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RESEARCH ARTICLE

A New Mathematical Approach to Calculate the Area of a Trapezoid

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ABSTRACT

This mathematical observation deals with the development of a completely new expression $[(S/4D)\times\sqrt{\{(S_1^2-D^2)\ (D^2-D_1^2)\}}]$ to calculate the area of a trapezoid (AOT) system. Because of the difficulties of the previous methods to find out the AOT, the proposed formula has thus indicated the best way to determine the AOT more precisely and easily beyond limitation, successfully. It is obvious that the derived formula is helpful to determine AOT more easily and precisely rather than existing method. Moreover, this developed theorem for the trapezoid system is observed for the 1st time.

Key words: Trapezoid, area, height, new formula

INTRODUCTION

It is obvious that the existing method from the past decade to determine the area of a trapezoidal field is assessed through the adding of two triangles. The area of a triangle is calculated through the Heron's formula.[1] Thus, this method is more intricate. In that case, the ordinary method to calculate the area of a trapezoid (AOT) is also conditional and time consuming.

In another case, when there is given four sides of a trapezoid system, the existing method to calculate the AOT is conditional.[2] This is because, we must need to determine the height. There is no formula to calculate the AOT without height. Therefore, there is a limitation of the existing formula.

In 1990, Peterson *et al.* presented several ways to find out the area of trapezoid, but he did not discuss the method to calculate the AOT beyond height of the trapezoid.[3] In 2014, Manizade and Mason also discussed the way to find out the AOT through the ordinary method.[4] That's why, in every case, the determination of the AOT is conditional. To eliminate the limitation of the existing method, we have tried to calculate the AOT. Moreover, we have been successful to determine AOT through mathematical evaluation. In this case, we have developed a new formula to find out the AOT. In this mathematical study, we proceed through a mathematical approach to eliminate the height from the existing formula. In this sense, we have obtained a mathematical correlation between the height and sides of a trapezoid system. Hence, it is cleared that we can easily eliminate the height to determine the AOT. Thus, our proposed method provides a completely new formula to calculate the AOT without calculating height. Therefore, our proposed method is more convenient to compute the AOT in a better way. Moreover, our proposed formula is also evaluated through numerical analysis in comparison with existing formula. Moreover, our proposed formula displays an alternative and interesting way to calculate the AOT in the absence of height finding. Finally, it is obvious that, in case of given four sides of a trapezoid, we must need to determine the height of the trapezoid.[2,5] Moreover, we do not need to rely on the propaganda. To sum up, this mathematical study is completely successful and precise for developing the new formula for the trapezoid system with zero limitation in comparison with existing method.

THEORY AND DISCUSSION

Methodology

The mathematical observation is studied on the basis of Heron's formula and ordinary formula.[1,2] Let PQRS be a trapezoid, we have to develop the formula for calculating the area.

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PQRS is a trapezoid [Figure 1] to which PQ and RS are parallel sides; PS and QR are oblique sides. By cutting the ST equal to PQ from RS side, that is, PQ = ST

Now, PQ || RS, that is, PQ || ST, and PQ = ST. Hence, PQTS is a parallelogram. PS || QT and PS=QT.

Let, RS = a, PQ = b, PS = c, QR = d. Hence, RT = RS-ST = a-b = e (let), height QN = h (let)

The area of the trapezoid is, $(1/2)\times(a+b)\times h$; according to ordinary method.

The area of the trapezoid is, $(S/4D) \times \sqrt{\{(S_1^2-D^2)(D^2-D_1^2)\}}$; according to the proposed method, where, $S = a + b$, $S_1 = c + d$, $D = a-b$, $D_1 = c-d$

Proof: In triangle QRT, QN is height (QN = h)

The half of the perimeter of QRT triangle,

$s = \{(a-b) + d + c\}/2$

$= (c+ d + e)/2$; $a-b = e$

The area of the triangle QRT is

$\sqrt{\{s(s-c)(s-d)(s-e)\}}$

$= \sqrt{[\{(c + d + e)/2\} \{(c + d+ e)/2 - c\} \{(c +d +e)/2 - d\} \{(c + d + e)/2 - e\}]}$

$= (1/4) \times \sqrt{\{(c + d + e)(c +d-e)(e + c - d)(e - c + d)\}}$

$= (1/4) \times \sqrt{[\{(c + d)^2-e^2\} \{e^2 - (c-d)^2\}}$

$= (1/4) \times \sqrt{\{(S_1^2-D^2)(D^2-D_1^2)\}}$, where $c + d = S_1$, $e = a-b = D$, $c-d = D_1$

Now, if the height of triangle is h then, $(1/2) \times h \times (a-b) = (1/4) \times \sqrt{\{(S_1^2-D^2)(D^2-D_1^2)\}}$

Or, $h = \{1/(2D)\} \times \sqrt{\{(S_1^2-D^2)(D^2-D_1^2)}$

Therefore, the area of the trapezoid is, $(1/2) \times (a + b) \times h$

$= (1/2) \times S \times \{1/(2D)\} \times \sqrt{\{(S_1^2-D^2)(D^2-D_1^2)\}}$

$= (S/4D) \times \sqrt{\{(S_1^2-D^2)(D^2-D_1^2)\}}$

CALCULATION AND RESULTS

Let the two parallel sides of a trapezoid be 91 cm and 51 cm, the transverses sides are 37 cm and 13 cm. We have to find out the area of the trapezoid.

Solution in ordinary method[2,5]

If ABCD is a trapezoid [Figure 2] to which the parallel side AB is 91 cm and DC is 51 cm

And the other two sides AD are 37 cm and BC are 13 cm.

Let the height be DF = CE= h cm and EB = x,

AF = AB-BE-EF

= 91-x-51

= (40-x) cm

In right-angled triangle EBC, $BC^2 = CE^2+EB$,

$$\text{Or, } 13^2 = h^2+x^2$$

$$\text{Or, } h^2 = 13^2-x^2 \qquad (1)$$

Again, right-angled triangle ADF, $AD^2 = AF^2+DF^2$

$$\text{Or, } 37^2 = (40-x)^2+h^2$$

$$\text{Or, } h^2 = 37^2- (40-x)^2 \qquad (2)$$

Now, we get from the Equations (1) and (2), $13^2-x^2 = 37^2-(40-x)^2$

Or, $80x = 400$

Or, $x = 5$

Now, $h^2 = 13^2 - 5^2$

Or, $h = 12$

The area of the trapezoid equals, $(1/2) \times (AB+CD) \times h$

$= (1/2) \times (91+51)\times 12$

$= 852\ cm^2$

Again, by applying the proposed method

Here [Figure 3], the summation of the parallel two sides is, S = AB + CD

= 91 + 51

= 142 cm

The difference, D = AB - CD, where AB > CD

= 40 cm

The summation of the two oblique sides is, S_1 = AD + CB

= 37+13

= 50 cm

The difference D_1 = AD - CB, AD > CB

= 37–13

= 24 cm

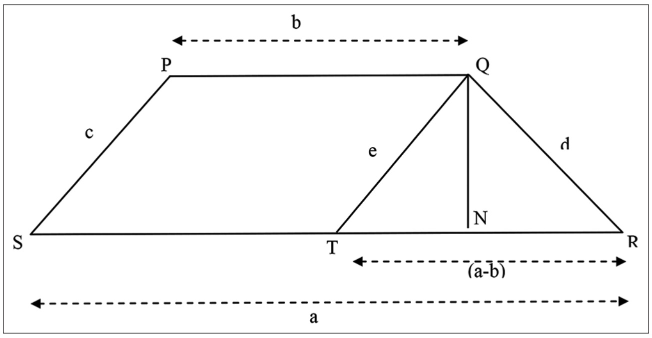

Figure-1: PQRS Trapezoid

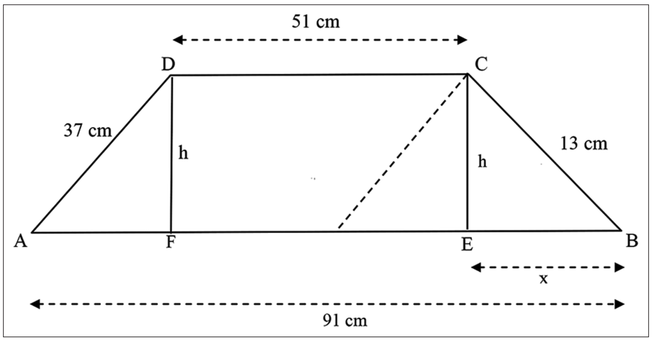

Figure-2: ABCD Trapezoid

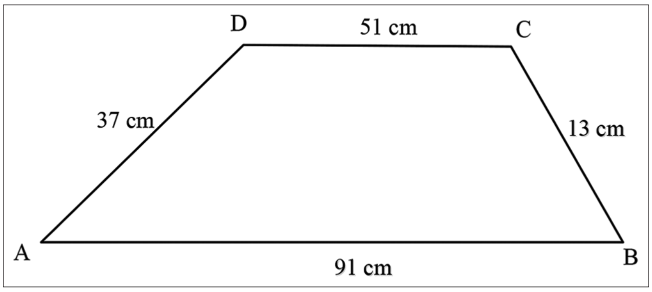

Figure-3: ABCD Trapezoid

Hence, the area of the trapezoid is $(S/4D) \times \sqrt{(S_1^2-D^2)(D^2-D_1^2)}$
$= \{142/(4\times40)\} \times \sqrt{(50^2-40^2)(40^2-24^2)}$
$= (142/160) \times \sqrt{(900\times1024)}$
$= (142/160) \times 960$
$= 852$ cm^2

Thus, this mathematical analysis observes the importance and application of the proposed formula through the comparison with the ordinary formula with the help of an example. This evaluation gives a numerical evidence for our proposed method and bitterness of the ordinary method as well. Hence, it is obvious that the proposed formula is completely correct.

CONCLUSION

Before this study, there was actually no alternative method to determine the AOT in such an effective and easiest way. This paper initiatively focuses on the results and gives also numerical evidence in comparison with the existing formula. Due to the massive difficulties for using ordinary method, the newly proposed method would be much more initiative for research and mathematical community, and could be an interesting new work where the value of AOT actually used as well.

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