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RESEARCH ARTICLE

Modification of 2-Connected with Gallai's Property under 18 Vertices Including Mobius Strip

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ABSTRACT

The studies of graph, structure and mathematical molecule have pair-wise relationship among each other is said to be the graph theory. It is important part of discrete mathematics, which is made of by vertex called the nodes or points and edges so called links or lines, if the graph has symmetrically lines or links, created as a undirected graph whereas the links joined asymmetrically creates direct graph. There are so many kinds of graph; Hypo-Hamiltonian graph has its own identity, it can be defined as a graph if it has not itself a Hamiltonian cycle but each graph developed by eliminating with a single vertex from the graph is Hamiltonian. Naeem Ahmed Kalhoro has worked on "A Two-Connected Graph with Gallai's Property." In his research paper, he has applied the property and has found the longest path and cycle in the graph which contains 12 vertices and in the modification of above work, we have developed a graph on 18 vertices and have tried to find the longest cycle and path in the proposed graph.

Key words: Gallai's property, Graph theory hypo-hamiltonian, Hamiltonian, Hypo-traceable

INTRODUCTION

The graph theory is extensively used in modern models such as wireless operating system, mobile networks, electronic and non-electric wires, and even though in data structure. It makes human easy, it can be used in several types of relationship among human being and living organism, social, chemical, and physical relationship are the best examples of graph theory.

In mathematics and modern technological perspective graph theory plays an important rule, due his wide range application, the researcher taking high attention in this field. Graph theory probably applicable in so many fields specially computer science, logical design, and graphing etc. In this investigation, we have found that The Peterson Graph which is most popular example of Hypo-Hamiltonian graph is undirected graph. This is developed by Julius Petersen in 1898 which is smallest bridgeless cubic graph which does not contains three edge coloring. It is generally made in pentagon with as a pentagon inside with five spokes. In 1966, a great mathematician, namely, Tibor Gallai idea of property about missing vertices in the graph before the manufacture of hypo Hamiltonian graph and by his given idea, it became famous as a property of Gallai's. Hence, many authors and mathematician have given their idea and have asked and replied the question about Gallai's property in 1969 Walther[1] was first to replied on the property and developed a planar graph covering twenty-five nodes and were sustaining the criteria of Gallai's property. Furthermore, another mathematician named Walther has given very important contribution and another mathematician Walther and Voss play an important role in this research.[2] In this research work, Zamfirescu has also given his contribution in the prove of property,[3] He developed a molecule which were containing 12 nodes and had been estimated the point that this type of 12 order graph keeps smallest opportunity of this kind of developed molecule or diagram. Later than Zamfirescu modified the question, he asked is there any path or cycle in the developed graph. Various best examples are that replying Zamfirescu's questions has been published.

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A Hamiltonian cycle is defined as a cycle which passes every vertex of graph for single time and will end at identical started node is said to be Hamiltonian cycle and another hand, the Hamiltonian path is defined as the longest path which start from a vertex and visit every vertex but does not end at the started point or vertex is called Hamiltonian path. Furthermore, a molecule or graph is said to be perceptible if it has Hamiltonian track or path and a grid or graph has Hamiltonian cycle, it is called Hamiltonian graph, including the definition of Hypo-Hamiltonian, it can be defined as a graph if it has not itself a Hamiltonian cycle but each graph developed by eliminating with a single vertex from the graph is Hamiltonian. A Hungarian researcher and mathematician in 1966 had operated in combinative, specifically in theory of graph, and was unprecedented associate and turncoat of Paul Erdős. There is the student mentor relation between.[4] W. Schmitz developed a lowest number of vertices containing 17 in the case of the planar graph.[5] Zamfirescu had developed the first two connected planar graph.[6] with 82 nodes. The popular deepest illustration of such kinds of structure or graph has 26 vertices nowadays,[7] contrary-wise the deepest planar sample awake to nowadays has 32 order.[6] In 1972, a great researcher, namely, Zamfirescu developed the idea about Gallai's property. He answered that if $p_j^i = \infty\ (p_i^{-j} = \infty)$ and the graph is not planar or *i–connected* graph such self-sufficiently set of *j* points persevere distinct from the lengthiest path condition $p_j^i \neq \infty\ (p_i{}^j \neq \infty)$. For example, $p_j^i p_i^{-j}$ displays shortest numeral of nodes of a *j–connected* graph just like the autonomously the group of *j* selected nodes be the disjoint from the lengthiest path. Equally the cases C_i^j *and* C_i^{-j} are plainly applied for longest circuits as the exchange of longest path. For finding the best reactions on the elevated enquiries about the matters on the work of researcher Zamfirescu's, had been passed out by the researcher Schmitz,[5] A great mathematician named Walther[8] and developed the example of $C_2^2 \leq 220$ *and* $C_2^1 \leq 105$,[1] An other mathematician, namely, Grunbaum,[9] Hatzel,[10] and Zamfirescu,[6] have also seen the studied and their consequences.[11,12] Naeem and Jumani have contributed in this work with developed two graphs of 18 and 22 vertices which satisfying the Gallai's property in 2019[13] and in his other paper.[14] A research paper on a connected graph contains 20 vertices for finding the cycle and path by similar property and satisfying the Gallai's property has been published by.[15-17]

RESULTS AND DISCUSSION

In this research paper, we will develop 2-connectedand including 18 nodes graph is satisfying Gallai's property o developed graph.

Theorem 1: Consider the non-planar graph G in Figure 1 which consists 18 vertices. The structure of graph is the combination of the shapes of

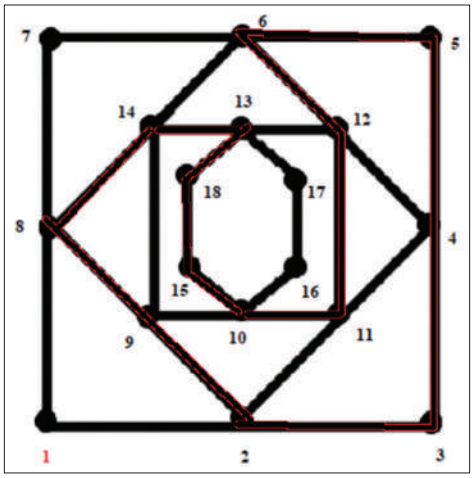

Figure 1: Proposed graph

Table 1: Longest cycle C (G)=15

Cycle	Detail of vertex in longest cycle	Missed vertex
F	1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 13, 14, 16, 17	7, 11, 15, 18
G	2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 18	1, 7, 16, 17
H	2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 16, 17	1, 7, 15, 18
I	2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 16, 17	1, 12, 15, 18
J	2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 18	1, 12, 16, 17
K	2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 16, 17	1, 7, 15, 18
L	2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 16, 17	1, 14, 15, 18
M	2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 18	1, 7, 16, 17
N	2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 18	1, 14, 16.17
E	2, 3, 4, 5, 6, 8, 9, 10, 12, 13, 14, 15, 18	7, 11, 16, 17

Table 2: Longest cycle C (G)=14

Cycle	Detail of vertex in longest cycle	Mixed vertex
A	1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 13, 16, 17	7, 11, 14, 15, 18
B	1, 2, 3, 4, 5, 6, 8, 9, 10, 13, 14, 15, 17	7, 11, 12, 16, 18
C	1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 13, 15, 18	7, 11, 14, 16, 17
D	1, 2, 3, 4, 5, 6, 8, 9, 10, 13, 14, 15, 18	7, 11, 12, 16, 17
O	2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 16, 17	1, 11, 12, 15, 18

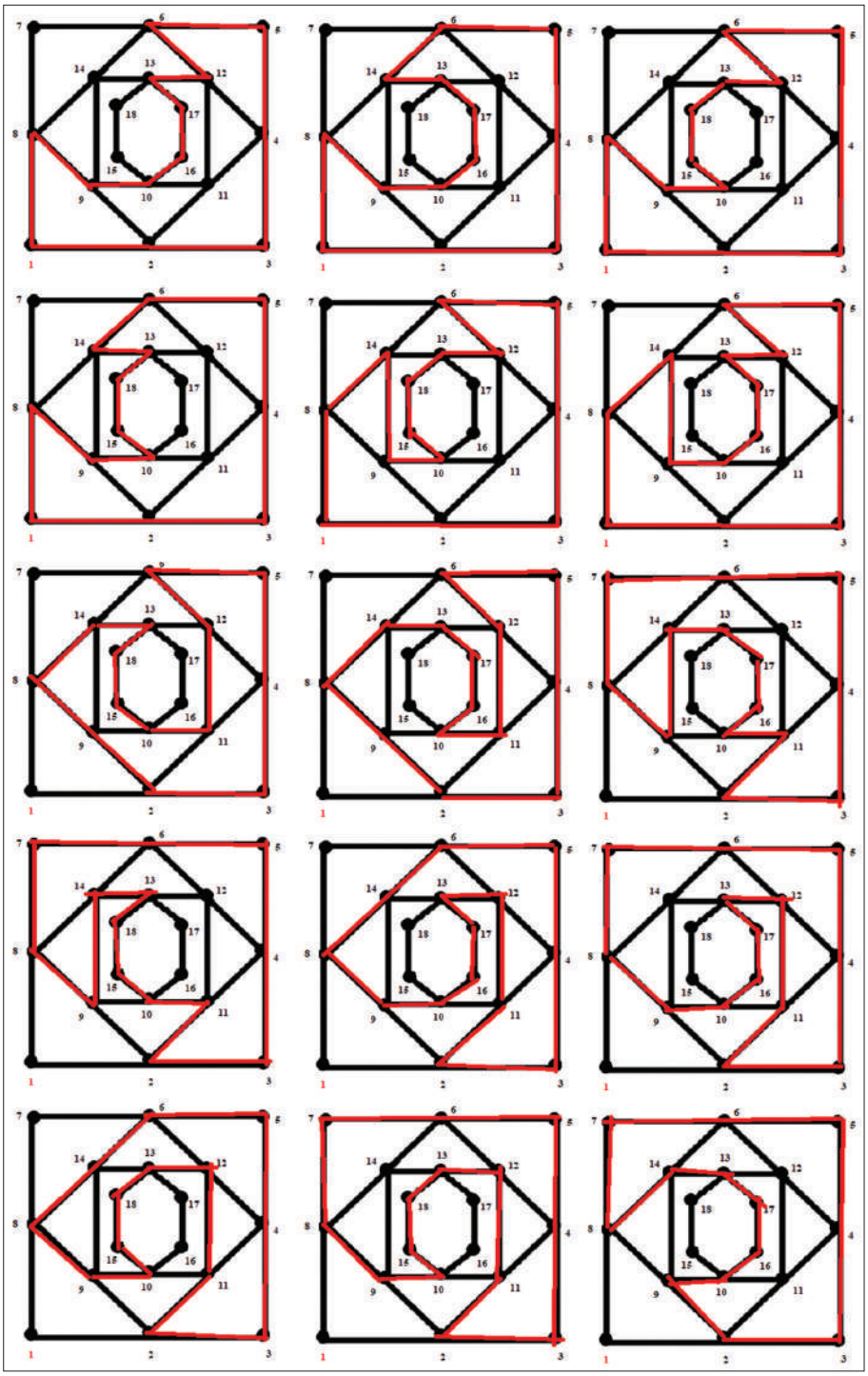

Figure 2: Longest cycle in the graph

Table 3: Longest Path *P* (G)=16

Path	Detail vertex of longest path	Missed vertex	Focus vertex
A	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17	15, 18	1
B	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18	16, 17	1
C	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17	15, 18	3
D	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18	16, 17	3
E	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17	15, 18	5
F	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18	16, 17	5
G	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17	15, 18	7
H	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18	16, 17	7
I	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17	15, 18	6

Table 4: Detail of vertices of longest cycle

Cycle	Detail vertex of longest cycle	Missed vertex	Focus vertex
A	1, 15, 14, 16, 17, 18, 9, 10, 11, 13, 12, 5, 6, 7, 8, 2	3, 4	1
B	1, 15, 14, 16, 17, 18, 9, 10, 11, 13, 12, 5, 6, 4, 3, 2	7, 8	1
C	5, 12, 13, 11, 10, 9, 18, 17, 16, 14, 15, 1, 2, 8, 7, 6	3, 4	5
D	5, 12, 13, 11, 10, 9, 18, 17, 16, 14, 15, 1, 2, 3, 4, 6	7, 8	5
E	15, 14, 16, 17, 18, 9, 10, 11, 13, 12, 5, 6, 7, 8, 2, 1	3, 4	15
F	15, 14, 16, 17, 18, 9, 10, 11, 13, 12, 5, 6, 4, 3, 2, 1	7, 8	15

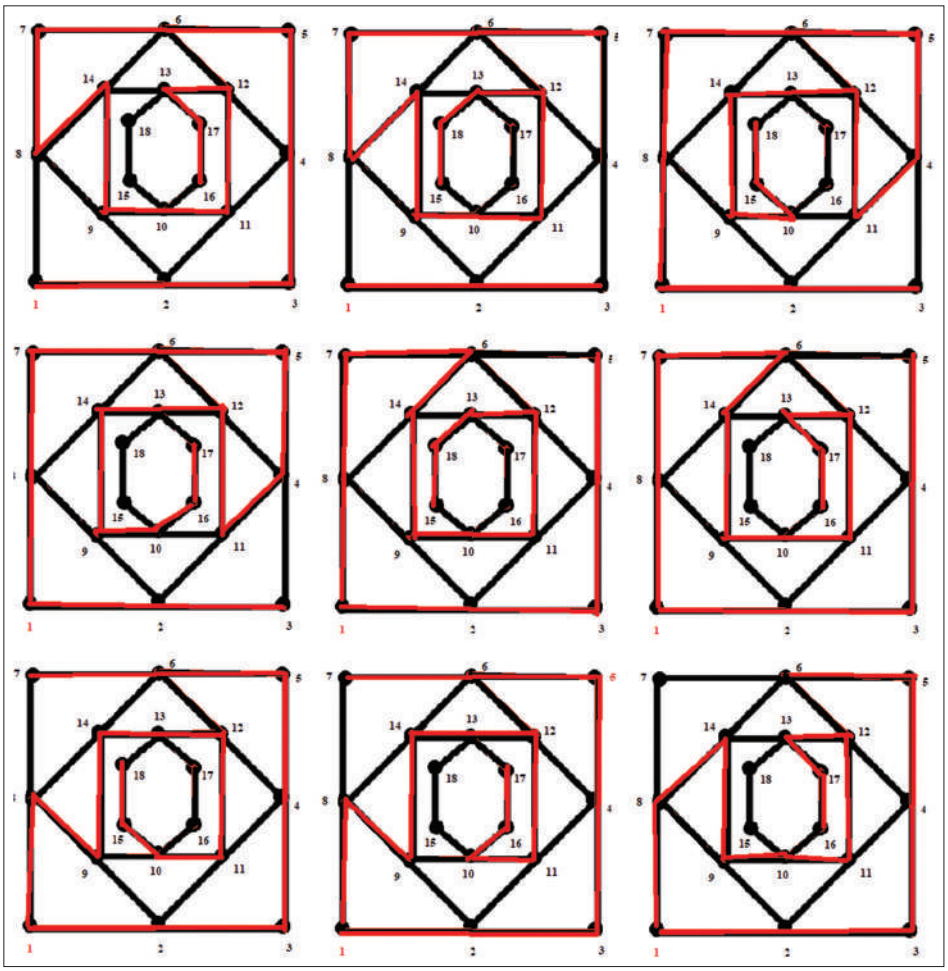

Figure 3: Longest path P(G)=16

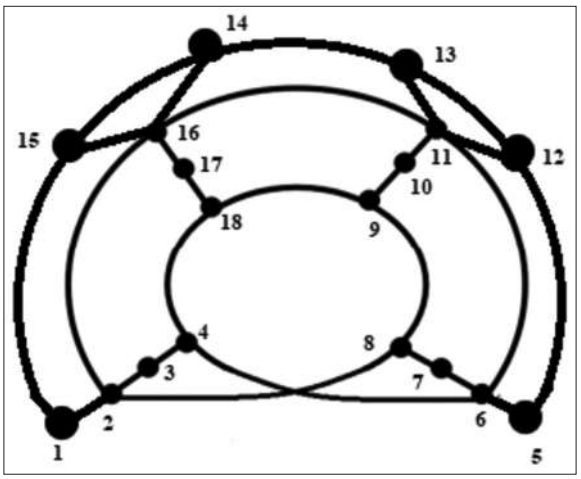

Figure 4: Mobius strip

square, rectangle, and hexagonal. Avoiding the vertices from the molecule, we investigated the 14 longest cycles *C(G)=15* and also *C(G)=14* in the graph and some copies are shown in Figure 2.

In this investigation, we have found some missing vertices as shown in Table 1 from the proposed graph and drawing them on the graph to find the longest cycles shown in the Figure 2. (A to O)

$C(G)=15$ $P(G)=16C(G)=14$ [Figure 2]The longest path of the proposed graph $P(G)=16$ by missing two vertices is shown in Table 3.

Theorem 2: In this theorem, we have developed a

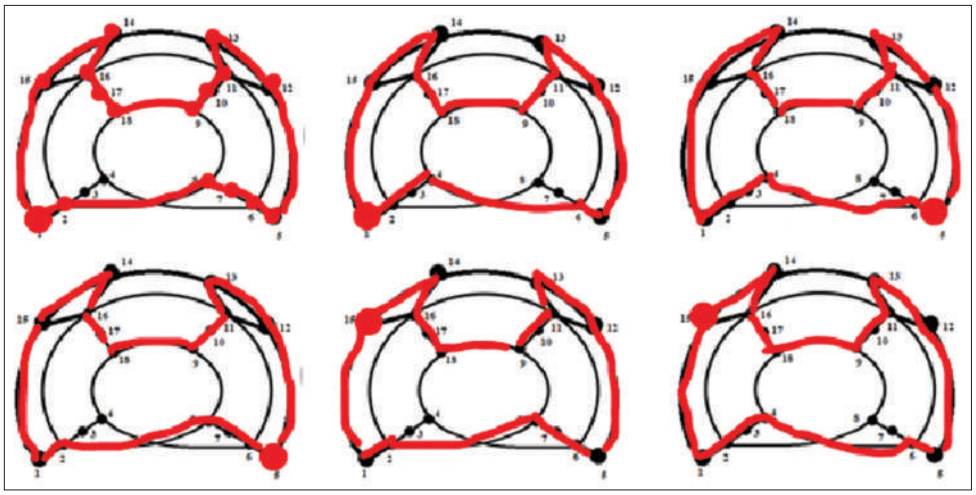

Figure 5: Longest cycle

Mobius strip which is sustaining and proving the Gallai's property [Figure 4].

Developed a Moebius band (or Mobius strip) which contains 12 vertices and longest cycle order was $C(G)=10$ [1] but in above graph, the order of highest cycle is 16 shown in Figure 5 and highest path is $P(G)=18$. [Tables 2 to 4].

CONCLUSION

Investigational outcomes display that the established a 2-conneted non-planar graph having missed nodes having lengthiest cycle exposed in the above tables.

In this investigation, we have learnt about the exaltation of 2-connected non-planar graph which have 18 nodes or vertices and satisfying Gallai's property.

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