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RESEARCH ARTICLE

On the Characteristics of Difference of Some Linear Positive Operators

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ABSTRACT

Recently, many researchers have concerned the problems based on difference of two linear positive operators having same basis functions. In this paper, we deal with the difference of two summation integral type linear positive operators, one having different basis functions and other Durrmeyer type operators. We also obtained the difference of Baskakov operators and beta operators.

Key words: Baskakov operators, Beta-Szasz operators, Difference of operators, Linear positive operators, Modulus of continuity, Szasz Durrmeyer type operators

2020 Mathematics Subject Classification: 41A25, 41A30.

ESTABLISHMENT

Some days ago Acu and Rasa,[1] Aral *et al.*,[2] Gonska and Kovacheva[3] have innovated new type of results in approximation theory on difference of operators. They considered the difference of operators to generalize the problem proposed by Lupas[4] on polynomial differences. After that, Gupta[5],[6] gave some results on the matter having same as well as different basis functions. Motivated by their work, I found the difference of two summation integral type operators in which one has different basis functions and other with same basis functions. We take interval I $\subseteq$ R and C(I)={f: I→R is continuous}. Also C_B (I) as the space of all f∈C(I) such that

$$\|f\|=\sup\{|f(x)|:x\in I\}<\infty$$

Now we take a positive linear functional F(f):C(I)→R such that $F(e_0)=1$. Also we take the following notations throughout the paper.

$$\rho^F \equiv F(e_1);\eta_r^F = F(e_1-\rho^F e_0)^r; r\in N$$

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BASIC INEQUALITY

Here, we give an inequality established by Acu and Rasa[1] for f(x)∈C(I).

Lemma1[1] If f∈C(I) and f''∈C_B (I), we have

$$\left|F(f)-f\left(\rho^F\right)\right|\le\frac{\eta_2^F}{2}\|f''\|$$

GENERAL CONCEPT OF OPERATORS

Let us consider two linear positive functionals $F_n^k, G_n^k : C(I)\to R, k\in N^0$ such that

$$F_n^k(e_0)=G_n^k(e_0)=1$$

Again, let $\alpha_{n,k}$, $\beta_{n,k}$∈C(I) such that

$$\sum_{k\in N^0}\alpha_{n,k}(x)=\sum_{k\in N^0}\beta_{n,k}(x)=e_0$$

then, we can have

$$\sum_{k\in N^0}\alpha_{n,k}(x)F_n^k(f),\ \sum_{k\in N^0}\beta_{n,k}(x) G_n^k(f)\in C(I) \tag{3.1}$$

We take D(I) the set of all f∈C(I) for which (3.1) holds then we can define two linear positive operators U_n, V_n: D(I)→C(I) as

$$U_n(f,x) := \sum_{k\in N^0} \alpha_{n,k}(x) F_n^k(f); V_n(f,x) := \sum_{k\in N^0} \beta_{n,k}(x) G_n^k(f)$$

FUNDAMENTAL OF DIFFERENCE OF OPERATORS

Here, we give the fundamental activity theorem to understand the difference of two operators.
Theorem 1[7] If f∈D(I) with f''∈ C_B (I), we have

$$|(U_n - V_n)(f;x)| \le \chi(x) \| f'' \| + 2\omega_1 (f, \delta_1(x)) + 2\omega_1(f, \delta_2(x))$$

Where

$$\chi(x) = \frac{1}{2}\sum_{k\in N^0}\left(\alpha_{n,k}(x)\eta_2^{F_n^k} - \beta_{n,k}(x)\eta_2^{G_n^k}\right)$$

$$\delta_1^2(x) = \sum_{k\in N^0} \alpha_{n,k}(x)\left(\rho^{F_n^k} - x\right)^2$$

$$\delta_2^2(x) = \sum_{k\in N^0} \beta_{n,k}(x)\left(\rho^{G_n^k} - x\right)^2$$

Its proof can be seen in Gupta and Acu.[7]

APPLICATIONS

Here, we have given two applications related to our considered subject.

Difference of two summation integral type operators

We find the difference between two summation integral type operators-
(1) Beta-Szasz operators and (2)Szasz Durrmeyer operators.
First operators we take Beta-Szasz operators as in[8] which are defined for x∈[0,∞) as

$$A_n(f(y),x) = \sum_{k=0}^{\infty} b_{n,k}(x)\int_0^{\infty} s_{n,k}(t) f(t)\,dt \qquad (5.1)$$

whose basis functions are

$$b_{n,k}(x) = \frac{1}{B(k+1,n)}\frac{x^n}{(1+x)^{n+k+1}}$$ and

$$s_{n,k}(y) = e^{-ny}\frac{(ny)^k}{k!}$$

From here, we get the moments

$$A_n(e_0,x) = 1, A_n(e_1,x) = \frac{1+(n+1)x}{n}, A_n(e_2,x) = \frac{(n+2)(n+1)x^2 + 4(n+1)x + 2}{n^2}$$

Other operators as in[9] we take Szasz Durrmeyer operators which are defined as

$$B_n(f(y),x) = n\sum_{k=0}^{\infty} s_{n,k}(x)\int_0^{\infty} s_{n,k}(y) f(y)\,dy$$

whose basis function $s_{n,k}$ is defined above. Also from here, we get the moments

$$B_n(e_0,x) = 1, B_n(e_1,x) = \frac{1+nx}{n}, B(e_2,x) = \frac{n^2x^2 + 4nx + 1}{n^2}$$

Now writing both the operators in the following standard form

$$A_n(f,x) = \sum_{k\in N^0} \alpha_{n,k}(x) F_n^k(f)$$

$$B_n(f,x) = \sum_{k\in N^0} \beta_{n,k}(x) G_n^k(f)$$

and comparing we have

$$F_n^k(f) = G_n^k(f) = n\int_0^{\infty} s_{n,k}(y) f(y)\,dy, \alpha_{n,k}(x) = \frac{1}{n} b_{n,k}(x), \beta_{n,k}(x) = s_{n,k}(x)$$

From here, we get-

$$F_n^k(e_0) = G_n^k(e_0) = 1$$

$$\rho^{F_n^k} = \rho^{G_n^k} = \frac{k+1}{n}$$

$$\eta_2^{F_n^k} = \eta_2^{G_n^k} = F(e_1 - \rho^{F_n^k} e_0)^2 = \frac{k+1}{n^2}$$

$$\chi(x) = \frac{1}{2}\sum_{k=0}^{\infty}\left(\alpha_{n,k}(x) - \beta_{n,k}(x)\right)\eta_2^{F_n^k} = \frac{1}{2}\sum_{k=0}^{\infty}\left(\frac{1}{n}b_{n,k}(x) - s_{n,k}(x)\right)\eta_2^{F_n^k}$$

$$= \frac{1}{2n^2}\left[(2n+1)x+2\right]$$

$$\delta_1^2(x) = \sum_{k\in N^0}\alpha_{n,k}(x)\left(\rho^{F_n^k} - x\right)^2 = \frac{1}{n}\sum_{k\in N^0}b_{n,k}(x)\left(\rho^{F_n^k} - x\right)^2$$

$$= \frac{(n-1)^2 x^2 + (3-2n)x + 1}{n^2}$$

$$\delta_2^2(x) = \sum_{k\in N^0}\beta_{n,k}(x)\left(\rho^{G_n^k} - x\right)^2 = \sum_{k\in N^0}s_{n,k}(x)\left(\rho^{G_n^k} - x\right)^2$$

$$= \frac{2n(n-1)x^2 + (3-2n)x + 1}{n^2}$$

Therefore according to Theorem 1, the difference of our operators (5.1) and (5.2) is

$$\left|(A_n - B_n)(f;x)\right| \le \chi(x)\,\|f''\| + 2\omega_1(f,\delta_1(x)) + 2\omega(f,\delta_2(x)) \le \frac{1}{2n^2}\left[(2n+1)x+2\right]f'' + 2\omega_1\left(f,\sqrt{\frac{(n-1)^2x^2+(3-2n)x+1}{n^2}}\right) + 2\omega_1\left(f,\sqrt{\frac{2n(n-1)x^2+(3-2n)x+1}{n^2}}\right)$$

Hence, we obtain the required result.

Difference of Two Summation Type Operators

We consider here the estimates for the difference in Baskakov operators and Beta operators.
The Baskakov operators are given by

$$P_n(f,x) = \sum_{k=0}^{\infty}p_{n,k}(x)f\left(\frac{k}{n}\right) := \sum_{k=0}^{\infty}\alpha_{n,k}(x)F_n^k(f)$$

Where $p_{n,k}(x) = \binom{n+k-1}{k}\frac{x^n}{(1+x)^{n+k}}$ and

$$F_n^k(f) = f\left(\frac{k}{n}\right).$$

Obviously, moments for Baskakov operators are

$$P_n(e_0,x) = 1, A_n(e_1,x) = x, A_n(e_2,x) = \frac{x\left[1+(n+1)x\right]}{n}$$

Also $\rho^{F_n^k} = F_n^k(e_1) = \frac{k}{n}$ and therefore $\eta_2^{F_n^k} = F(e_1 - \rho^{F_n^k}e_0)^2 = 0$.

Now, the Beta operators are given by

$$Q_n(f,x) = \sum_{k=0}^{\infty}b_{n,k}(x)f\left(\frac{k}{n}\right) := \sum_{k=0}^{\infty}\beta_{n,k}(x)G_n^k(f)$$

Where $b_{n,k}(x) = \frac{1}{B(k+1,n)}\frac{x^n}{(1+x)^{n+k+1}}$ and

$$G_n^k(f) = f\left(\frac{k}{n}\right).$$

Moments for these operators are

$$Q_n(e_0,x) = 1, Q_n(e_1,x) = \frac{(n+1)x}{n}, Q_n(e_2,x) = \frac{(n+1)x\left[(n+2)x+1\right]}{n^2}$$

Also $\rho^{F_n^k} = G_n^k(e_1) = \frac{k}{n}$ and therefore $\eta_2^{G_n^k} = F(e_1 - \rho^{G_n^k}e_0)^2 = 0$.

Therefore

$$\chi(x) = \frac{1}{2}\sum_{k\in N^0}\left(\alpha_{n,k}(x)\eta_2^{F_n^k} - \beta_{n,k}(x)\eta_2^{G_n^k}\right) = 0$$

$$\delta_1^2(x) = \sum_{k\in N^0}p_{n,k}(x)\left(\rho^{F_n^k} - x\right)^2 = P_n\left(\left(\frac{k}{n} - x\right)^2, x\right) = \frac{x(x+1)}{n}$$

$$\delta_2^{\,2}(x) = \sum_{k\in N^0} b_{n,k}(x)\left(\rho^{G_n^k} - x\right)^2 = Q_n\left(\left(\frac{k}{n}-x\right)^2, x\right) = \frac{x\left[(n+2)x+(n+1)\right]}{n}$$

Applying Theorem 1, we get

$$\left|(P_n - Q_n)(f,x)\right| \le 2\omega_1\left(f, \sqrt{\frac{x(x+1)}{n}}\right) + 2\omega_1\left(f, \sqrt{\frac{x\left[(n+2)x+(n+1)\right]}{n^2}}\right)$$

Thus, we have obtained the required difference of Baskakov and Beta operators. Taking appropriate f(x) defined on some interval we can show the results as classical one.

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