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RESEARCH ARTICLE

AN ASSESSMENT ON THE SPLIT AND NON-SPLIT DOMINATION NUMBER OF TENEMENT GRAPHS

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ABSTRACT

One of the primary fields of mathematics, graph theory is an intriguing and exciting subject. A network is represented mathematically by a graph, which identifies the connections between nodes and edges. One of the fascinating areas of mathematics is Graph Theory. A large area of graph theory is called dominance in graphs. Claude Berge formalised dominance as a theoretical field in graph theory. Domination is one of the fascinating and active areas of Graph Theory research. In this paper, we first provide basic definitions, outlining both core ideas and certain dominant concepts. In specifically, Tenement graphs, a novel type of graph, are defined. Tenement graphs' Split and Non-Split Domination Numbers are addressed.

Keywords: Tenement graphs, Split domination number, Non-split domination number.

INTRODUCTION

A network is represented mathematically by a graph, which identifies the connections between nodes and edges[1]. A graph contains points and edges which consists of objects such as edges, arcs, and lines[6]. Around the 1950s, the research of Domination in Graphs got underway. In 1958, Claude Berge formalised dominance as a theoretical domain in graph theory. The phrases "dominating set" and "dominance number" were later introduced by Oystein Ore in his book on graph theory published in 1962[4]. In a graph G, the term "dominating set" refers to a set D of vertices where each vertex has adjacent in D. Applications for dominance in facility location issues are numerous[10].

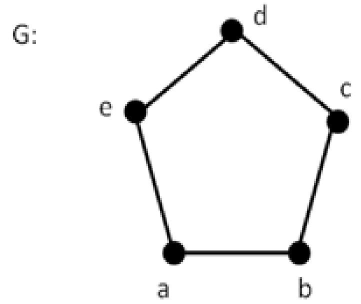
BASIC DEFINITIONS

Definition 2.1:

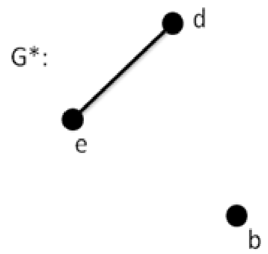
If every vertex in G is taken over by a minimum of one vertex in set D, then set D is a dominating set of G[2]. The dominance number of a graph is the size of the lowest dominant set and is represented by the symbol γ(G)[6].

Definition 2.2:

If the induced subgraph "V-D" is not connected, a dominating set D is said to be split. Split Domination number[7] is the term used to describe the minimal cardinality of a **Split dominating set**. Its symbol is γ (G). G* is used to identify the induced subgraph.
Example:



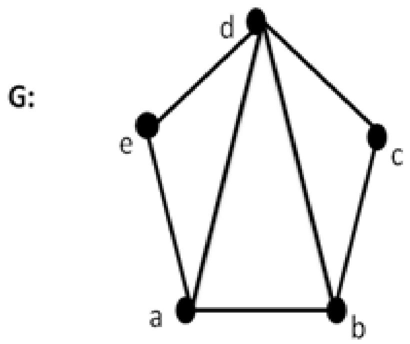
Vertex set, V ={a,b,c,d,e }
Minimum dominating set, D = {a,c}
V-D = {b,d,e}
The induced subgraph is denoted as G*.



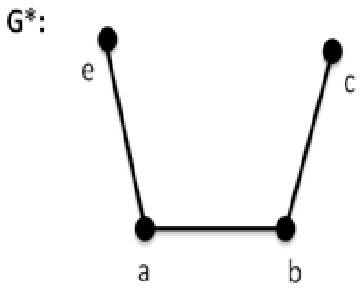
The induced subgraph G* is disconnected. Therefore D is said to be a split dominating set
Hence, $\boldsymbol{\gamma_s}$ **(G)=2**

Definition 2.3:

If the induced subgraph <V-D> is connected, a dominating set D of G is a non-split dominating set[8]. The non-split dominating set's least cardinality is known as the **Non-split dominating number**[9] and is indicated by the symbol ns(G).G* is used to identify the induced subgraph.
Example:



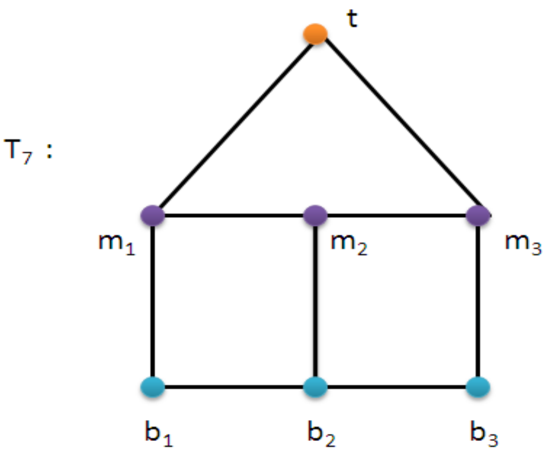
Vertex set, V = { a,b,c,d,e}
Minimum dominating set, D = {d}
Induced subgraph <V-D> is denoted as G*



The induced subgraph G* is disconnected. Therefore D is said to be non split dominating set
$\gamma_{ns}(G) = 1$

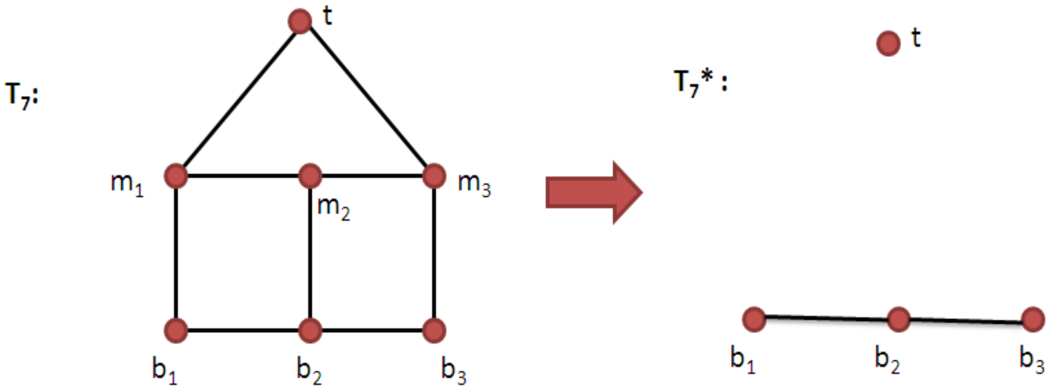
Definition 2.4:

Tenement graph is a graph which consists of exactly three vertex sets namely Top vertex set T, Mid vertex set M and Bottom vertex set B such that three non-adjacent vertices are of degree 2 and all other vertices are of degree 3. It is denoted as T_P graph where p is the number of vertices.



Top vertex set, T = {t}
Mid vertex set, M = $\{m_1,m_2,m_3\}$
Bottom vertex set, B= $\{b_1,b_2,b_3\}$
Here, three non-adjacent vertices have degree two and all the other vertices have degree three.

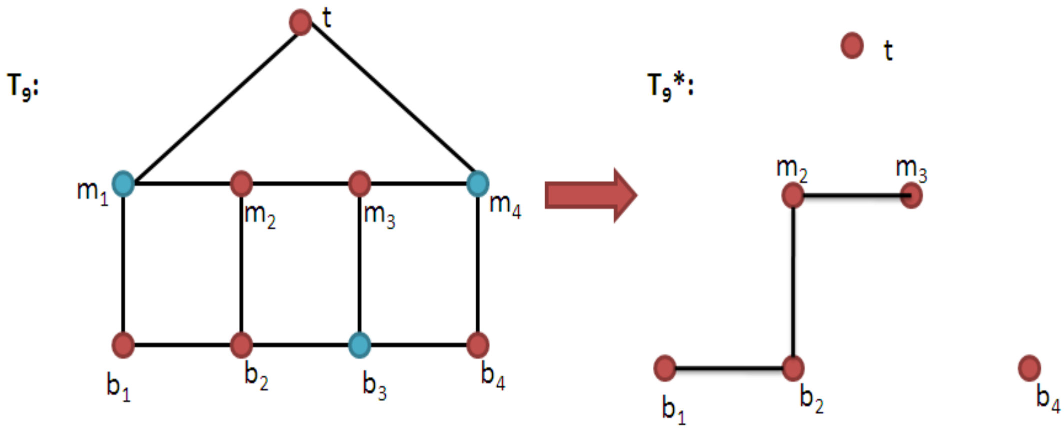
SPLIT DOMINATION NUMBER OF TENEMENT GRAPHS



p = 7, q = 9
Vertex set, V = $\{t,m_1,m_2,m_3,b_1,b_2,b_3\}$
Split dominating set D = $\{ m_1,m_2,m_3\}$

Induced subgraph <V-D> is denoted as T_7*

$\gamma_s(T_7) = 3$

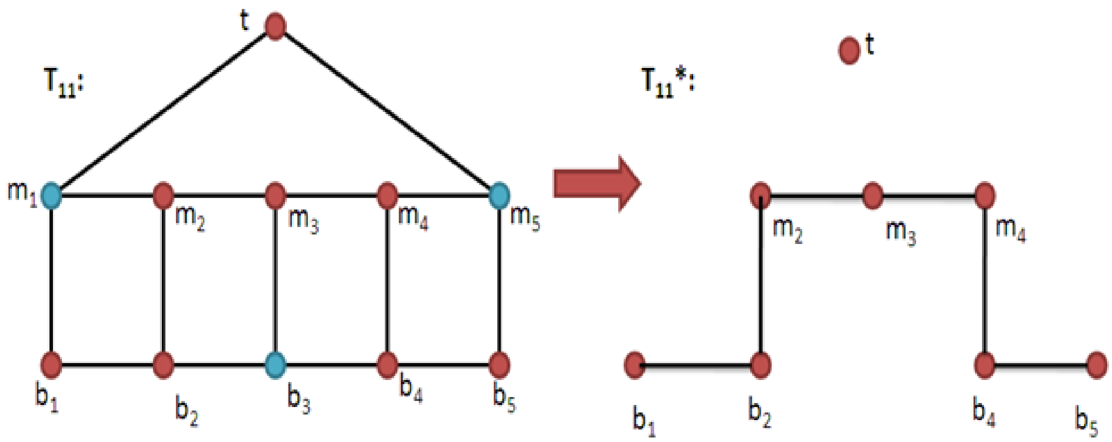


p=9; q=12 ;

Vertex set, V = {t,m_1,m_2,$m_{3,}$,m_4,b_1,b_2,$b_{3,}$,b_4}

Minimum Split dominating set D = { m_1,b_3,m_4**}.**

The Induced subgraph <V-D> is denoted as T_9*. Hence, **$\gamma_s(T_9) = 3$**



p=11;

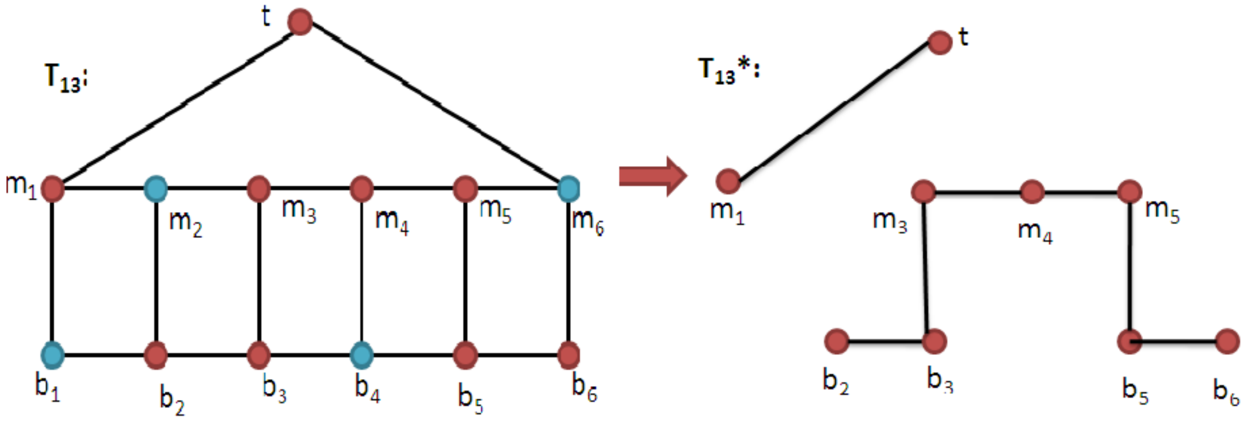
q=15

Vertex set, V = {t,m_1,m_2,m_3,m_4,m_5,b_1,b_2,b_3,b_4,b_5}

Split dominating set D = { m_1,b_3,m_5}.

The Induced subgraph <V-D> is denoted as T_{11}*

$\gamma_s(T_{11}) = 3$



p=13,q=18

Vertex set, V = {t,m_1,m_2,m_3,m_4,m_5,m_6,b_1,b_2,b_3,b_4,b_5,b_6 }

Split dominating set D = { m_2,b_1,b_4,m_6}. The Induced subgraph <V-D> is denoted as T_{13}*

$\gamma_s(T_{13}) = 4$

Continuing like this, we get as follows

Tenement graph (T_p)	Split domination number $\gamma_s(T_p)$
T_7	3
T_9	3
T_{11}	3
T_{13}	4
T_{15}	5
T_{17}	5
T_{19}	5
T_{21}	6
T_{23}	7
T_{25}	7
T_{27}	7
T_{29}	8
T_{31}	9
:	:

RESULTS:

For T_P such that p is odd,

$\gamma_s(T_p)=\lfloor p/3 \rfloor -0$; $9\le p\le 17$ & p=23

$\gamma_s(T_p)=\lfloor p/3 \rfloor -1$; $19\le p\le 21$, $29\le p\le 31$ & p=25

$\gamma_s(T_p)=\lfloor p/3 \rfloor -2$; $33\le p\le 41$, p=27

Theorem 3.1 :

For every Tenement graph, Mid vertex set M is a split dominating set

Proof:

Let T_p be a tenement graph with order p and size q. Let V be the set of all vertices of the graph T_P such that |V|= p

Three vertex sets, designated as top vertex set T, middle vertex set M, and bottom vertex set B, make up each Tenement graph. We know that |T|=1 and Order of Mid vertex set is equal to order of Bottom vertex set

(i.e) |M|=|B|

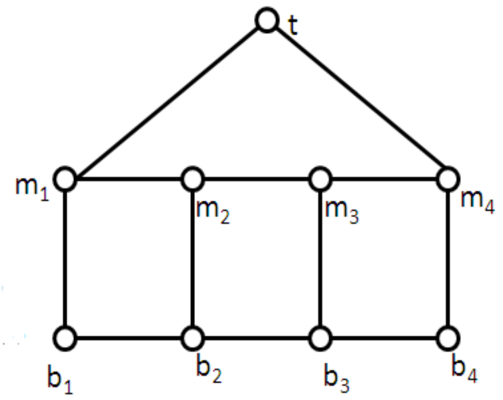
Additionally, each vertex in M is next to the matching vertex in B.
This suggests that every one vertex'm' in M is related to a vertex 'b' in B.
M therefore outweighs B. M also outweighs T.
When M is absent, T and B become disjointed since M dominates both of them.
The induced subgraph <V-M> is a disconnected graph
This implies M is a split dominating set
Hence the proof

Theorem 3.2:

Every Mid vertex set M in T_p is split dominating but |M| is not a split Domination number for all Tenement graphs

Proof:

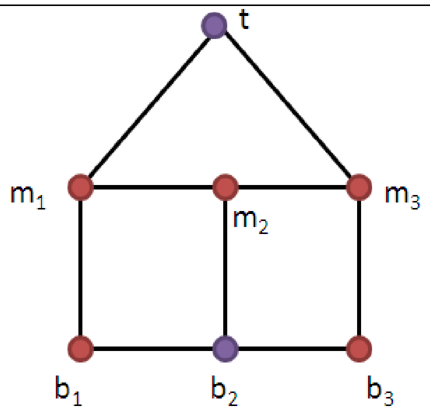
Consider the tenement graph T_9



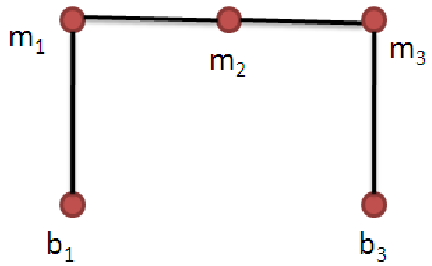
Here $M=\{m_1,m_2,m_3,m_4\}$
By the previous theorem 3.1, M is a split dominating set in T_9
But there exists another set $D=\{m_1,b_3,m_4\}$which dominates all other vertices and also <V-D> is a disconnected graph
The smallest possible cardinality of a split dominating set is referred to as a split domination number by the definition.
Although M and D are both split dominant sets in this case, |M|=4 and |D|=3
|D|<|M|. Therefore, |D| is the split domination number of T_9
Hence, |M| is not a split domination number for all Tenement graphs

NON SPLIT DOMINATION NUMBER OF TENEMENT GRAPHS

T_7:

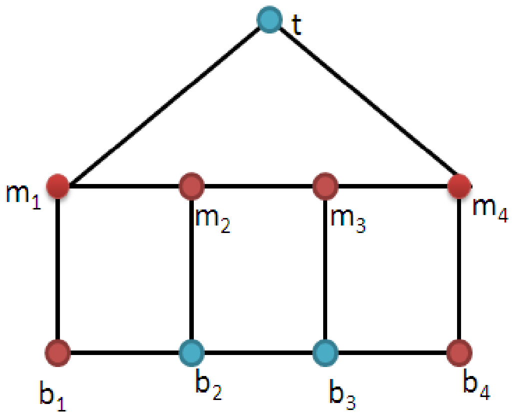


T_7^* :

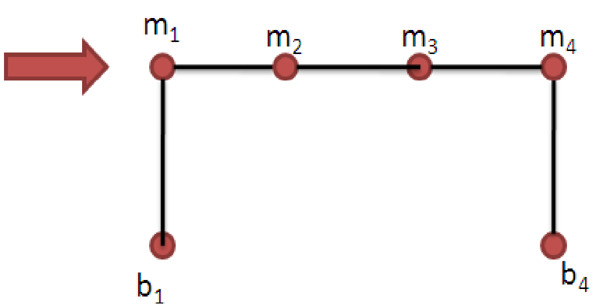


p = 7, q = 9
Vertex set, V = $\{t, m_1, m_2, m_3, b_1, b_2, b_3\}$
Minimum Non Split dominating set D = $\{t, b_2\}$
Induced subgraph <V-D> is denoted as $\mathbf{T_7^*}$ and it is connected
$\boldsymbol{\gamma_{ns}(T_7) = 2}$

T_9:

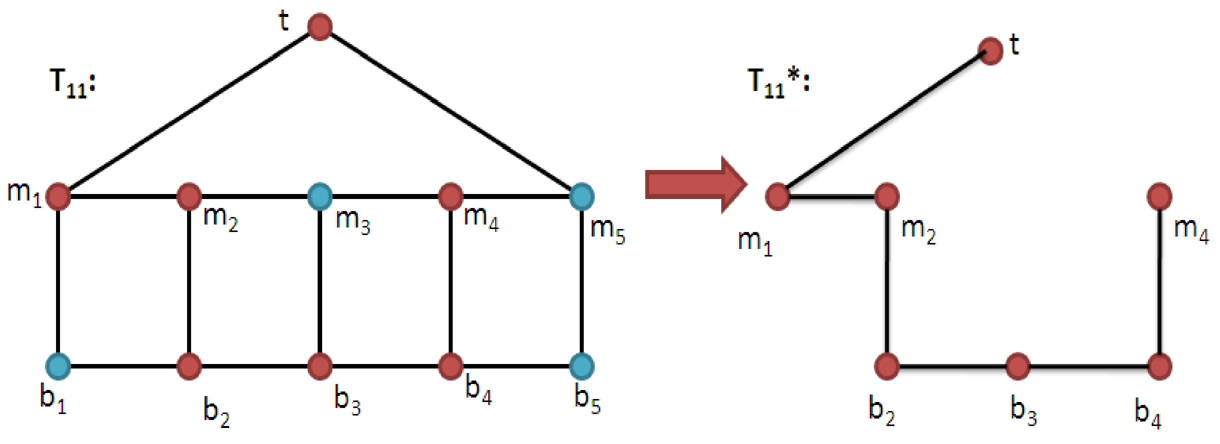


T_9^*:



p=9; q=12
Vertex set, V = $\{t, m_1, m_2, m_3, m_4, b_1, b_2, b_3, b_4\}$
Minimum Non Split dominating set D = $\{ t, b_2, b_3\}$
Induced subgraph <V-D> is denoted as $\mathbf{T_9^*}$ and it is connected

$\boldsymbol{\gamma_{ns}(T_9) = 3}$



p=11; q=15
Vertex set, V = {t,m_1,m_2,m_3,m_4,m_5,b_1,b_2,b_3,b_4,b_5}
Minimum Non Split dominating set D = { b_1,b_5,m_3,m_5}
Induced subgraph <V-D> is denoted as **T_{11}*** and it is connected. **$\gamma_{ns}(T_{11}) = 4$**
Continuing like this, we get as follows

Tenement graph (T_p)	Non Split domination number $\gamma_{ns}(T_p)$
T_7	2
T_9	3
T_{11}	4
T_{13}	4
T_{15}	4
T_{17}	5
T_{19}	6
T_{21}	6
T_{23}	6
T_{25}	7

RESULTS:
For T_P such that p is odd,
$\gamma_{ns}(T_p)=\lceil p/3 \lceil -0;\ 9\le p\le 11$
$\gamma_{ns}(T_p)=\lceil p/3 \lceil -1;\ 13\le p\le 21$
$\gamma_{ns}(T_p)=\lceil p/3 \lceil -2;\ 23\le p\le 27$

CONCLUSION

In this paper, we explored split and non-split dominance numbers as well as the concept of a dominance number. The newly defined Tenement graph became familiar to us. Tenement graphs' Split and Non-split Domination Numbers are calculated and listed. Additionally, various theorems relating to the split domination number of Tenement graphs were covered.

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