AJMS
Asian Journal of Mathematical Sciences

RESEARCH ARTICLE

ON MODEL OF SHIP MOVEMENT CONTROL WITH ACCOUNT POSSIBILITY OF CHANGING OF MASS OF CARGO

*** E.L. Pankratov**

Nizhny Novgorod State Agrotechnical University, 97 Gagarin avenue, Nizhny Novgorod, 603950, Russia

Corresponding Email: elp2004@mail.ru

Received: 10-01-2026; Revised: 19-02-2026; Accepted: 05-03-2026

ABSTRACT

In this paper we presents a model of vessel motion control with the possibility to change the cargo mass. In the framework of the model we consider external forces acting on the vessel, the magnitude of which can be variable. We also present an analyti-cal approach for analysis of the considered model. The approach gives a possibility to take into account changing of parameters of the in time, as well as the nonlinearity of the model.

Keywords: vessel motion control; control model; analytical approach for analysis.

INTRODUCTION

Vessel controllability is a topical issue to decrease quantity of accidents and optimizing cargo transportation. In this regard, a forecast of vessel movement is necessary. One of the classic methods for choosing a vessel's route is to calculate its possible trajectory depending on various parameters [1-5]. Another topical issue is increasing the buoyancy and stability of a vessel. In this paper we intro-duce a vessel movement model that is more general than those introduced in liter-ature. We also presents an analytical approach to analyze the model

METHODOLOGY

Motion of a vessel can be described on the basis of Newton's second law [1-7]. In the framework of this law, changing of the projections of the corresponding velocities is described by the following equations

$$(m+m_x)\frac{d v_x}{d t}=F_{x1}+F_{x2};\ (L_{xx}+L_x)\frac{d\omega_x}{d t}=M_{kr}-\rho g V x_1+\sum_{n=1}^{N}P_n x_{2n}+M_{x1}+M_{x2}$$

$$(m+m_y)\frac{d v_y}{d t}=F_{y1}+F_{y2};\ (L_{yy}+L_y)\frac{d\omega_y}{d t}=M_{diff}-\rho g V y_1+\sum_{n=1}^{N}P_n y_{2n}+M_{y1}+M_{y2}\quad(1)$$

$$(m+m_z)\frac{d\,v_z}{d\,t}=\rho\,g V-F_g-\sum_{n=1}^{N}P_n+F_{z1}+F_{z2};$$

$$(L_{zz}+L_z)\frac{d\,\omega_z}{d\,t}=M_z-\rho\,g V z_1+\sum_{n=1}^{N}P_n z_{2n}+M_{z1}+M_{z2}.$$

where d x /d t = vx; d y /d t = vy; d z /d t = vz; d θ /d t = ωx; d ψ /d t = ωy; d φ /d t = ωz; m is the vessel mass; x and y are the horizontal movements of the vessel; z is the vertical movement of the vessel; ψ is the vessel trim angle; θ is the vessel heel angle; mx, my and mz are the attached mass of water; Lx, Ly and Lz are the attached moments of water; ωx, ωy and ωz are the projections of angular velocity on axes Ox, Oy and Oz; vx, vy and vz are the projections of vessel velocity on similar axes; ρ is the water density; g is the acceleration due to gravity; V is the vessel volumetric displacement; Fg is the vessel gravity (weight displacement); $\sum_{n=1}^{N}P_n$ is the total weight of cargo accepted on board; Fx1, Fy1 and Fz1 are the components of water resistance force to hull movements; Mx1, My1 and Mz1 are the moments of water resistance to hull movements; Fx2, Fy2 and Fz2 are the components of disturbing force; Mx2, My2 and Mz2 are the moments due to wave action; Lxx, Lyy and Lzz are the central moments of inertia of vessel mass relative to coordinate axes; x1= x1c-x1g, y1= y1c-y1g, z1= z1c-z1g, x1c, y1c and z1c are the coordinates of the vessel's center, x1g, y1g and z1g are the coordinates of the vessel's center of gravity; Mkr is the heeling moment of external forces; Mdiff is the trimming moment of external forces. Integration of the left and right parts of the equations of system (1) over time leads to the following result

$$v_x=\int_0^t\frac{F_{x1}+F_{x2}}{m+m_x}d\tau\ ;\ \omega_x=\int_0^t\frac{M_{kr}-\rho\,g V x_1+\sum_{n=1}^{N}P_n x_{2n}+M_{x1}+M_{x2}}{L_{xx}+L_x}d\tau\ ;$$

$$v_y=\int_0^t\frac{F_{y1}+F_{y2}}{m+m_y}d\tau\ ;\ \omega_y=\int_0^t\frac{M_{diff}-\rho\,g V y_1+\sum_{n=1}^{N}P_n y_{2n}+M_{y1}+M_{y2}}{L_{yy}+L_y}d\tau\ ; \qquad (2)$$

$$v_z=\int_0^t\frac{\rho\,g V-F_g-\sum_{n=1}^{N}P_n+F_{z1}+F_{z2}}{m+m_z}d\tau\ ;$$

The initial values of the coordinates and speeds of movement, as well as the angles of rotation and the corresponding speeds were considered as zero, which can be obtained by choosing the origin. Repeated integration of the equations of system (1) over time allows us to obtain the equations of motion of the vessel in the final form

$$x=\int_0^t(t-\tau)\frac{F_{x1}+F_{x2}}{m+m_x}d\tau\ ;\ \theta=\int_0^t(t-\tau)\frac{M_{kr}-\rho\,g V x_1+\sum_{n=1}^{N}P_n x_{2n}+M_{x1}+M_{x2}}{L_{xx}+L_x}d\tau\ ;$$

$$y=\int_0^t (t-\tau)\frac{F_{y1}+F_{y2}}{m+m_y}d\tau\,;\ \psi=\int_0^t (t-\tau)\frac{M_{diff}-\rho\,g V y_1+\sum_{n=1}^{N}P_n y_{2n}+M_{y1}+M_{y2}}{L_{yy}+L_y}d\tau\,; \quad (3)$$

$$z=\int_0^t (t-\tau)\frac{\rho\,g V-F_g-\sum_{n=1}^{N}P_n+F_{z1}+F_{z2}}{m+m_z}d\tau\,;$$

$$\varphi=\int_0^t (t-\tau)\frac{M_z-\rho\,g V z_1+\sum_{n=1}^{N}P_n z_{2n}+M_{z1}+M_{z2}}{L_{zz}+L_z}d\tau\,.$$

DISCUSSION

In this section we analyze the equations of motion (3) for different values of considered parameters. Figs. 1 and 2 show dependences of the coordinate z, angles θ and ψ on various parameters. Figs. 3 and 4 show similar dependences of the velocities vx, as well as the frequencies ωx and ωy.

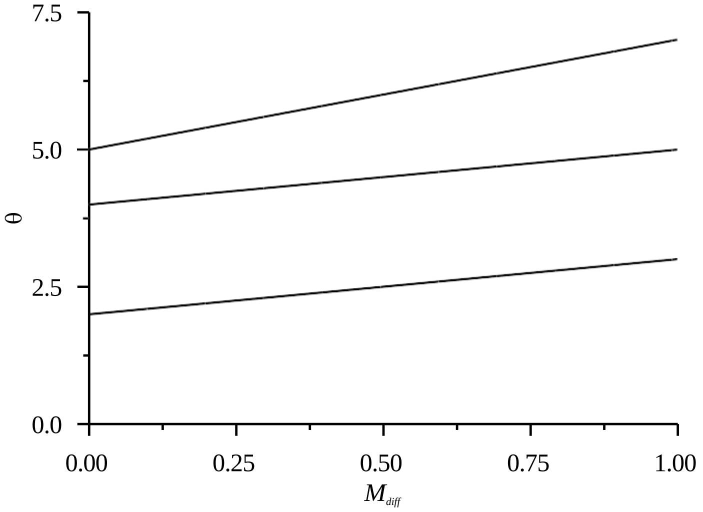

Fig. 1. Typical dependences of the ship's roll angle θ on the heeling moment of external forces Mkr. Similar will be the dependences of this angle on the moments Mx1 and Mx2, the weights of the cargo P. The dependences of the z coordinate on the density of water ρ, the acceleration of gravity g, the volumetric displacement of the vessel V, the forces Fz1 and Fz2

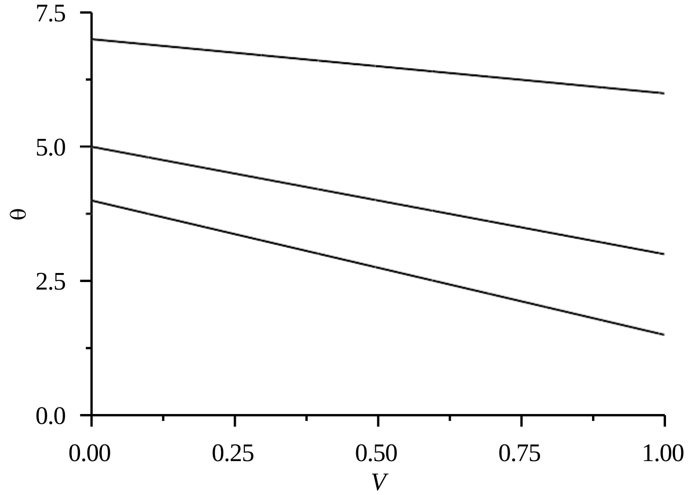

Fig. 2. Typical dependences of the ship's roll angle θ on the volumetric displacement of the ship V. The dependences of this angle on the density of water ρ and the acceleration of gravity g will be similar. The dependences of the z coordinate on the gravity Fg of the weights of the cargo received on the ship P

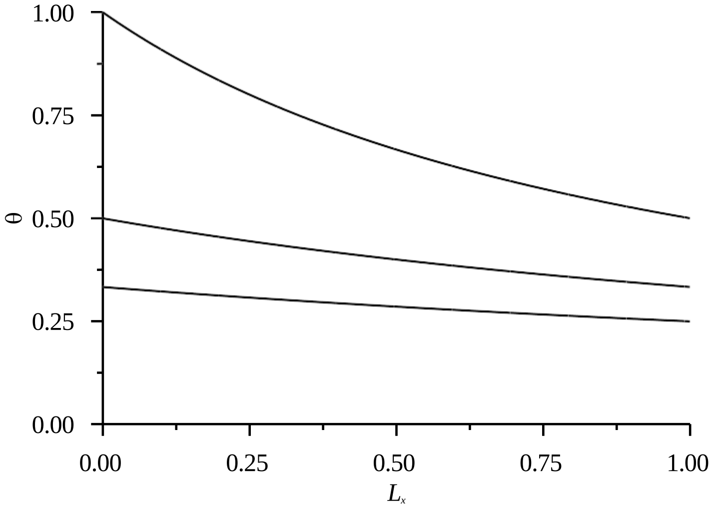

Fig. 3. Typical dependences of the ship's roll angle θ on the central moment of inertia of the ship's mass Lxx. The dependence of this angle on the added moments of water Lx will be similar. The dependences of the trim angle of the ship ψ of the same moments will be similar. Also, the dependences of the z coordinate on the masses m and mz will be similar

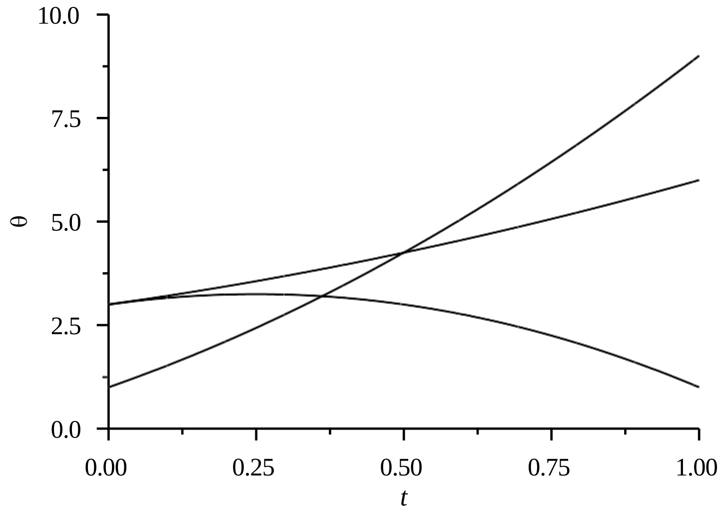

Fig. 4. Typical dependences of the ship's roll angle θ on time t. The time depend-ences of the ship trim angle □ and the z coordinate are similar

CONCLUSION

We presents a model of vessel motion control with the possibility to change the cargo mass. The model gives a possibility to take into account external forces acting on the vessel, the magnitude of which can be variable. We also present an analytical approach for analysis of the considered model. The approach gives a possibility to take into account changing of parameters of the in time, as well as the nonlinearity of the model.

REFERENCE

1. Yu.I. Yudin, I.I. Sotnikov. Mathematical models of plane-parallel movement of a ship. Classification and critical analysis. Bulletin of the Moscow State Technical University. Vol. 9 (5). P. 200-208 (2006).

2. S.V. Smolentsev, D.V. Isakov. Simulation of ship movement based on a simplified kinematic model. Bulletin of the State University of Maritime and River Fleet named after Admiral S.O. Makarov. Vol. 52 (6). P. 1111-1121 (2018).

3. V.A. Kostyukov, A.M. Maevsky, B.V. Gurenko. Mathematical model of a surface mini-ship. Don's Engineering Bulletin. Issue 3. Article No. 3297 (2015).

4. V.A. Zuev, E.M. Gramuzov. New approaches to modeling the ice environment during model tests of ships.Proceedings of the Nizhny Novgorod State Technical University by R.E. Alekseev. Issue 4, pp. 107-114 (2015).

5. O.V. Tarovik, A.G. Topazh, A.B. Krestyantsev, A.A. Kondratenko. Modeling Arctic sea transport systems: the basis of an interdisciplinary approach and practical experience. Arctic: ecology and economics. Issue 1.P. 86-101 (2017).

6. V.E. Kuz'michev. Laws and formulas of physics (Kiev: Naukova Dumka, 1989).

7. E.L. Pankratov. On approach for prognosis the movement of a car in the framework of a two-dimensional model. International journal of applied mathematics and machine learning. Vol. 15 (2). P. 99-106 (2021).