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RESEARCH ARTICLE

Analysis of Nigerian Naira Exchange Rates against US Dollar, British Pounds, and Euro Currency Using Mean Reverting Model

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ABSTRACT

This work is aim at analyzing Nigerian Naira exchange rate against American Dollar, British Pounds, and the Euro currency. Monthly average exchange rates from May 2015 to April 2020 were used for the study. Augmented Dickey-Fuller was used to determine the presence of mean reversion in the data. The solution to the model is gotten using Ito's formula. Mean reversion rate, long-run mean, and volatility were calculated from the historic data using Microsoft Excel. The concept of half-life was used to determine the time it will take for the rates to revert to their long-run mean. The data analysis reveals that the data are stationary. The analyses also reveal that American Dollar will be the cheapest currency followed by Euro in the near future. Based on the result, the researchers recommend mean reverting model in modeling exchange rates. The researchers also recommend that investors should invest using Dollar or Euro more than they use pounds

Key words: Exchange rates, mean reversion models, half-lives

INTRODUCTION

In the past few years, the price of dollar has become a source of concern to every Nigerian. Investors, traders, civil servants, and even peasant farmers have all felt the impact of the rise and fall of the Nigerian currency against other countries' currencies especially within the past 2 years. Nevertheless, the Nigerian government on their own part has developed a lot of policies to help fight the situation but it seems that all the policies work for some times and then crashed. Noko[1] has it that when there are fluctuations in exchange rate, various economic activities are usually affected such as; the purchasing power, balance of payment, price of goods and services, import structure, export earning, government revenue, and external reserve among others. Ayodele[2] asserted that exchange rate has a negative impact on the GDP because as it increases, the economic growth is negatively affected, while inflation rate exerts a positive impact on GDP, indicating that firms are more willing to produce when inflation rate is high and vice versa.

It has now become imperatively necessary that the naira exchange rate against these currencies be analyzed to determine the trend and also provide a simulation equation that will guide investors on the way forward, that is when to either invest or hold their peace[3,4].

Mean reverting processes are naturally attractive to model commodity *prices* since they embody the economic argument that when prices are "too high," demand will reduce and supply will increase, producing a counter-balancing effect[5]. When prices are "too low" the opposite will occur, again pushing prices back toward some kind of long-term mean[6]. Mean reverting processes are also useful for modeling other processes, observed or unobserved, such as interest rates or commodity "convenience yield." Hence, this work is aimed at using this mean reverting model to analyze the behavior of naira exchange rate against the three major foreign currencies[7,8].

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Statement of the problem

The exchange rate is perhaps one of the most widely discussed topics in Nigeria today. This is not surprising given its macro-economic importance especially in a highly import dependent economy as Nigeria[9].

Since September 1986, when the market determined exchange rate system was introduced through the second tier foreign exchange market, the naira exchange rate has exhibited the features of continuous depreciation and instability[10].

This instability and continued depreciation of the naira in the foreign exchange market have resulted in declines in the standard of living of the populace, increased cost of production which also leads to cost push inflation[11]. It has also tended to undermine the international competitiveness of non-oil exports and make planning and projections difficult at both micro and macro levels of the economy. A good number of small and medium scale enterprises have been strangled out as a result of low dollar/naira exchange rate and so many other problems resulting from fluctuations in exchange rates[12].

Hence, this work is aimed at using the mean reversion model to analyze the behavior of naira exchange rate with the aim of developing simulation equations that will help to determine future exchange rates in the country.

MATERIALS AND METHODS

Data

The data that are used for this study are monthly average exchange rates of dollar, pounds, and euro from May 2015 to April 2020 (60 observations) gotten from the archives of the central bank of Nigeria (CBN). In the case of dollar, IFEM and DAS are ignored while BDC is used. This is because BDC is the one available to customers at any point in time.

The Model

Mean reversion model is given by

$$dS_t=\lambda(\mu-S_t)dt+\sigma S_t\, dB_t \qquad (1)$$

Where

- B_t is a Brownian Motion, so $dB_t \sim N(0\sqrt{dt})$
- λ is the speed of mean reversion
- μ is the "long run mean" to which the process tends to revert
- σ as usual is a measure of the process volatility.

The mean that is the first moment of the model is given by (see[13,14])

$Es(t)=\mu+(S_0-\mu)e^{-\lambda t}$ which is equal to μ as $t \to \infty$

While the variance is given by

$Var\, s(t)=\dfrac{\sigma^2}{2\lambda}\left(1-e^{-2\lambda t}\right)$ which is equal to $\dfrac{\sigma^2}{2\lambda}\, as\, t \to \infty$

Data analysis

Testing for mean reversion

Equation (1) can be discretized as follows

$$\Delta S_t=\beta_0+\beta_1 S_{t-1}+\sigma S_{t-1}\,\varepsilon_t,\ \varepsilon_t \sim N(0,1) \qquad (2)$$

Where $\beta_0=\lambda\mu\Delta t$, $\beta_1=-\lambda\Delta t$ and $\Delta t=\dfrac{1}{n}$. This implies that observations of the exchange rates through time can be considered as observations of the linear relationship between ΔS_t and S_t in the presence of noise $\sigma S_{t-1}\,\varepsilon_t$.G[15].

To test for evidence of mean reversion, Augmented Dickey-Fuller test (ADF) is used to perform unit root test. The Dickey-Fuller version used is the type one version that is constant and no trend[16]. The Microsoft Excel real statistics regression analysis tool is used to determine the calculated tau, while the tabular tau is gotten from ADF table.

The following formal hypothesis is tested

H_0: $\beta = 0$ (series is non-stationary, i.e., unit root)

H_1: $\beta < 0$ (series is stationary, i.e., mean reverting)

Rate of mean reversion, long-run mean, and volatility

The mean reversion rate, long-run mean, and the volatility are estimated from the historic data using excel regression tool. In[17] the Mean reversion rate (λ)= negative of the slope

Mean reversion level/the long-run mean = intercept divided by mean reversion rate

Volatility is gotten using STEYX function in excel which is the residual standard deviation. All these were gotten using Microsoft Excel regression tool.

Half life

This is the average time that will be taken for the rates to revert half way to its long-term level or long-run mean. From equation (1), let us consider the ODE

$ds_t=\lambda(\mu-s_t)dt$ which gives

$$s_t=\mu+(S_0-\mu)e^{-\lambda t} \qquad (3)$$

Hence, the half-life can be deduced thus: $s_{\left(t_{1/2}\right)} - \mu = \frac{\left(S_0 - \mu\right)e^{-\lambda t}}{2}$ hence, we have that

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

Where λ is the rate or speed of mean reversion.

Simulation equations and predictions

To get the simulation equation, we make use of the fact that equation (2) can further be simplified as $S_t=S_{t-1}+\beta_0+\beta_1 S_{t-1}+\sigma S_{t-1} \varepsilon_t$ which in turn gives

$$S_t=\beta_0+(1+\beta_1+\sigma\varepsilon_t) S_{t-1} \qquad (4)$$

Equation (3) is then used for the simulation equation.

For predictions, the expected value, that is, the mean is used.

RESULTS

Solution to the model

To solve equation (1), we first derive the Ito's formula thus

Let f(s, t) be a continuous and differentiable function. Then, by Taylor's series expansion for two variables:

$$df\left(s,t\right) = \frac{\partial f}{\partial s}ds + \frac{1}{2}\frac{\partial^2 f}{\partial s^2}\left(ds\right)^2 \qquad (5)$$

but, $ds=\lambda(\mu-s)dt+\sigma sdB$

hence, $(ds)^2=(\lambda(\mu-s)dt+\sigma sdB)^2$

$= (\lambda\mu dt-\lambda sdt+\sigma sdB)(\lambda\mu dt-\lambda sdt+\sigma sdB)$

$=\lambda^2 \mu^2 (dt)^2-2\lambda^2 \mu s(dt)^2+2\lambda\mu\sigma sdtdB-2\lambda s^2 \sigma dtdB+\lambda^2 s^2 (dt)^2+\sigma^2 s^2 (dB)^2$

Since $(dB)^2 \rightarrow dt$ *as* $dt \rightarrow 0$, we have that $(ds)^2=\sigma^2 s^2 dt$(Xeurong (2007)

Hence,

$$df\left(s,t\right) = \frac{\partial f}{\partial s}\left(\lambda\mu dt - \lambda sdt + \sigma sdB\right) + \frac{1}{2}\frac{\partial^2 f}{\partial s^2}\sigma^2 s^2 dt$$

$$= \lambda\mu\frac{\partial f}{\partial s}dt - \lambda s\frac{\partial f}{\partial s}dt + \sigma s\frac{\partial f}{\partial s}dB + \frac{1}{2}\sigma^2 s^2\frac{\partial^2 f}{\partial s^2}dt$$

Finally, we say that:

$$df\left(s,t\right) = \sigma s\frac{\partial f}{\partial s}dB + \left(\lambda\mu\frac{\partial f}{\partial s} - \lambda s\frac{\partial f}{\partial s} + \frac{1}{2}\sigma^2 s^2\frac{\partial^2 f}{\partial s^2}\right)dt \qquad (6)$$

We then proceed to the main solution thus:

Taking f = ln s hence; $\frac{\partial f}{\partial s} = \frac{1}{s}$ and $\frac{\partial^2 f}{\partial s^2} = -\frac{1}{s^2}$ so substituting we get

$$d\left(\ln s\right) = \sigma s\frac{1}{s}dB + \left(\lambda\mu\frac{1}{s} - \lambda s\frac{1}{s} + \frac{1}{2}\sigma^2 s^2\left(-\frac{1}{s^2}\right)\right)dt$$

$$= \left(\frac{\lambda\mu}{s} - \lambda - \frac{\sigma^2}{2}\right)dt + \sigma dB$$

Hence,

$$d(lns) = \frac{\lambda\mu}{s}dt + \sigma dB - \left(\lambda + \frac{\sigma^2}{2}\right)dt \qquad (7)$$

Now, stochastically integrating both sides from 0 to t, we have

$$\int d(lns) = \int\frac{\lambda\mu}{s}dt + \int\sigma dB - \int\left(\lambda + \frac{\sigma^2}{2}\right)dt$$

$$lns = \lambda\mu\int\frac{1}{s}dt + \sigma B_{(t)} - \left(\lambda + \frac{\sigma^2}{2}\right)t$$

$$= -\left(\lambda + \frac{\sigma^2}{2}\right)t + \sigma B_{(t)} + \lambda\mu\int\frac{1}{s}dt$$

$$= -\left(\lambda + \frac{\sigma^2}{2}\right)t + \sigma B_{(t)} + \lambda\mu\int_s^t\left[-\left(\lambda + \frac{\sigma^2}{2}\right) + \sigma dB_{(t)}\right]dt$$

Hence,

$$\ln s = -\left(\lambda + \frac{\sigma^2}{2}\right)t + \sigma B_{(t)} +$$

$$\lambda\mu\int\left[-\left(\lambda + \frac{\sigma^2}{2}\right)\left(t - s\right) + \sigma\left(B_{(t)} - B_{(s)}\right)\right]ds$$

Now, taking the exponent of both sides and letting $s(0) = s_0$, we have

$$S_{(t)} = S_0 exp\left[-\left(\lambda + \frac{\sigma^2}{2}\right)t + \sigma B_{(t)}\right] +$$

$$\lambda\mu\int_0^t exp\left[-\left(\lambda + \frac{\sigma^2}{2}\right)\left(t - s\right) + \sigma\left(B_{(t)} - B_{(s)}\right)\right]ds \qquad (8)$$

Equation (8) is the solution to the equation.

Unit root test[18]

Table 1 shows the unit root test of the naira exchange rate of the three currencies at 5% level of significance

H_0: The exchange rates has unit root

From Table 1, the test statistic for all the currencies is smaller than the critical value; hence, the null hypothesis is rejected in all[19]. Therefore, we conclude that the exchange rates are stationary, that is, mean reverting

Rate of mean reversion, long-run mean, and volatility

Using Excel 2007 and making use of the data stated above, the mean reversion rate, long-run mean, and volatility of the exchange rates of the three currencies are given in Table 2.

Half-lives

The half-lives of the exchange rates of the three currencies are as follows

Dollar

$$H(t_{1/2}) = \frac{\ln 2}{\lambda} = \frac{0.69315}{0.1} = 6.9315$$

Pounds

$$H(t_{1/2}) = \frac{\ln 2}{\lambda} = \frac{0.69315}{0.04} = 17.32875$$

Euro

$$H(t_{1/2}) = \frac{\ln 2}{\lambda} = \frac{0.69315}{0.04} = 17.32875$$

Table 1: Unit root test for exchange rates

Currency	Test statistic	Critical value (5%)
Dollar	–2.94451	–2.891
Pounds	–3.1894	–2.891
Euro	–3.2789	–2.891

Table 2: Rate of mean reversion, long-run mean, and volatility

Currency	Mean reversion rate(λ)	Long-run mean	Volatility
Dollar	0.1	380.85	0.06
Pounds	0.04	433.47	0.03
Euro	0.04	398.32	0.03

Simulation equations and predictions

The simulation equation is gotten using equation (3.3)

Dollar

$S_t=\beta_0+(1+\beta_1+\sigma\varepsilon_t)S_{t-1}$

$$\beta_0 = \lambda\mu\Delta t = 0.1\times 380.85\times\frac{1}{60} = 0.6348$$

$$\beta_1 = -\lambda\Delta t = -0.1\times\frac{1}{60} = -0.0017$$

Volatility(σ)=0.06

Hence, the simulation equation is

$S_t=0.6348+(1–0.0017+0.06\varepsilon_t)\ S_{t-1}$

$=0.6348+(0.9983+11.93\varepsilon_t)\ S_{t-1}$

Where ε_t is a random number.

Pounds

$S_t=\beta_0+(1+\beta_1+\sigma\varepsilon_t)\ S_{t-1}$

$$\beta_0 = \lambda\mu\Delta t = 0.04\times 433.47\times\frac{1}{60} = 0.28898$$

$$\beta_1 = -\lambda\Delta t = -0.04\times\frac{1}{60} = -0.00067$$

Volatility(σ)=0.03

Hence, the simulation equation is

$S_t=0.28898+(1–0.00067+0.03\varepsilon_t)\ S_{t-1}$

$=0.28898+(0.9993+0.03\varepsilon_t)\ S_{t-1}$

Where ε_t is a random number.

Euro

$$\beta_0 = \lambda\mu\Delta t = 0.04\times 398.32\times\frac{1}{60} = 0.2655$$

$$\beta_1 = -\lambda\Delta t = -0.04\times\frac{1}{60} = -0.00067$$

Volatility(σ)=0.03

Hence, the simulation equation is

$S_t=0.2655+(1–0.00067+0.03\varepsilon_t)\ S_{t-1}$

$=0.2655+(0.9993+0.03\varepsilon_t)\ S_{t-1}$

Where ε_t is a random number.

Predictions

Using the mean, we can predict the naira exchange rate of the currencies. Hence, in the next 40 months taking S_0 to be value for April 2020, we have the following:

Dollar

$ES_{(t)}=\mu+(S_0-\mu)e^{-\lambda t}$

Taking the values in table above, we have

$ES_{(40)}= 380.85+(420.15-380.85)e^{-0.1\times 40}$

$$= 380.85 + (39.3) \times \frac{1}{2.71828^4}$$

$$= 380.85 + \frac{39.3}{54.5980}$$

=381.5698

$\cong N381.57K$

Pounds

$ES_{(t)}=\mu+(S_0-\mu)e^{-\lambda t}$

Furthermore, taking the values in table above, we have

$ES_{(40)}=433.47+(447.92-433.47)e^{-0.04\times 40}$

$$= 433.47 + (14.45) \times \frac{1}{2.71828^{1.6}}$$

$$= 433.47 + \frac{14.45}{4.9530}$$

$\cong N436.39K$

Euro

$ES_{(t)}=\mu+(S_0-\mu)e^{-\lambda t}$

Furthermore, taking the values in table above, we have

$ES_{(40)}=398.32+(392.12-398.32)e^{-0.04\times 40}$

$$= 398.32 + (-6.2) \times \frac{1}{2.71828^{1.6}}$$

$$= 398.32 - \frac{6.2}{4.9530}$$

=397.0682

$\cong N397.6K$

DISCUSSION

From Table 1, one can observe that the exchange rates of Nigerian naira as against all the three currencies are stationary. This means that mean reversion model can be used to model the exchange rates

The result in Table 2 shows that though the exchange rates are stationary, their mean reversion rates are small especially for pounds and euro.

The implication of the half-lives is that apart from noise term, it will take dollar 6.9315 months to revert to half of its long-run mean of 380.85. Hence, the dollar exchange rate will take approximately 14 months to go back to a rate very close to N380:85k per dollar.

Furthermore, it will take pounds 17.32875 months to revert back to half its mean exchange rate of 433.47. Hence, it will take approximately 35 months for the exchange rate to go back to a rate very close to N433:47k per pound.

Moreover, it will also take Euro 17.32875 months to revert back to half its long-run mean exchange rate of 398.32. Hence, Euro currencies will also take approximately 35 months to go back to the exchange rate of N398:32k per Euro.

From predictions, we will observe that, in the near future, dollar will have the cheapest exchange rate followed by euro, then pounds.

RECOMMENDATIONS

Based on the results, the researcher recommends as follows:

i. Mean reversion model should be used in modeling exchange rate since it best models the concept
ii. Investors should make use of dollar and Euro currencies instead of the pounds since dollar and Euro will always be cheaper than the pounds
iii. Since exchange rates follow mean reversion process, investors are advised to hold their peace whenever it is on the high side since what goes up will surely come down.

CONTRIBUTIONS TO KNOWLEDGE

i. To the best of our knowledge, mean reversion model has not been used by any other researcher to model exchange rates; hence, this work has shown that mean reversion model can be used to model exchange rates
ii. Equation (5) that is the extension of Ito's formula for mean reversion model is developed by the researcher
iii. The step-by-step method of solving mean reversion model has not been done by any other researcher

iv. Simulation equation and predictions are developed for the three currencies.

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