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RESEARCH ARTICLE

On the Application of Dynamic Programming Fixed-Point Iterative Method in the Determination of the Shortest Route/Path between Umuahia and Abuja, all in Nigeria

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ABSTRACT

In this research, dynamic programming seeks to address the problem of determining the shortest path between a source and a sink by the method of a fixed-point iteration well defined in the metric space (X,d), d the distance on X=U the connected series of edges that suitably works with the formula

$$x_{n+1} = f(x_n) = x^* = F(X) = dist(S_0, S_k) = \min[U_j, i]$$

$$= \sum_{i,j}^{n} \min[u_j, i] = \min \sum_{i,j}^{n} [u_i + d_{ij}, i], U_{ij} \geq 0, S_0 = source, S_k = sink$$

Such that

$$d_{ij} + d_{kj} \leq d_{ik},\ i \neq k, j \neq k, i \neq j$$

With the pivot row and pivot column being row k.

Then, evaluation of the shortest route between Umuahia and Abuja by the above method revealed it to be 702 km by going from Umuahia through Enugu through Ankpa through Lokoja and then to Abuja (i.e., TED GS. by the Backward dynamic method)

Key words: Dynamic programming, Dijkstra's algorithm, greedy and Prim's algorithm, complete metric space, source and node, pseudo-contractive fixed-point method

INTRODUCTION

There are few basics of dynamic programming problems which must be discussed before the details. Those basics are discussed below.

A route is defined as a course of travel, especially between two distant points/locations while shortest is sound to be a relatively smallest length, range, scope, etc., than others of its kind, type, etc. Therefore, we say that the shortest route is the relatively smallest of its kind, especially between two distant points/locations. The route must be accessible/useable by a motor vehicle, the route may be single or double lane. The routes may possess bus stops, junctions, interconnected streets, or venues for join. The routes may be traced or not but should be wide enough to be used by a motor vehicle. The routes may short or interconnect with another route that started from Umuahia to end at Abuja that is the route should have a source and a sink.

Therefore,[1,2] "the application of shortest route/path in Dynamic programming" can be seen as the practical use of the shortest course of travel by road users among other routes of its kind especially between two points/locations.

It is, however, disturbing to note that much of the available routes from Umuahia to Abuja have one traveling challenge or the other such as long distance, police menace, traffic jams, and bad road networks.

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Perhaps, it is necessary to answer some questions to really appreciate the issue on ground. These questions are; How do road users view the various existing routes from Umuahia to Abuja? Which of the routes are preserved by road users? What route should one undertake to minimize time and distance of travel/from the researcher's observation, it was clear that the route quality such as shortest distance freedom from police menace and traffic jam as well as good road networks, all contributed to affect road users decision of route to make use of when traveling from Umuahia to Abuja in Nigeria.

Definitely, it is important to note that –

a. This work is limited to time and distance of travel by motor vehicle on road excluding the effects of traffic jams, police menace, bad road network, etc., and – number of routes from Umuahia to Abuja.
b. This study will be of major significance to travelers and transporters (who are major beneficiaries).
c. The study will help us appreciate the importance and practical use of dynamic programming in determining the shortest route of travel when traveling from one location to the other.
d. To carry out the study, the following hypothesis were formulated for investigation
   i. Any part of the shortest route from Umuahia to Abuja is itself a shortest path.
   ii. Any part of an optimal path is itself optimal.

The above two hypothesis [2,3] are also known as "the principle of optimality."

iii. Walk: A walk is simply a route, in the graph along a connected series of edges. *BCAD* is a walk from *B* to *D* through *C* and *ABDE* is a walk from *A* to *E* in a walk edges and vertices may be repeated.
iv. Trail: When all the edges of a walk are different, the walk is called a trail. *BCD* is a trail from *B* to *D*. *A* closed trail is one in which the start and finish vertices are the same. *ADECDBA* is a closed trail.
v. Path: This is a special kind of trail if all the vertices of a trail are distinct then the trail is a path *ABCE* is a path, all edges and all vertices are distinct in a path.
vi. Cycle: A cycle ends where it starts and all the edges and vertices in between are distinct *ABDA* and *ABCEDA* are as vertices have been repeated.
vii. Tree: This is a connected graph which contains no cycles.

Note that a tree with n vertices has $n-1$ edges.

viii. Vertex degree: The degree of a vertex is the number of edges touching the vertex.
ix. Directed graph or diagraph: It is a graph in which each of a diagraph is called an arc.
x. Weight: The edges of a graph are often given a number which can represent some physical property, for example, length, cost time, and profit. The general term for this number is weight.
xi. Network: A graph whose edges have all been weighted is called a network.
xii. Stage and state: The stage tells us how "Far" the vertex in question is from the destination vertex while the states refer directly to the vertices.
xiii. Action: This refers to possible choices at each vertex.
xiv. Value: The numbers calculated for each state at each stage are referred to as values.
xv. The optimal value: The optimal value is the label which is assigned to the vertex. The value is also known as the Bellman function.

Major Introduction (Methods of Determining the Shortest Route/Path)

There abound several methods of determining the shortest route/path from one location/point to the other in this section we shall do well to review some of the existing methods of finding the shortest route.

The Dynamic Programming Technique

The network [2,3] below can help us explain the dynamic programming technique.

To find the shortest or longest path from *S* to *T* in the above network, we begin at the destination vertex *T*. The vertices next to *T* best route from these are examined. These are stage vertex the best route from these to *T* is noted. We now move to the next set of vertices, moving away from *T* towards *S*, that is, the Stage 2 vertices. The optimal route from these vertices to *T* is found using the already calculated optimal route from the Stage 1 vertices. Then, this process is repeated until the start vertex, *S*, is reached. The optimal route from *S* to *T* can then be found the principle of optimality is used at

each stage, the current optimal path is developed from the previously found optimal path.
Since the method involves starting with the destination vertex and working back to start vertex, it is often called backward dynamic programming.

Dijkstra's Algorithm

Dijkstra's algorithm [3,4] is a method of determining the shortest path between two vertices. The shortest path is found stage by stage. In finding the shortest route to a vertex, we assign to the vertex various numbers. These numbers are simply the length of various paths to that vertex. As there may be many possible paths to a vertex, several different numbers may be assigned to it. Of all possible numbers assigned to a vertex, the smallest one is important. We call this smallest number a label. The label gives the length of the shortest path to the vertex, suppose we wish to find the shortest path from *S* to *T* in a network the algorithm can be presented in three steps. Since the algorithm can be applied to both graphs and diagraphs, the word "arc" can be replaced "edge" in the following steps.
$\min\left[U_j, i\right] = \min\left[U_i + d_{ij}, i\right]; d_{ij} \geq 0,$ outlined in the following details below
Step 1 – assign a label O to *S*.
Step 2 – This is the general step. Look at a vertex which has just been assigned to
Label, say the vertex is *A* through a single are, say that this vertex is *B* to *B* assign the number given by (label of *A+weightAB*). If a vertex is reachable by more than 1 route assign to it the minimum possible such number. Repeat this process with all vertices that have just been assigned a label and all vertices that are reachable from them. When all reachable vertices have been assigned a number, the minimum number is converted into a label. Repeat step 2 until the final vertex *T* is assigned a label.
Step 3 – Steps I and 2 have simply found the length of the shortest route this step finds the actual shortest route, we begin at the destination vertex *T* an arc *AB* is included whenever the condition label *B* label of *A*= weight of *AB* holds true. This route may not be unique.

Greedy and Prim's Algorithm

These algorithms [4,5] are used mainly by television and telephone companies in competing the cities by a cable so that their Carle television and telephone facilities are made available to them. That is, these algorithms help to solve problems known as minimum connector problem, which means connecting cities with minimum amount of cable.

$d_{ij} + d_{jk} < d_{ik}$

In graph theory terms, the cities are vertices and the cable is edge. If the vertices are connected in such a way that a cycle exists, then at least one edge could be removed leaving the vertices still connected. Recalling that a connected graph which contains no cycles called a tree, it is clear that the best way of connecting all the vertices would be to find a tree which passes through very vertex. The networks below illustrate this.
A tree which passes through all the vertices of a network is called a spanning tree. Spanning tree which has the shortest total length is a minimum spanning tree. There may be more than 1 minimum spanning tree. The problem faced by the television or telephone companies is to find a minimum spanning tree of the network.
There [4,5] are two algorithms which may be used to find a minimum spanning tree
(i) The greedy algorithm
(ii) Prim's algorithm

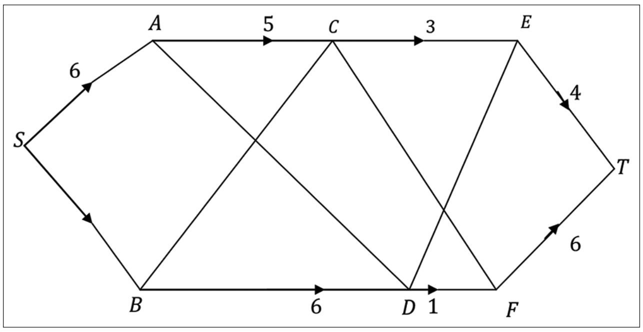

Figure 1: Dijkstra's Algorithm

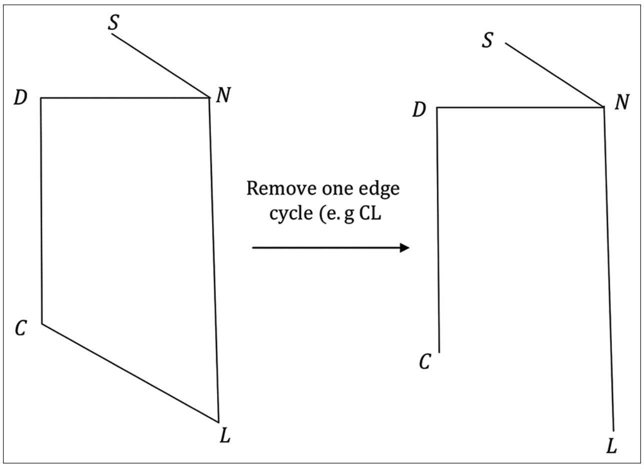

Figure 2: Greedy and Prim's Algorithm

They are essentially the same algorithm and really only differ in the way they are set on.

The greedy algorithm builds up the tree adding one vertex and one edge with each application. Any vertex can be used as a starting the vertex added at each stage unused vertex nearest to any vertex which is already a part of the tree and that the edge added is the shortest available edge. The greedy algorithm may be summarizing as follows:

Step 1 –Choose any vertex as a starting vertex.

Step 2 – Connect the starting vertex to the nearest vertex.

Step 3 – Connect the nearest unused vertex to the tree.

Step 4 – Repeat step 3 until all vertices have been included.

Prim's algorithms uses a tabular format making it more suitable for computing purposes since, as mentioned earlier, greedy and Prim's algorithms are basically the same it will be enough to illustrate how the greedy and Prim's algorithms are used by working through a specific example in chapter three.

BASIC RESULTS

Preliminaries

Let X be a non-empty set and d or ρ a function defined on $X\times X$ into the set of real numbers R such that

$$d(\ldots): X \times X \rightarrow R$$

satisfying the following conditions

- $d(x,y)=0$ if and only if $x=y$
- $d(x,y)=d(y,x)$ for all $x,y \in X$
- $d(x,y)\leq d(x,z)+d(z,y)$ for all $x,y,z \in X$

The number $d(x,y)$ is called the distance between x and y, d is called the metric, and the pair (X,d) is called the metric space.

Definition 2.[4,5]: A subset A of a metric space is said to be bounded if there is a positive constant M such that $d(x,y) \leq M$ for all $x,y \in A$.

Definition 2.2[4,5]: A subset A of a metric space is called a closed set if every convergent sequence in A is its limit in A.

Definition 2.3[4,5]: A subset of a metric space is called compact if every bounded sequence has a convergent subsequence.

Definition 2.4[4,5]: A mapping from one metric space into another metric space is called continuous if $x_n\to x$ implies that $T(x_n)\to Tx$ that is $\lim d(x_n,x)=0\Rightarrow \lim d(T(x_n),Tx)=0$

Theorem 2.1[5,6]: Every bounded and closed subset of R^n is compact

Definition 2.5[5,6]: A sequence in a metric space $X=(X,d)$ is said to converge or to be convergent if there is an $x \in X$ such that

$$\lim d(x_n, x) = 0$$

x is called the limit of $\{x_n\}$ and we write

$$\lim_{n\to\infty} x_n = x$$

Or simply

$$x_n \to x$$

If $\{x_n\}$ is not convergent, it is said to be divergent

Lemma 2.2: Let $X=(X,d)$ be a metric space, then

a. A convergent sequence in X is bounded and its limit is unique

b. If $x_n\to x$ and $y_n\to y$ in X, then $d(x_n,y_n)\to d(x,y)$

Definition 2.6[5,6]: A sequence $\{x_n\}$ in a metric space $X=(X,d)$ is said to be Cauchy if for every $\varepsilon>0$, there is an $N=N(\varepsilon)$ such that

$$d(x_m, x_n) < \varepsilon \text{ for every } m,n>N$$

The space X is said to be complete if every Cauchy sequence in X converges.

Theorem 2.2[5,6]: The Euclidean space, R^n is a complete metric space

Definition 2.7[5,6]: A metric can be induced by a norm if a norm on X defines the metric d on X as $d(x,y)=\|x-y\|$ and the normed space so defined is denoted by $(X,\|.\|)$ or simply X.

Definition 2.8[5,6]: Let (X,d) be a continuous complete metric space with the metric $d(X_1,X_2)$ induced by the norm $\|x_1-x_2\|$. If $T: X\to X$ is a map such that

$$Tx = d(x_1, x_2) = x_1 - x_2 = x \forall x_1, x_2 \in X$$

Then x is a fixed point of the set X

Definition 2.9[5,6]: If $\|.\|$ be a norm induced by the metric d such that the operator $T: X\to X$ is such that $\|Tx_1-Tx_2\|\leq K\|x_1-x_2\|$ $\forall$ x_1, $x_2 \in X$ and $k>1$, then such a Lipschitzian map is called a contractive map and non-expansive or a pseudo contractive map if, on the other hand, $k=1$, but if $k>1$, the map becomes a strong pseudo-contraction

Main Result

The above-mentioned definitions and results served as a guide in developing the facts below which form the basis of our main result used in determining the shortest route problem solutions.

Facts

i. The domain of existence of the shortest route path dynamic programming problem is the complete metric space with the set $X=R$, a closed and bounded set.
ii. The fixed-point iterative operator is continuous in the domain of the closed set R and converges at a unique sink (x_n+1) where the initial iterate x_0 is the source.
iii. The distance function sometimes is linear and sometimes non-linear, hence, the reason for the use of the metric induced by the norm $d(x_1,x_2)=\|x_1-x_2\|$
iv. That the shortest route problem of the dynamic programming problem satisfies the strong pseudo-contractive condition of the fixed-point iterative method
v. That the shortest route method of the dynamic programming problem is a reformulation of the modified Krasnoselskii method of the fixed-point iterative method for strongly pseudo-contractive maps

Theorem 2.3

Let (X,d) be a complete metric space and T a strongly pseudo-contractive iterative map of the shortest route problem in (X,d) induced by the norm ‖x_1-x_2 ‖ well posed in the Banach space such that the solution method

$$Tx = \min\left[U_j, i\right] = \sum_{ij}^{n} \min\left[U_j, i\right]$$

$$= \sum_{ij}^{n}\left[U_i + d_{ij}, i\right], d_{ij} \geq 0$$

has the unique fixed point

$$d_{ij} + d_{kj} < d_{ik}$$

with $i \to k$ becoming $i \to j \to k$ and $i \neq k$, $j \neq k$, $i=j$; the pivot row with pivot column being row k and the triple operation, $i \to j \to k$ holding in each element d_{ij} in D_k-1 ∀ i,j such that when $d_{jk}+d_{kj} \leq d_{ij}$ ($i \neq k$, $j \neq k$, $i \neq j$) is satisfied, then we

i. Create D_k by replacing d_{ij} in D_{k-1} with $d_{ik}+d_{kj}$
ii. Create S_k by replacing S_{ij} in S_{k-1} with k and setting k in k+1 and repeating step k.

Proof of Main Result

Let (X,d) be a complete metric space, the closed and bounded distance function space of the dynamic programming containing all the various paths linking the various nodes beginning from the source to the sink. We aim to establish that the dynamic programming method of the shortest route is a strongly pseudo-contractive iterative method of the modified Mann.

Hence,

$$Tx = \min\left[U_j, i\right] = \sum\nolimits_{ij}^{n}\left[U_j, i\right] = \sum\nolimits_{ij}^{n}\left[U_i + d_{ij}, i\right];$$

$$d_{ij} \geq 0 \Rightarrow \sum K d_{ij} \geq 0$$

$\Rightarrow \sum K d_{ij} \geq 0$ provided $K \geq 0$

Where, K is the contraction factor. If $K \geq 0$, then the iterative method is strongly pseudo-contractive and so the modified Mann's iterative method in this case, the Dijkstra's or the greedy and the Prim's method becomes the suitable iterative method for use. Hence, the proof.

APPLICATION – (SHORTEST ROUTE BETWEEN UMUAHIA AND ABUJA)

In this section, I shall only apply this network to three of the six reviewed algorithms or methods of application of the shortest route which include.

(i) Dynamic programming technique, (ii) Dijkstra's algorithm, and (iii) greedy and Prim's algorithm

The diagram below, give the routes of study

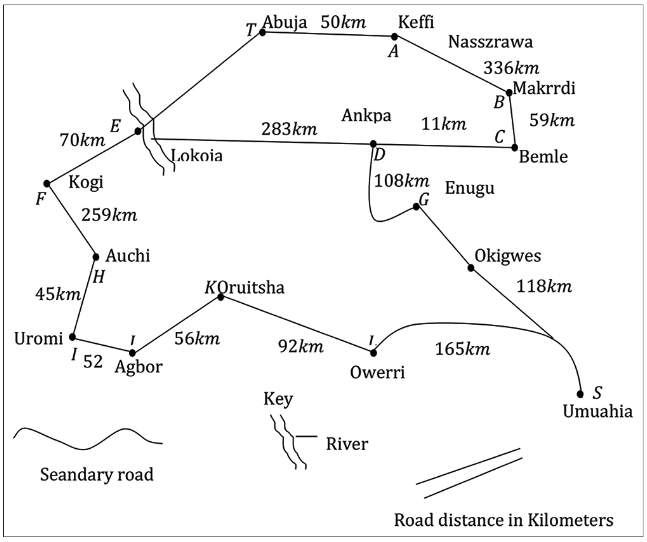

Figure 3: Backward dynamic programming techniques for finding the distance between Umuahia and Abuja by Dijkstra's algorithm

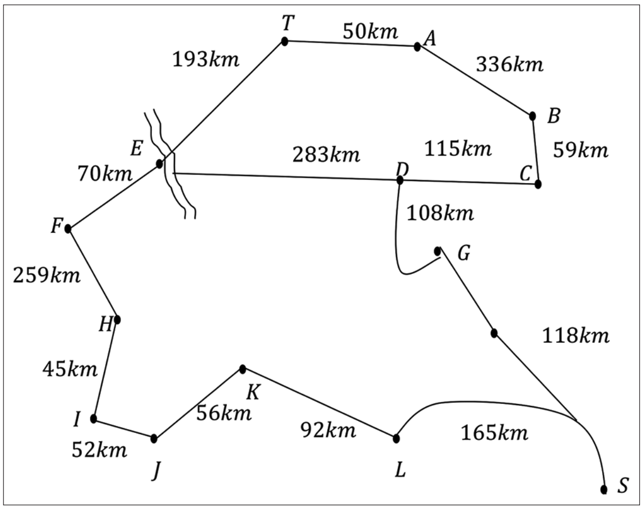

Figure 4: Backward dynamic programming technique for finding the distance between Umuahia and Abuja using Greedy and Prims

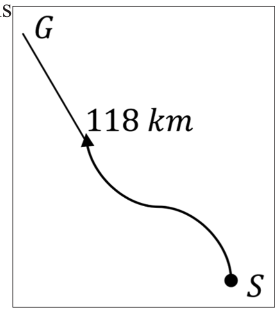

Figure 5: Distance between Umuahia and Enugu

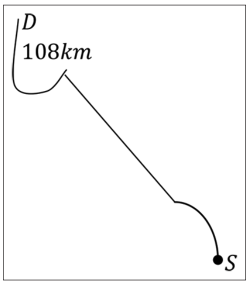

Figure 6: Distance between Enugu and Anikpa

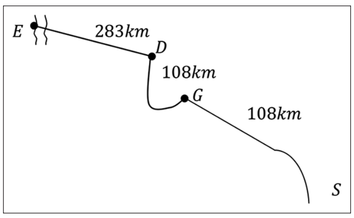

Figure 7: Distance between Umuahia and Anikpa

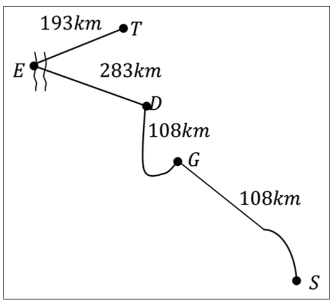

Figure 8: Distance between Umuahia and Lokoja

Application by Backward Dynamic Programming Technique

For ease of reference, we repeat the network drawn in Figure 3.

There are four possible routes from *T* to *S,* that is, from the end of the network to the beginning. As stated by backward dynamic programming, we now begin at *J* and

i. *TABCDGS*
ii. *TABCDEFHIJKLS*
iii. *TEDGS*
iv. *TEFHIJKLS*

i. *TABCDGS=TA+AB+BC+CD+DG+GS*
=50+336+59+115+108+18
=786 km

ii. *TABCDEHIJKLS=TA+AB+BC+CD+DE+EF+FH+HI+IJ+JK+KL+LS*
=50+336+59+115+283+70+259+45+52+56+92+165
=1582 km

iii. *TEDGS=FE+ED+DG+GS*
=193+283+108+118
=702 km

iv. *TEFHLJKLS=TE+EF+FH+HI+JK+KL+LS*
=193+70+259+45+52+56+92+165
=932 km

Since, we are looking for the shortest route, we close path *TEGS* because it gave us the smallest optional route, given by min (786, 1582, 702, 932).

Dijkstra's Algorithm

It is worthy of note that from the network given to us Figure 3 whereas backward dynamic programming begins at *T* and ends at *S*, Dijkstra's algorithm will be instead begin at *S* and ends in *T* so, there exist four possible networks from *S* to *T* which include.

i. *SGDCBAT*

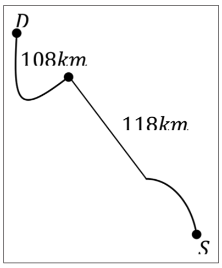

Figure 9: Outline of the cumulative shortest routes between Umuahia and Abuja

ii. *SLKJIHFEDCBAT*
iii. *SGDET*
iv. *SLKJIHFET*

i. *SGDCBAT*

$$= SG + GD + DC + CB + BA + AT$$
$$= 118 + 108 + 115 + 59 + 336 + 50$$
$$= 786km,$$

ii. *SLKJIHFEDCBAT*

$$= SL + LK + KJ + JI + IH + HF + FE + ED + ED$$
$$+ DC + CB + BH + AT$$
$$= 165 + 92 + 56 + 52 + 45 + 259 + 70 + 283 + 115$$
$$+ 159 + 336 + 50$$
$$= 1582km$$

iii. *SGDET*

$$= SG + GD + DE + ET$$
$$= 118 + 108 + 283 + 193$$
$$= 702km$$

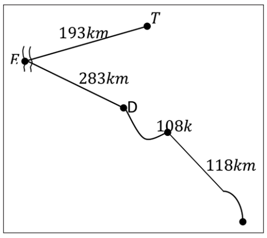

Figure 10: Total Shortest route distances between Umuahia and Abuja

iv. *SLKJIHFET*

$$= SL + LK + KJ + JF + IH + HF + FE + ET$$
$$= 165 + 92 + 56 + 52 + 45 + 259 + 70 + 193$$
$$932km$$

Furthermore, at this stage, we are now sure that the shortest distance from *S* to *T* is 702 km.
We also know the actual path which achieves this shortest length by inspection since there are not money diversity in the routes to use which is given as *SGDETieS→G→D→E→T* which diagram mathematically can be shown below as

Table 1: The Dijsktra's Algorithm table for shortest route between Umuahia and Abuja

	S	L	K	J	I	H	F	E	D	C	B	A	T
S	--	165	257	313	365	410	669	739 509	1022, 226	1137 341	1196 400	1532 736	1582 932,702 786
L	165	-	92	148	200	245	504	574 674	857 391	972 506	1031 565	1367 901	1417,767 951, 867
K	257	92	-	56	108	153	412	482 766	765 483	880 598	939 657	1275 993	1325,675 1043, 959
J	313	148	56	-	52	97	356	426 822	709 539	824 654	883 713	1219 1049	1269,619 1099 1015
I	365	200	108	52	-	45	304	374 874	657 591	772 706	831 965	1167 1101	1217, 567, 1151,1067
H	410	245	153	97	45	-	259	329 919	612 636	727 751	786 810	1122, 1146	1172, 522, 1196,1112
F	669	504	412	356	304	259	-	70 1178	353 895	468 1010	527 1069	863, 1405	913,263 1455, 1371
E	739	574	482	426	374	320	70	-	283 965	398 1080	457 1139	793 1475	843, 193, 1525
D	226	391	483	539	591	636	895	965	-	115	174	570	1158, 560
C	341	504	598	654	706	751	1010	1080	1363	-	59	395	1273, 445
B	400	565	657	713	765	810	1069	1139	1422	1537	-	336	1332, 386
A	736	901	993	1049	1101	1146	1405	1475	1758	1873	1932	-	50, 1668
T	786 702 932 258	951 867, 1417, 767	1043 675, 1043, 959	1099 6192 1015, 1269	1151 567 1069, 1217	1196 522, 1112, 1172	1455 913, 263, 1271	1525 193 843	506 1158	445, 1273	1332, 386	50, 668	-

Table 2: The Greedy and Prims shortest route algorithm table of distance between Umuahia and Abuja

	S	D	E	T		D	E	T
S	-	226 1022	739, 509	932,702, 786,15582	D	-	739, 509	932, 702,786, 1582
					E	283	-	193, 843
D	226	-	283	476 560	T	476, 560	193, 843	-
E	739, 509	283	-	193 843				
T	932, 702, 786, 1582	476, 560	193, 843	-				

Figure 11: A re-examination of the shortest route using the above Greedy and Prims Algorithm by considering only points S – D as in i – v below

Prim's and Greedy Algorithm

Applying Greely's algorithm in the figure, that is, Figure 3, we have being considering we use the following procedure
Choosing any vertex as a starting vertex, say *S*

The nearest vertex to *S* is G
The nearest vertex to *G* is *D*
Also,
The nearest vertex to *D* is *E*
The nearest vertex to *E* is *T*
The total length of the figure is 702 km which the shortest route of the path *SGDET*.

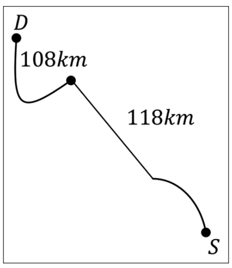

Figure 12: A re-examination of the shortest route problem of Umuahia to Abuja using the above Greedy and Prims table as in i – iv above

APPLICATION USING PRIM'S ALGORITHM

Prim's algorithm uses the table formulated below to find the shortest route
From a close study of the table above, we come up

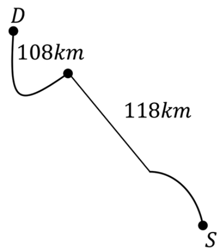

Figure 13: A re-examination of the shortest route diagram from Umuahia to Abuja using Greedy and Prims table as in i – iv above

with the following resolutions;

i. There is a zero distance from *S* to *S*, therefore, we eliminate row and column *S*.
ii. The nearest distance from *S* is *L* so, we eliminate row and column *I*.
iii. We resolve to eliminate points *LJFHEDCAT*, *LJIHEDCBAT*, and *LJIHEDCAT*
iv. We now illustrate, the shortest route chosen as follow.
v. We represent the shortest route from one point to another with the least number derivable in that route.

But, for purpose of clarity; we reduce the table to this since the shortest route is the aim.
Repeating Figure 9.

Applying the principles (i), (ii), (iii), (iv), and (v) above. The smallest number in the 1st row is 283. Hence, we eliminate *D* and the shortest route diagram is *SD*

DISCUSSION, CONCLUSION, AND SUGGESTION

Discussion of Finding

Our findings on choice of routes, road users within Umuahia-Abuja metropolis make use of, showed that most Umuahia-Abuja road users have one problem or the other traveling on road. The problems range from potholes, traffic jams, and long-distance routes. Existing routes are even being governed by markets and traders thereby increasing traffic jams.

Our finding also discovered that road users and motor vehicles are increasing at geometric progression while the route/networks are increasing in arithmetic progression or even Istinated not increasing flood, during rainy, and time of travel.

Thus, making other factors affecting movements from one point to the other constraint and focusing one distance, we will certainly agree that shortest routes makes travel interesting. These follow the hypotheses that any part of shortest/congest path is itself a shortest/congest path and we say that only part of an optional route is itself optional.

Suggestions

Based on our findings in the course of this study, the researcher suggests as follows:

1. That routes be create from one geographic location to another by any responsible authority, especially government.
2. Maintenance activity/works should be done on a regular basis on the existing routes.
3. Road directions and warnings should be positioned at strategic functions to enable travelers locate their destinations from their source and have enough information to prevent accidents.
4. Branched network should be attached to reduce the rate of traffic gains on our routes.
5. Police menace on our roads (routes) of travel should be discouraged.
6. Road users should be cautions as they use the roads.
7. Safety providing agencies should make themselves available in every route of travel within Umuahia.
8. Road users should make the shortest path their route of travel to minimize length and time of travel.

CONCLUSION

In the transportation world today, the routes are regarded as king in the sense that they provide channels/links between two geographical points/locations [Figures 1 to 12 and Tables 1 to 2 as in section 1 and 2]. The routes do not just come into existence they are created or built by man to facilitate movement from on point to the other.

Although these routes cannot catapult any one from one geographical location to the other on their own, when they exist and good once, even without locomotion machines like motor vehicles, one can still makes a Forney by foot.

Government on their own should make building and maintenance of road and networks paramount projects. It is expected that wise travelers having known that there exists short and long route may decide to choose traveling through the shortest route.

Suggestions for Further Research

This work has examined the application of shortest route in dynamic programming considering the factor of minimum distance. Further studies could still be carried out to understand more factors which could likely determine the minimum or maximum distance between two location and other applications excluding the ones used in this work to determine the shortest path/route between one location/point to the other.

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