du \right. \\
 & \left. - \frac{v}{h} \left( \int_0^t \int_0^r u^2 v_{out} du d\tau + \int_0^t \int_0^r u^2 v_{out} du d\tau \right) - r^3 (2P_k + P_a) \frac{t h S}{18\rho} + \frac{1}{h} \int_0^t \int_0^r u^2 \int_0^h (h-v) v_r \frac{\partial v_z}{\partial u} dv du d\tau \right. \\
 & \left. + r^3 t S \frac{P_k - P_a}{6\rho} - \frac{v}{3h} \left( \int_0^t \int_0^r u \int_0^h v v_z dv du d\tau + \int_0^t \int_0^r u^2 \int_0^h \frac{\partial v_r}{\partial u} dv du d\tau + \int_0^t \int_0^r u^2 \int_0^h \frac{\partial v_z}{\partial z} dv du d\tau \right) \right. \\
 & \left. + v v_{in} t \frac{r^3}{3h} + \frac{1}{2h} \int_0^t \int_0^r u^2 \int_0^h v v_z^2 dv du d\tau \right] - \int_0^t \int_0^r u^2 \int_0^z (z-v) v_z dv du \left. \right\}
 \end{aligned}$$

Next, we solve this equation by averaging the functional corrections.<sup>[6-8]</sup> In the framework of this method, to obtain the first approximation of the required projections of the velocity of the material under consideration, we replace them with the yet unknown mean values  $v_r \rightarrow \alpha_{r1}$ ,  $v_z \rightarrow \alpha_{z1}$ . Then, the relations for these approximations take the following form,

$$\begin{aligned}
 v_{r1} = \alpha_{r1} + \frac{\Theta}{R^4} & \left[ \frac{v}{3} \left( -\alpha_{r1} r t \frac{z^2}{2} - \alpha_{r1} z r^2 t + r^2 \alpha_{r1} z t \right) + \alpha_{z1} r t - \alpha_{z1} r t - \alpha_{r1} r^3 \frac{z^2}{6} \right] \tag{4} \\
 v_{z1} = \alpha_{z1} + \frac{\Theta}{3R^4} & \left\{ v t r^3 (\alpha_{r1} + \alpha_{z1}) + r^3 z^2 \frac{tS}{2\rho} \left[ P_k - (P_k - P_a) \frac{z}{3h} \right] - r^3 z^2 t S \frac{P_k - P_a}{2\rho h} + \alpha_{z1} r^2 z^2 \right. \\
 & \times t \frac{v}{4} + z \left[ \alpha_{z1} r^3 \frac{h}{2} - t r^2 \frac{v}{h} (v_{out} + v_{in}) + t S r \frac{P_k - 3P_a}{6\rho} + v v_{in} t \frac{r^3}{h} + \alpha_{z1}^2 h \frac{r^2}{4} - \alpha_{z1} r^2 z t \frac{v}{h} \right] \\
 & \left. - v v_{in} t r^3 - \alpha_{z1}^2 r^3 t \frac{z^2}{4} - \alpha_{z1} r^2 \frac{z^2}{2} \right\}
 \end{aligned}$$

The average values of  $\alpha_{r1}$  and  $\alpha_{z1}$  are determined using the standard relations,

$$\alpha_{r1} = \frac{1}{\pi \Theta R^2 L} \int_0^R \int_0^L \int_0^L v_{r1} dz dr dt, \quad \alpha_{z1} = \frac{1}{\pi \Theta R^2 L} \int_0^R \int_0^L \int_0^L v_{z1} dz dr dt \tag{5}$$

Substitution of relations (4) into (5) leads to the following result,

$$\alpha_{r1} = \frac{3\alpha_{z1} v \Theta}{R^2 + v \Theta}; \quad \alpha_{z1} = \frac{v v_{\sigma E} h \Theta^2 R^5 + v_{\sigma KE} v h \Theta^2 R^5 - P_0 \Theta^2 R^2 h^2 S / 24 \rho}{15 \Theta^2 R^2 \left[ h R^2 \frac{v \Theta}{144} - h^3 \frac{v \Theta}{144} + h^3 \frac{R^2}{180} + v R^3 \frac{\Theta}{15} + \frac{v^2 \Theta^2 R^3}{5(R^2 + v \Theta)} \right]} \tag{6}$$

The first approximation of the required value of the compressor pressure (at the inlet to the pipeline) was determined from the condition that the output velocity of the material is equal to zero. In the final form, the relation for the required value of the compressor pressure in the first-order approximation by the method of averaging the functional corrections takes the following form,

$$P_{\kappa 1}(0) = \frac{18\rho}{h^2\Theta S(h^2 - R^2)} \left[ (v_{in} - \alpha_{z1})R^3h^2\Theta - v_h\Theta^2(\alpha_{r1} + \alpha_{z1})\frac{R^2}{3} - \alpha_{z1}hR\frac{\Theta v}{12} + v_{v_{in}}\Theta\frac{R^2}{3} \right. \\ \left. + h\Theta\frac{SP_0}{6\rho} - 2P_0R^2\frac{\Theta S}{9\rho} + \alpha_{z1}^2R^2\Theta\frac{h}{12} - hR\Theta\left(\alpha_{z1}R\frac{h}{6} - \Theta v_{in}\frac{v}{3} + v_{v_{in}}\Theta\frac{R}{3h} + \alpha_{z1}^2\frac{h}{12}\right) \right] \quad (6)$$

The second approximation of the velocity of free-flowing material is defined in the framework of the standard procedure for the method of averaging of functional corrections.<sup>[6-8]</sup> In the framework of this method, to obtain the first approximation of the required projections of the velocity of the material under consideration, we replace them by the following sums:  $v_r \rightarrow \alpha_{r2} + v_{r1}$ ,  $v_z \rightarrow \alpha_{z2} + v_{z1}$ , where  $\alpha_{r2}$  and  $\alpha_{z2}$  are the average values of the second approximation of velocities. Then, the relations for these approximations assume the following form,

$$v_{r2} = \alpha_{r2} + v_{r1} + \left( \frac{v}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial}{\partial r} \int_0^t \int_0^z (z-u)(\alpha_{r2} + v_{r1}) du d\tau \right] + \frac{v}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial}{\partial r} \int_0^t \int_0^z (\alpha_{z2} + v_{z1})(z-u) du d\tau \right] \right. \\ \left. + \frac{v}{3} \frac{\partial}{\partial r} \int_0^t \int_0^z (\alpha_r + v_{r1}) du d\tau + \frac{v}{3} \frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \int_0^t \int_0^z (z-u)(\alpha_{r2} + v_{r1}) du d\tau \right) \right] + (h-z) \right. \\ \left. \times (h-z) - \frac{\partial}{\partial r} \int_0^t \int_0^z (z-u)(\alpha_{r2} + v_{r1})^2 du d\tau - z \int_0^t \int_0^z (\alpha_{r2} + v_{r1}) \frac{\partial v_{z1}}{\partial u} du d\tau - \int_0^t \int_0^z u(\alpha_{r2} + v_{r1}) \right. \\ \left. \times \frac{\partial v_{z1}}{\partial u} du d\tau + \frac{z}{h} \int_0^h (h-u)(\alpha_{r2} + v_{r1}) du - \frac{v}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \int_0^t \int_0^h (h-u)(\alpha_{r2} + v_{r1}) du d\tau \right) + \int_0^t \int_0^h u \right. \\ \left. \times (\alpha_{r2} + v_{r1}) \frac{\partial v_{z1}}{\partial u} du d\tau - \int_0^t \int_0^h (\alpha_{z2} + v_{z1}) \frac{\partial v_{r1}}{\partial u} du d\tau - \frac{\partial}{\partial r} \left[ r \frac{\partial}{\partial r} \int_0^t \int_0^h (h-u)(\alpha_{z2} + v_{z1}) du d\tau \right] \right. \\ \left. \times \frac{v}{r} - \frac{v}{3} \frac{\partial}{\partial r} \int_0^t \int_0^h (\alpha_{r2} + v_{r1}) du d\tau - \frac{v}{3} \frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \int_0^t \int_0^h (h-u)(\alpha_{r2} + v_{r1}) du d\tau \right) \right] \right. \\ \left. + \frac{\partial}{\partial r} \int_0^t (h-u) \int_0^h (\alpha_{r2} + v_{r1})^2 du d\tau \right\} - \int_0^z (z-u)(\alpha_{r2} + v_{r1}) du \Bigg) \frac{1}{hR} \quad (7)$$

The average values of  $\alpha_{r2}$  and  $\alpha_{z2}$  have been determined standardly.<sup>[6-8]</sup>

$$\alpha_{r2} = \frac{2}{hR^2\Theta} \int_0^R \int_0^h \int_0^t (v_{r2} - v_{r1}) dz dr dt, \quad \alpha_{z2} = \frac{2}{hR^2\Theta} \int_0^R \int_0^h \int_0^t (v_{z2} - v_{z1}) dz dr dt \\ \alpha_{r2} = \frac{2}{hR^2\Theta} \int_0^R \int_0^h \int_0^t (v_{r2} - v_{r1}) dz dr dt, \quad \alpha_{z2} = \frac{2}{hR^2\Theta} \int_0^R \int_0^h \int_0^t (v_{z2} - v_{z1}) dz dr dt \quad (8)$$

Substituting relation (7) into relation (8) leads to the following result

$$\alpha_{r2} = \frac{b^2d}{a^2f} + \frac{be}{af} + \frac{b}{a} \sqrt{\left(\frac{db}{fa} + \frac{e}{f}\right)^2 - 4\frac{dc}{fa} + 4\frac{g}{f} - \frac{c}{a}}, \\ \alpha_{z2} = \frac{bd}{af} + \frac{e}{f} + \sqrt{\left(\frac{db}{fa} + \frac{e}{f}\right)^2 - 4\frac{dc}{fa} + 4\frac{g}{f}},$$

Where,

$$\begin{aligned}
 a = & h \left( \Theta R^2 \frac{h^3}{12} - \alpha_{z1} \Theta^2 \frac{h^4}{12} - \frac{\Theta^2}{R} \left[ S \frac{\Theta h^2}{16\rho} \left( P_a + \frac{P_a - P_\kappa}{3} \right) + \alpha_{r1} v \Theta \frac{h^3}{4} + \alpha_{z1} h^3 \Theta \frac{v}{3} - \alpha_{z1} \frac{h^5}{4} \right. \right. \\
 & + \frac{h^3}{3} \left[ \alpha_{z1} \frac{h^2}{2} - \Theta S (P_a - 2P_\kappa) \frac{h^2}{12} - v_{out} \frac{v \Theta}{6h} + (P_\kappa - P_a) \frac{h \Theta}{4\rho} - v \frac{\alpha_{z1}}{3R} + \frac{\Theta}{4} \alpha_{z1}^2 \right] + \frac{v}{3} v_{in} \times h^3 \Theta \\
 & - \alpha_{z1}^2 h^4 \frac{\Theta}{12} - \alpha_{r1} v \Theta \frac{h^3}{10} - \frac{h^5}{5} \left( \frac{\alpha_{z1}}{2} + \Theta S \frac{5P_\kappa - 4P_a}{6\rho h^3} - v_{out} \frac{v \Theta}{3h^2} - v \frac{\alpha_{z1}}{3Rh} + \frac{\Theta}{4} \alpha_{z1}^2 \right) + \alpha_{z1} \times \frac{h^5}{8} \\
 & - \frac{\Theta S}{24h\rho} \left( P_a + \frac{P_a - P_\kappa}{3} \right) - \left[ \alpha_{z1} \Theta^2 \frac{h^2}{6} - \Theta^3 h^2 (P_a - 2P_\kappa) \frac{S}{24} - v_{out} \Theta \frac{v^2}{6h} + (P_\kappa - P_a) \frac{\Theta^2 h}{6\rho} \right. \\
 & \left. - \alpha_{z1} \Theta^2 \frac{v}{9R} + \frac{\Theta^2}{6} \alpha_{z1}^2 \right] \frac{h^3}{3} + \frac{h^5}{5} \left( \frac{\alpha_{z1}}{6} \Theta^2 + \Theta^3 S \frac{5P_\kappa - 4P_a}{24\rho h^3} - v_{out} \frac{v \Theta^4}{12h^2} - v \frac{\alpha_{z1} \Theta^2}{9Rh} + \frac{\Theta^3}{8} \alpha_{z1}^2 \right) , \\
 & - \alpha_{z1} \Theta^2 \frac{v}{9R} + \frac{\Theta^2}{6} \alpha_{z1}^2 \left] \frac{h^3}{3} + \frac{h^5}{5} \left( \frac{\alpha_{z1}}{6} \Theta^2 + \Theta^3 S \frac{5P_\kappa - 4P_a}{24\rho h^3} - v_{out} \frac{v \Theta^4}{12h^2} - v \frac{\alpha_{z1} \Theta^2}{9Rh} + \frac{\Theta^3}{8} \alpha_{z1}^2 \right) \right. \\
 & - \alpha_{z1} h^3 \frac{v}{6} \Theta - v_{in} h^3 \Theta \frac{v}{3} + \alpha_{z1}^2 \Theta \frac{h^4}{16} - h^2 \frac{\Theta^2 S}{24\rho} \left( P_a + \frac{P_a - P_\kappa}{3} \right) - \alpha_{r1} v \Theta^3 \frac{h^3}{8} + \alpha_{z1} \Theta^2 \frac{h^5}{12} - \frac{\alpha_{z1}}{6} \\
 & \times v \Theta^3 h^3 - \frac{1}{3} v_{in}^2 v h^3 \Theta + \alpha_{vz1}^2 h^4 \frac{\Theta^3}{24} + \frac{\Theta^3 S}{48h\rho} \left( P_a + \frac{P_a - P_\kappa}{3} \right) - \alpha_{z1} \Theta^2 \frac{h^5}{24} + \alpha_{vz1} v \Theta^3 \frac{h^3}{12} + v \\
 & \times \alpha_{r1} \Theta^3 \frac{h^3}{20} + \frac{v_{in}}{12} \Theta^3 v z^3 - \Theta^3 \alpha_{vz1}^2 \frac{z^4}{32} \left. \right\} + R^2 v_{out} \Theta^2 \frac{h^2}{8} - 2h^3 \frac{\Theta^2}{3R} \left( \Theta S R^2 \frac{53P_a - 88P_\kappa}{1440\rho h^2} + \alpha_{z1} \frac{R^2}{24} \right. \\
 & \left. - v R^2 \frac{v_{out} \Theta}{36h^2} - \alpha_{z1} \frac{v R}{9h} + \alpha_{z1}^2 \frac{R^2 \Theta}{48h} + \alpha_{r1} v \frac{\Theta R^2}{24h^2} + v \alpha_{z1} \frac{\Theta R^2}{18h^2} + v R^2 \frac{v_{in} \Theta}{18h^2} \Theta \right) - \alpha_{z1} h^2 R^2 \frac{\Theta^2}{4}
 \end{aligned}$$

$$b = \Theta^2 R^2 \frac{h^3}{4} (v_{out} - v_{in}),$$

$$\begin{aligned}
 c = & \alpha_{r1} \Theta R^2 \frac{h^4}{4} + (v_{out}^2 - v_{in}^2) \Theta^2 R \frac{h^4}{6} + \alpha_{r1} \Theta R \frac{h^5}{8} + \alpha_{r1} v h^2 R^2 \frac{\Theta^2}{24} \\
 & + \alpha_{r1} v R \Theta^2 \frac{h^3}{48} - \alpha_{r1} \frac{\Theta^2}{12} h^2 (5P_\kappa - 4P_a) \frac{\Theta S}{36\rho} \left[ \alpha_{z1} \frac{h^5}{4} - v \Theta v_{out} \frac{h^3}{18} - \alpha_{z1} \frac{v h^4}{6R} + \alpha_{z1}^2 h^4 \frac{\Theta}{12} \right] \\
 & - \frac{\alpha_{r1} \Theta^2}{12R^2} h^4 \left[ \alpha_{z1} \frac{h^3}{4} + (17P_\kappa - 13P_a) \frac{\Theta S}{36\rho} - v_{out} v \Theta \frac{h^4}{18} - v \Theta h^2 \frac{\alpha_{z1}}{6R} + \alpha_{z1}^2 h^2 \frac{\Theta}{6} \right] + (v_{out}^2 - v_{in}^2) \\
 & \times \frac{v h^2 R}{108} - (v_{out}^2 - v_{in}^2) R \Theta^2 \frac{h^4}{72} + (5P_\kappa - 8P_a) \frac{\alpha_{r1} \Theta^3 R S}{1296\rho h} + \alpha_{z1} \alpha_{r1} (5P_a - P_\kappa) \Theta^3 \frac{R h^3 S}{144\rho} \\
 & + \Theta^2 R^2 \times (v_{out}^2 - v_{in}^2) \frac{h^3}{4} + \alpha_{r1} R \Theta^2 \frac{h^2}{8} \left( \alpha_{z1} \frac{h^2}{4} + \Theta S \frac{5P_\kappa - 4P_a}{18\rho h^4} - v_{out} \Theta \frac{v}{9} - \alpha_{z1} \frac{v h}{6R} + \alpha_{z1}^2 \frac{\Theta}{6} \right) \\
 & - \alpha_{r1} R \times \Theta^3 S h^3 \frac{8P_a + P}{10368\rho} + \left[ \alpha_{z1} \frac{h}{12} + (5P_\kappa - 4P_a) \frac{h S}{36\rho} + \frac{\alpha_{z1}^2 \Theta}{12h} - \Theta \frac{v v_{out}}{18h^2} - \frac{\alpha_{z1} v \Theta}{18Rh} \right] \\
 & + \alpha_{r1} v_{out} \Theta^2 \frac{h^3}{8} + h^3 (v_{out}^2 - v_{in}^2) \frac{\Theta^3}{24} + \Theta^3 (v_{out}^2 - v_{in}^2) \frac{h^4 P_a}{576} + h^4 \Theta^3 (v_{out}^2 - v_{in}^2) \frac{P_a - P_\kappa}{1440} - \alpha_{z1} \alpha_{r1} h^3 \Theta^2 \frac{R}{8}
 \end{aligned}$$

$$\begin{aligned}
 & -\alpha_{r1}\Theta^2\frac{h^2}{8}\times\left[\alpha_{z1}\frac{h^3}{4}+(3P_{\kappa}-2P_a)\frac{\Theta S}{18\rho h}-v_{out}\Theta\frac{hv}{18}-\alpha_{z1}\Theta\frac{vh^2}{6R}+\alpha_{z1}^2h^2\frac{\Theta}{6}\right]-\alpha_{z1}\alpha_{r1}h^4\Theta^2\frac{R}{8}-\alpha_{v,1}h^2 \\
 & \times\Theta^2\left[\alpha_{z1}\frac{h^3}{8}+(4P_{\kappa}+P_a)\frac{\Theta S}{36\rho}-v_{out}vh\frac{\Theta}{18}-\alpha_{z1}\frac{v\Theta h^2}{12R}-\alpha_{z1}^2h^2\frac{\Theta}{12}-\alpha_{z1}\frac{h^3}{4}+\alpha_{r1}vh\frac{\Theta}{3}\right. \\
 & \left.+\frac{4}{9}v_{in}vh\Theta+\frac{4}{9}hv\alpha_{z1}\Theta\right]-\alpha_{z1}R\left(\alpha_{z1}\frac{h^2}{4}+\Theta S\frac{5P_{\kappa}-4P_a}{18\rho h^2}-v_{out}v\frac{\Theta}{18}-\alpha_{z1}\frac{hv}{6R}+\alpha_{z1}^2\Theta\frac{h}{12}\right) \\
 & \times\Theta^2\frac{h^2}{12}-\alpha_{r1}\Theta R^2\frac{h^5}{8}-(v_{out}^2-v_{in}^2)\frac{h^2}{24}\left(\alpha_{z1}\frac{h^2}{6}+S\frac{8P_{\kappa}-7P_a}{36\rho h}-v_{out}\frac{v}{9}-\alpha_{z1}\frac{hv\Theta}{18R}+\alpha_{z1}^2\frac{h}{12}\right) \\
 & \times\Theta^3-\alpha_{r1}\Theta^2\frac{h^3}{24}\left[\alpha_{z1}\frac{h^2}{4}+(5P_{\kappa}-4P_a)\frac{\Theta S}{36\rho h}-\Theta\frac{v_{out}v}{18h^2}-\alpha_{z1}\frac{vh}{12R}+\alpha_{z1}^2\frac{\Theta h}{12}\right]-(v_{out}^2-v_{in}^2) \\
 & \times R\Theta^2\frac{h^4}{16}-\alpha_{z1}(6P_a-P_{\kappa})\Theta^3\frac{h^3SR}{720\rho}-h^3S\Theta^3(10P_a-P_{\kappa})\frac{v_{out}^2-v_{in}^2}{2160\rho}-\alpha_{r1}h^2\Theta^3S\frac{6P_a-P_{\kappa}}{1440\rho}
 \end{aligned}$$

$$\begin{aligned}
 d & = hR^2\Theta^2\frac{v}{4}+(22P_a-47P_{\kappa})\frac{hS\Theta^3}{4320\rho}-\alpha_{z1}R\Theta^2\frac{h^3}{8}-\alpha_{z1}\Theta^2\frac{h^5}{48}+h^2\Theta^2\frac{vv_{out}}{192}+\alpha_{z1}\frac{\Theta^2v}{72R} \\
 & \times h^3+\alpha_{z1}^2h^3\frac{5\Theta^3}{144}-\alpha_{z1}\Theta^2\frac{h^4}{16}-\alpha_{r1}v\Theta^3\frac{h^2}{18}-2\alpha_{z1}v\Theta^3\frac{h^2}{27}-v_{in}v\Theta^3\frac{h^2}{27}
 \end{aligned}$$

$$\begin{aligned}
 e & = \alpha_{z1}R^2\frac{\Theta^2}{8}+\frac{R}{16}\Theta^2\left[\alpha_{z1}\frac{h}{12}+(5P_{\kappa}-4P_a)\frac{\Theta S}{36\rho h^2}-v_{out}\frac{v\Theta}{18h}-v\frac{\alpha_{z1}}{6R}+\alpha_{z1}^2\frac{\Theta}{6}\right] \\
 & +v\frac{\Theta^2R^3}{18h^2}+\alpha_{z1}v\Theta^3\frac{2R^2}{9h}+\alpha_{z1}^2R\frac{\Theta^3}{48}+5R^2\frac{\Theta h^3}{128}-\alpha_{z1}^2\Theta^3\frac{R}{8}-\alpha_{z1}\Theta^2\frac{Rh}{16} \\
 & +R\Theta^3S\frac{5P_a-P_{\kappa}}{288\rho h^2}+\alpha_{r1}R^2\frac{v\Theta^3}{12h}+\alpha_{z1}^2\Theta^3\frac{R}{2}+\Theta^3vR\frac{v_{in}}{9h}-\alpha_{z1}\Theta^2R^2\frac{h^2}{8} \\
 & -\Theta^2\frac{R}{12}\left(\alpha_{z1}\frac{h}{4}+\Theta S\frac{5P_{\kappa}-4P_a}{18\rho h}-v_{out}\Theta\frac{v}{9h}-\alpha_{z1}\frac{\Theta v}{9R}+\frac{1}{2}\alpha_{z1}^2\Theta\right)+ \\
 & \Theta^2R\frac{h^2}{48}+\frac{R\Theta^2}{16h}\left[\alpha_{z1}\frac{h^2}{4}+(5P_{\kappa}-4P_a)\frac{\Theta S}{18\rho}-v_{out}\Theta\frac{v}{9}-\alpha_{z1}v\frac{\Theta h}{6R}+\alpha_{z1}^2\frac{\Theta}{6}\right] \\
 & -\alpha_{r1}v\Theta^2\frac{R}{8}-\frac{h}{9}v_{in}vR\Theta^3-\alpha_{z1}\Theta^3\frac{v}{18}+\alpha_{z1}\Theta^2\frac{R^2}{16}-\alpha_{z1}^2R\frac{\Theta^3}{24}-\alpha_{z1}\Theta^2\frac{hR}{32} \\
 & +R\Theta^2\frac{S(5P_a-P_{\kappa})}{2304\rho h^2}+\Theta^2v\times\frac{\alpha_{r1}R}{16h}+\Theta^3R\frac{v_{in}v}{18h}+\alpha_{v,1}R\frac{v\Theta^3}{12h}-h\Theta\frac{R^2}{12}
 \end{aligned}$$

$$f = h^2\Theta^2\frac{R^2}{16},$$

$$\begin{aligned}
 g & = v\Theta^2\frac{h^2}{2}\left[\frac{\Theta}{3}\left(\frac{4}{3}v_{in}v-\alpha_{z1}^2\times\frac{h}{2}+\alpha_{r1}v+\frac{4}{3}v\alpha_{z1}\right)\right. \\
 & \left.+\frac{13}{48}\left\{\alpha_{z1}\Theta^2\frac{h^2}{2}\left(\frac{h}{2}-\frac{v\Theta}{3R}\right)+\frac{\Theta^3}{18}\left[(5P_{\kappa}-4P_a)\frac{S}{\rho}-2v_{out}\frac{v}{h}+3\alpha_{z1}^2\right]\right\}\right],
 \end{aligned}$$

$$\begin{aligned} & \times v h^2 + \alpha_{r1} h^2 \Theta^2 R^2 \frac{v}{4} + v \Theta^3 R \frac{h^2}{12} (v_{out}^2 - v_{in}^2) + (5438 P_a - 3 P_\kappa) \frac{v \Theta^3 S}{2880 \rho} + \alpha_{r1} v h^2 \Theta^2 \frac{R^2}{16} \\ & + \alpha_{z1} v R^4 \Theta^2 \frac{13}{96} + v_{in} v R^2 \Theta^2 \frac{h^2}{3} - v v_{out} h^2 \Theta^2 \frac{R^2}{12} + R \Theta^2 h^2 S \frac{P_a - P_\kappa}{2160 \rho} + \alpha_{z1} \Theta^2 R^2 \frac{h^4}{24} + R \Theta^2 \\ & \times \frac{h^3}{48} \left\{ \alpha_{z1} \frac{h}{2} \left( \frac{h}{2} - \frac{v \Theta}{3 R} \right) + \frac{\Theta}{3} \left[ (5 P_\kappa - 4 P_a) \frac{S}{6 \rho} + \frac{\alpha_{z1}^2}{2} - v_{out} \frac{v}{3 h} \right] \right\} \\ & + \Theta R \frac{h^3}{24} \left[ \frac{\Theta}{3} \left( \alpha_{r1} v + \frac{4}{3} v \times \alpha_{z1} + \frac{4}{3} v_{\alpha x} v - \frac{\alpha_{z1}^2}{2} \right) - \alpha_{z1} \frac{h^2}{4} \right] \end{aligned}$$

The second-order approximation of the required value of the compressor pressure (at the inlet of the pipeline) was determined from the condition that the output velocity of the free-flowing material is equal to zero. In the final form, the relation for the required value of the compressor pressure in the second-order approximation by the method of averaging the functional corrections takes the form,

$$\begin{aligned} P_k = \frac{3 \rho}{h S \Theta} & \left( \alpha_{z2} h R + P_\alpha h^2 \frac{\Theta S}{6 \rho} + \alpha_{z2} \frac{v \Theta h}{3 R} + P_\alpha \frac{\Theta h S}{2 \rho} + \frac{4}{3} \alpha_{z2} v \Theta - \alpha_{z2}^2 h \frac{\Theta}{2} + \right. \\ & \left. v \alpha_{r2} \Theta + \frac{4}{3} v_{in} \Theta v + \alpha_{z2} \frac{h^2}{2} - h^2 P_\alpha \frac{\Theta S}{6 \rho} - \alpha_{z2} \frac{h^2}{2} - \alpha_{z2} \Theta h \frac{v}{3 R} - P_\alpha S \frac{\Theta h}{2 \rho} + \alpha_{z2}^2 h \frac{\Theta}{2} \right) \end{aligned}$$

In the framework of this work, the required velocities of free-flowing material have been determined in the second-order approximation by the method of averaging the functional corrections. In the same approximation, the pressure of the compressor is obtained, which makes it possible to increase energy saving during pneumatic transport. This approximation is usually sufficient to obtain qualitative conclusions and obtain some quantitative results. The results of the analytical calculations were verified by comparing them with the results of numerical simulation.

## DISCUSSION

In this section, we analyze dependencies of the compressor pressure, which makes it possible to increase energy saving during pneumatic transport, from certain parameters. Figure 2 shows the dependence of the optimum value of the compressor pressure on the axial coordinate for different pipe lengths  $h$ . Figure 3 shows the dependence of the optimum value of the compressor pressure on the axial coordinate

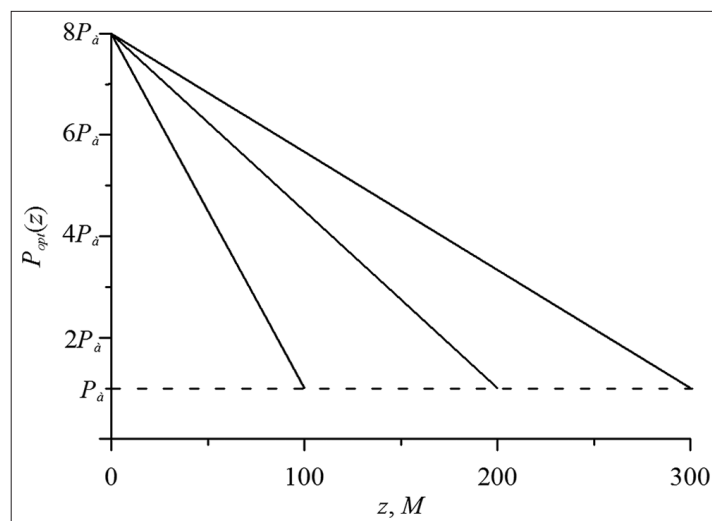
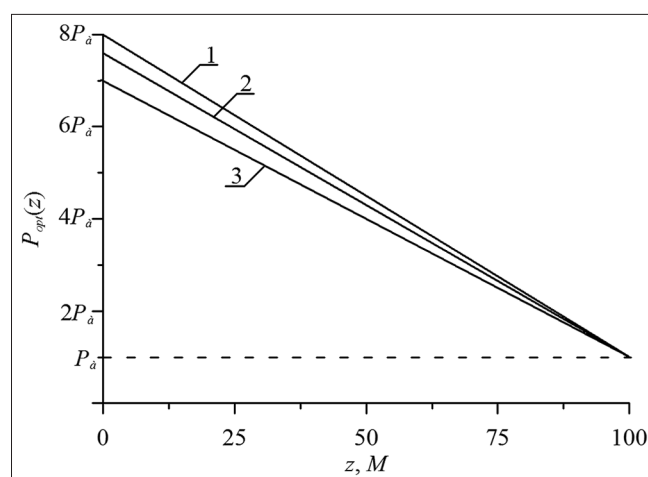


Figure 2: Dependences of the optimal value of the compressor pressure from the axial coordinate for different pipe lengths  $h$



**Figure 3:** Dependences of the optimal value of the compressor pressure from the axial coordinate for different diameters of the pipeline  $R$ . The increase in the number of dependence corresponds to the decreasing of the pipeline radius (1, 1.5, and 2 m)

for different diameters of the pipeline  $R$ . The increase in the number of the dependence corresponds to the reduction in the radius of the pipeline (1, 1.5, and 2 m).

## CONCLUSION

In this paper, we consider the possibility of determination of a pressure value in devices for pneumatic transport to increase energy saving. An analytical approach has been introduced for analysis of transport of free-flowing materials to estimate the transport velocity and the choice of the required pressure value. The dependencies of the optimum value of the compressor pressure on the parameters were investigated.

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