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Some Results Regarding To New Triple Laplace Transform

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ABSTRACT

In this paper, the new triple Laplace transform is defined. The main results and theorems on the new triple Laplace transform are investigated. The theory of fractional differential equations based on new triple Laplace transform is developed in this work.

Keywords: Laplace transform; fractional calculus; fractional integral; fractional derivatives; partial fractional derivatives.

AMS Subject Classification: 45-xx, 65-xx.

1. INTRODUCTION

Fractional calculus has attracted many researchers in the last decades. The impact of this fractional calculus on both pure and applied branches of science and engineering has been increased. Many researchers started to approach with the discrete versions of this fractional calculus benefiting to get many aspects in the real life [1-3]. In [4], the authors introduced a new well-behaved simple approach of fractional derivatives called conformable derivative, which occurs naturally and obeys the Leibniz rule and chain rule.

In [5-6], the conformable calculus and their new properties have been analyzed for real valued multivariable functions. In [7], conformable gradient vectors are defined, and a conformable sense Clairaut's Theorem have been proven. In [8-10], the researchers have worked on the linear ordinary and partial differential equations based on the conformable derivatives. Namely, two new results on homogeneous functions involving their conformable partial derivatives are introduced, specifically, homogeneity of the conformable partial derivatives of a homogeneous function and the conformable version of Euler´s Theorem [3].

The conformable Laplace transform was initiated in [11] and studied and modified in [12]. The conformable Laplace transform is not only useful to solve local conformable fractional dynamical systems but also it can be employed to solve systems within nonlocal conformable fractional derivatives that were defined and used in [10]. Finally, it is also a remarkable fact a large number of studies in the theory and application of fractional differential equations based on this new definition of derivative, which have been developed in a short time. We come up with the idea to study the nonlinear partial fractional differential equations by defining a function in 3 space. Therefore, a new triple Laplace transform is defined.

This paper is divided into the following sections: In section 2, some basic definitions are introduced. In section 3, the main results and theorems on the new triple Laplace transform are investigated. In section 4, a conclusion of our research work is provided.

2. PRELIMINARIES

In this section, some basic definitions on conformable partial derivatives are introduced.
Definition 2.1. ([4])

For a function $f:(0,\infty)\to R,$ the conformable derivative of order $\alpha\in(0,1]$ of f at x>0 is defined by,

$$T^{\alpha}f(x)=\lim_{h\to 0}\frac{f(x+hx^{1-\alpha})-f(x)}{h}. \quad (2.1)$$

Definition 2.2. ([5])

Given a function $f:R^{+}\times R^{+}\times R^{+}\to R,$ the conformable partial fractional derivatives (CPFDs) of orderα,and γ of the function $(x$,,) is defined as follows:

$$\partial_x^{\alpha}f=\lim_{h\to 0}\frac{f(x+hx^{1-\alpha},y,t)-f(x,y,t)}{h},$$
$$\partial_y^{\beta}f=\lim_{k\to 0}\frac{f(x,y+ky^{1-\beta},t)-f(x,y,t)}{k},$$
$$\partial_t^{\gamma}f=\lim_{\lambda\to 0}\frac{f(x,y,t+\lambda t^{1-\gamma})-f(x,y,t)}{\lambda}. \quad (2.2)$$

Where 0<α,,≤1,x,y,t>0.

Theorem 2.1. Let $\alpha,\beta,\gamma\in(0,1],a,b\in R$ and $l,m,n\in N$; Then, we have the following:

1) $\frac{\partial^{\alpha}}{\partial x^{\alpha}}\big(au(x,y,t)+bv(x,y,t)\big)=a\frac{\partial^{\alpha}}{\partial x^{\alpha}}u(x,y,t)+b\frac{\partial^{\alpha}}{\partial x^{\alpha}}v(x,y,t),$

2) $\frac{\partial^{\alpha+\beta+\gamma}}{\partial x^{\alpha}\partial y^{\beta}\partial t^{\gamma}}(x^{l}y^{m}t^{n})=lmn\,x^{l-\alpha}y^{m-\beta}t^{n-\gamma},$

3) $\frac{\partial^{\alpha}}{\partial x^{\alpha}}\left(\left(\frac{x^{\alpha}}{\alpha}\right)^{l}\left(\frac{t^{\gamma}}{\gamma}\right)^{n}\right)=l\left(\frac{x^{\alpha}}{\alpha}\right)^{l-1}\left(\frac{t^{\gamma}}{\gamma}\right)^{n},$

$\frac{\partial^{\gamma}}{\partial t^{\gamma}}\left(\left(\frac{x^{\alpha}}{\alpha}\right)^{l}\left(\frac{y^{\beta}}{\beta}\right)^{m}\left(\frac{t^{\gamma}}{\gamma}\right)^{n}\right)=n\left(\frac{x^{\alpha}}{\alpha}\right)^{l}\left(\frac{y^{\beta}}{\beta}\right)^{m}\left(\frac{t^{\gamma}}{\gamma}\right)^{n-1},$

4) $\frac{\partial^{\alpha}}{\partial x^{\alpha}}\left(\sin\left(\frac{x^{\alpha}}{\alpha}\right)\cos\left(\frac{t^{\gamma}}{\gamma}\right)\right)=\cos\left(\frac{x^{\alpha}}{\alpha}\right)\cos\left(\frac{t^{\gamma}}{\gamma}\right),$

5) $\frac{\partial^{\beta}}{\partial y^{\beta}}\left(\sin\left(\frac{x^{\alpha}}{\alpha}\right)\cos\left(\frac{y^{\beta}}{\beta}\right)\cos\left(\frac{t^{\gamma}}{\gamma}\right)\right)=-\sin\left(\frac{x^{\alpha}}{\alpha}\right)\sin\left(\frac{y^{\beta}}{\beta}\right)\cos\left(\frac{t^{\gamma}}{\gamma}\right).$ (3.2)

Proof: By the definition of CFPD,

$$\frac{\partial^\alpha}{\partial x^\alpha}\big(au(x,y,t)+bv(x,y,t)\big) = \lim_{h\to 0}\frac{au(x+hx^{1-\alpha},y,t)-au(x,y,t)+bv(x+hx^{1-\alpha},y,t)-bv(x,y,t)}{h} = a\frac{\partial^\alpha}{\partial x^\alpha}u(x,y,t)+b\frac{\partial^\alpha}{\partial x^\alpha}v(x,y,t). \tag{4.2}$$

Similarity, we can prove the results 2-5.

Theorem 2.2. Let $\alpha,\beta,\gamma\in(0,1]$ and $f(x,y,t)$ be a differentiable at a point for $x,y,t>0$. Then,

1) $\partial_x^\alpha f = \frac{\partial^\alpha}{\partial x^\alpha} f(x,y,t) = x^{1-\alpha}\frac{\partial f(x,y,t)}{\partial x} = x^{1-\alpha}\partial_x f$,

2) $\partial_y^\beta f = \frac{\partial^\beta}{\partial y^\beta} f(x,y,t) = y^{1-\beta}\frac{\partial f(x,y,t)}{\partial y} = y^{1-\beta}\partial_y f$,

3) $\partial_t^\gamma f = \frac{\partial^\gamma}{\partial t^\gamma} f(x,y,t) = t^{1-\gamma}\frac{\partial f(x,y,t)}{\partial t} = t^{1-\gamma}\partial_t f$. (5.2)

Proof: By the definition of CFPD,

$\frac{\partial^\alpha}{\partial x^\alpha}(f(x,y,t)) = \lim_{h\to 0}\frac{f(x+hx^{1-\alpha},y,t)-f(x,y,t)}{h}$, taking $hx^{1-\alpha}=n$ we have

$$\partial_x^\alpha f = \lim_{n\to 0}\frac{f(x+n,y,t)-f(x,y,t)}{nx^{\alpha-1}} = x^{1-\alpha}\lim_{n\to 0}\frac{f(x+n,y,t)-f(x,y,t)}{n} = x^{1-\alpha}\frac{\partial f(x,y,t)}{\partial x}.$$

Similarity, we can prove the results 2-5.

Definition 2.3. Let $(x,\ t)$ be a real valued piecewise continuous function of x,t defined on the domain D of $R^+\times R^+\times R^+$ of exponential order α, β and γ, respectively. Then, the new triple Laplace transform (NTLT) of $(x,)$ is defined as follows:

$$L_x^\alpha L_y^\beta L_t^\gamma\big(u(x,y,t)\big) = U_{\alpha,\beta,\gamma}(p,q,s) = \int_0^\infty\int_0^\infty\int_0^\infty e^{-p\left(\frac{x^\alpha}{\alpha}\right)-q\left(\frac{y^\beta}{\beta}\right)-s\left(\frac{t^\gamma}{\gamma}\right)}u(x,y,t)x^{\alpha-1}y^{\beta-1}t^{\gamma-1}dxdydt, \tag{6.2}$$

where$p,,\in\mathbb{C}$ are Laplace variables and $\alpha,\beta,\gamma\in(0,1]$.
The new inverse triple Laplace transform, denoted by $(x,)$, is defined by

$$u(x,y,t) = L_p^{-1}L_q^{-1}L_s^{-1}\left(U_{\alpha,\beta,\gamma}(p,q,s)\right) = \frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} e^{p\left(\frac{x^\alpha}{\alpha}\right)}\left[\frac{1}{2\pi i}\int_{\beta-i\infty}^{\beta+i\infty} e^{q\left(\frac{y^\beta}{\beta}\right)}\left[\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{s\left(\frac{t^\gamma}{\gamma}\right)}U_{\alpha,\beta,\gamma}(p,q,s)ds\right]dq\right]dp. \tag{7.2}$$

3. RESULTS AND THEOREMS

In this section, the main results and theorems on the new triple Laplace transform are investigated.

Theorem3.1.

If $L_x^{\alpha}L_y^{\beta}L_t^{\gamma}\left(u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)\right)=U_{\alpha,\beta,\gamma}(p,q,s), L_x^{\alpha}L_y^{\beta}L_t^{\gamma}\left(v\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)\right)=V_{\alpha,\beta,\gamma}(p,q,s)$ and

A,B and C are constants then

1) $L_x^{\alpha}L_y^{\beta}L_t^{\gamma}\left(Au\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)+Bv\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)\right)=AL_x^{\alpha}L_y^{\beta}L_t^{\gamma}\left(u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)\right)+$

   $BL_x^{\alpha}L_y^{\beta}L_t^{\gamma}\left(v\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)\right)=AU_{\alpha,\beta,\gamma}(p,q,s)+BV_{\alpha,\beta,\gamma}(p,q,s).$

2) $L_x^{\alpha}L_y^{\beta}L_t^{\gamma}(c)=\frac{c}{pqs}$,where c is constant.

3) $L_x^{\alpha}L_y^{\beta}L_t^{\gamma}\left(\left(\frac{x^\alpha}{\alpha}\right)^l\left(\frac{y^\beta}{\beta}\right)^m\left(\frac{t^\gamma}{\gamma}\right)^n\right)=\frac{\Gamma(l+1)\Gamma(m+1)\Gamma(n+1)}{p^{l+1}q^{m+1}s^{n+1}}$,Where (.) Is the gamma function. Note that $\Gamma(n+1)=n!\,, n=0,1,2,3,\dots$.

4) $L_x^{\alpha}L_y^{\beta}L_t^{\gamma}\left(e^{a\left(\frac{x^\alpha}{\alpha}\right)+b\left(\frac{y^\beta}{\beta}\right)+c\left(\frac{t^\gamma}{\gamma}\right)}u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)\right)=U_{\alpha,\beta,\gamma}(p-a,q-b,s-c).$

5) $L_x^{\alpha}L_y^{\beta}L_t^{\gamma}\left(\sin\left(A\frac{x^\alpha}{\alpha}\right)\sin\left(B\frac{y^\beta}{\beta}\right)\sin\left(C\frac{t^\gamma}{\gamma}\right)\right)=\frac{ABC}{(p^2+A^2)(q^2+B^2)(s^2+C^2)}.$

6) $L_x^{\alpha}L_y^{\beta}L_t^{\gamma}\left(cos\left(A\frac{x^\alpha}{\alpha}\right)cos\left(B\frac{y^\beta}{\beta}\right)cos\left(C\frac{t^\gamma}{\gamma}\right)\right)=\frac{pqs}{(p^2+A^2)(q^2+B^2)(s^2+C^2)}.$

7) $L_x^{\alpha}L_y^{\beta}L_t^{\gamma}\left(\left(\frac{x^\alpha}{\alpha}\right)^l\left(\frac{y^\beta}{\beta}\right)^m\left(\frac{t^\gamma}{\gamma}\right)^n u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)\right)=(-1)^{l+m+n}\frac{d^{l+m+n}}{dp^l dq^m ds^n}\left(U_{\alpha,\beta,\gamma}(p,q,s)\right).$

Proof: The results 1-6 can be easily proved by using the definition of new triple Laplace transform (NTLT). Here only we see the proof of result 7. So, by definition of NTLT (equation(6.2)),

$$U_{\alpha,\beta,\gamma}(p,q,s)=L_x^{\alpha}L_y^{\beta}L_t^{\gamma}\left(u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)\right)=$$

$$\int_0^\infty\int_0^\infty\int_0^\infty e^{-p\left(\frac{x^\alpha}{\alpha}\right)-q\left(\frac{y^\beta}{\beta}\right)-s\left(\frac{t^\gamma}{\gamma}\right)}u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)x^{\alpha-1}y^{\beta-1}t^{\gamma-1}dxdydt.$$

Differentiating with respect to p; l- times. We get

$$\frac{d^l}{dp^l}U_{\alpha,\beta,\gamma}(p,q,s)$$

$$=\frac{d^l}{dp^l}\int_0^\infty\int_0^\infty\int_0^\infty e^{-p\left(\frac{x^\alpha}{\alpha}\right)-q\left(\frac{y^\beta}{\beta}\right)-s\left(\frac{t^\gamma}{\gamma}\right)}u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)x^{\alpha-1}y^{\beta-1}t^{\gamma-1}dxdydt$$

$$=$$

$$\int_0^\infty\int_0^\infty\int_0^\infty\frac{d^l}{dp^l}e^{-p\left(\frac{x^\alpha}{\alpha}\right)-q\left(\frac{y^\beta}{\beta}\right)-s\left(\frac{t^\gamma}{\gamma}\right)}u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)x^{\alpha-1}y^{\beta-1}t^{\gamma-1}dxdydt=$$

$$\int_0^\infty\int_0^\infty\int_0^\infty(-\left(\frac{x^\alpha}{\alpha}\right))^l\,e^{-p\left(\frac{x^\alpha}{\alpha}\right)-q\left(\frac{y^\beta}{\beta}\right)-s\left(\frac{t^\gamma}{\gamma}\right)}u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)x^{\alpha-1}y^{\beta-1}t^{\gamma-1}dxdydt.$$

Now, we again differentiate with respect to q and s; m and n- times respectively, we obtain the simplification as follows:

$$\frac{d^{l+m+n}}{dp^l dq^m ds^n}\left(U_{\alpha,\beta,\gamma}(p,q,s)\right)$$

$$=\int_0^\infty\int_0^\infty\int_0^\infty(-\left(\frac{x^\alpha}{\alpha}\right))^l\,(-\left(\frac{y^\beta}{\beta}\right))^m\,(-\left(\frac{t^\gamma}{\gamma}\right))^n\,e^{-p\left(\frac{x^\alpha}{\alpha}\right)-q\left(\frac{y^\beta}{\beta}\right)-s\left(\frac{t^\gamma}{\gamma}\right)}u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)x^{\alpha-1}y^{\beta-1}t^{\gamma-1}dxdydt$$

$$=(-1)^{l+m+n}\int_0^\infty\int_0^\infty\int_0^\infty e^{-p\left(\frac{x^\alpha}{\alpha}\right)-q\left(\frac{y^\beta}{\beta}\right)-s\left(\frac{t^\gamma}{\gamma}\right)}\left(\frac{x^\alpha}{\alpha}\right)^l\left(\frac{y^\beta}{\beta}\right)^m\left(\frac{t^\gamma}{\gamma}\right)^n u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)x^{\alpha-1}y^{\beta-1}t^{\gamma-1}dxdydt,$$

which implies,

$$\frac{d^{l+m+n}}{dp^l dq^m ds^n}\left(U_{\alpha,\beta,\gamma}(p,q,s)\right)=(-1)^{l+m+n}L_x^\alpha L_y^\beta L_t^\gamma\left(\left(\frac{x^\alpha}{\alpha}\right)^l\left(\frac{y^\beta}{\beta}\right)^m\left(\frac{t^\gamma}{\gamma}\right)^n u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)\right).$$

Now, we multiply $(-1)+m+n$, on the both the sides, we get the required result. ■

Theorem 3.2. For α, β, $\gamma\in$ (0,1]. Let $u(x,y,t)=u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)$ be the real valued piecewise continuous function x, $andt$ of the domain D on (0,∞)×(0,∞)×(0,∞). The NTLT (new triple Laplace transform) of the conformable partial fractional derivatives of order α, and γ, then

1) $L_x^\alpha L_y^\beta L_t^\gamma\left(\frac{\partial^\alpha}{\partial x^\alpha}\left(u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)\right)\right)=pU(p,q,s)-U(0,q,s),$
2) $L_x^\alpha L_y^\beta L_t^\gamma\left(\frac{\partial^\beta}{\partial y^\beta}\left(u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)\right)\right)=qU(p,q,s)-U(p,0,s),$

3) $L_x^\alpha L_y^\beta L_t^\gamma\left(\frac{\partial^\gamma}{\partial t^\gamma}\left(u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)\right)\right)=sU(p,q,s)-U(p,q,0),$

4) $L_x^\alpha L_y^\beta L_t^\gamma\left(\frac{\partial^{2\alpha}}{\partial x^{2\alpha}}\left(u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)\right)\right)=p^2U(p,q,s)-pU(0,q,s)-U_x(0,q,s),$ (3.1)

5) $L_x^\alpha L_y^\beta L_t^\gamma\left(\frac{\partial^{3\alpha}}{\partial x^{3\alpha}}\left(u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)\right)\right)=p^3U(p,q,s)-p^2U(0,q,s)-pU_x(0,q,s)-U_{xx}(0,q,s).$

Proof: Here we go for proof of result (1), and the remaining results can be proved. To obtain newtriple Laplace transform of the fractional partial derivatives, we use integration by parts and theorem 2. By applying the definition of NTLT, we have

$$L_x^\alpha L_y^\beta L_t^\gamma\left(\frac{\partial^\alpha}{\partial x^\alpha}\left(u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)\right)\right)$$

$$=\int_0^\infty\int_0^\infty\int_0^\infty e^{-p\left(\frac{x^\alpha}{\alpha}\right)-q\left(\frac{y^\beta}{\beta}\right)-s\left(\frac{t^\gamma}{\gamma}\right)}\frac{\partial^\alpha u}{\partial x^\alpha}x^{\alpha-1}y^{\beta-1}t^{\gamma-1}dxdydt$$

$$=\int_0^\infty\int_0^\infty e^{-q\left(\frac{y^\beta}{\beta}\right)-s\left(\frac{t^\gamma}{\gamma}\right)}\left(\int_0^\infty e^{-p\left(\frac{x^\alpha}{\alpha}\right)}\frac{\partial^\alpha u}{\partial x^\alpha}x^{\alpha-1}dx\right)y^{\beta-1}t^{\gamma-1}dydt \tag{3.2}$$

Since we have theorem 2.2, $\frac{\partial^\alpha}{\partial x^\alpha}f(x,y,t)=x^{1-\alpha}\partial_x f$. We use this result in to equation (3.2). Therefore, we have

$$\int_0^\infty\int_0^\infty e^{-q\left(\frac{y^\beta}{\beta}\right)-s\left(\frac{t^\gamma}{\gamma}\right)}\left(\int_0^\infty e^{-p\left(\frac{x^\alpha}{\alpha}\right)}\frac{\partial u}{\partial x}dx\right)y^{\beta-1}t^{\gamma-1}dydt, \tag{3.3}$$

The integral inside the bracket is given by,

$$\int_0^\infty e^{-p\left(\frac{x^\alpha}{\alpha}\right)}\frac{\partial u}{\partial x}dx=pU(p,y,t)-U(0,y,t)\,.$$
(3.4)

By substituting equation (3.4) in equation (3.3), and simplifying, we have

$$L_x^\alpha L_y^\beta L_t^\gamma\left(\frac{\partial^\alpha}{\partial x^\alpha}\left(u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)\right)\right)=pU(p,q,s)-U(0,q,s).$$

In general, the above results in theorem 4 can be extended as follows:

$$L_x^\alpha L_y^\beta L_t^\gamma\left(\frac{\partial^m}{\partial x^m}\left(u\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\gamma}{\gamma}\right)\right)\right)=p^mU(p,q,s)-\sum_{k=0}^{m-1}p^{m-1-k}U_t^{(k)}(0,q,s)\,.$$

4. CONCLUSION

In this work, the definition of new triple Laplace transform is defined and investigated results and theorems. This new triple Laplace transform can be applied to find the solution of linear and nonlinear homogeneous and nonhomogeneous partial fractional differential equations.

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