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Analytical Expressions For Levitation of Atmospheric Drops Under The Influence of A Laser Impulse And Interrupt Time Periods

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ABSTRACT

The derivation of the force expressions obtained for a particle is dependent on the shape of the particle and the shape of the radiating beam. In our derivation we consider only spherical particles located in a Gaussian beam profile. This choice will enable us to compare numerical results with previously published experimental results of others. Here the expressions for the 3rd transmitted and reflected rays are presented. Finally, the velocity of a drop of liquid in a flow of air and gas under the influence of a laser beam was found. An analytical expression for laser impulse and interrupt time periods is presented.

Keywords: Optical levitation, Droplets, Laser beam, Axial and radial forces, ray on a dielectric sphere,

INTRODUCTION

Climate change is a subject which is never far from the news[1], although both the quantity and quality of media coverage can only be described as highly variable[2]. Many of the world's leaders, including Ban Ki-moon, have described it as the greatest (moral) challenge facing our generation,. The effective struggle against global warming will only be possible with a responsible collective answer, that goes beyond particular interests and behavior and is developed free of political and economic pressures[3],….On climate change, there is a clear, definitive and ineluctable ethical imperative to act…. The establishment of an international climate change treaty is a grave ethical and moral responsibility[4],.

There are two constituents of our atmosphere which have very significant impacts on the flows of radiation through it – modulating these flows – and hence have major influences on the Earth's climate. These are the radiation-active gases active gases, which trap terrestrial (longwave) radiation, warming the surface via the greenhouse effect, and clouds, which mainly affect solar (shortwave) radiation, being a major contributor to the planetary albedo[5]. However, they also have important effects on longwave radiation, of which they are relatively efficient absorbers and emitters. This is especially, but perhaps surprisingly, the case with high-altitude cirrus clouds which make an important contribution to the greenhouse effect[6],. Of the two, it is the effects of clouds which are more readily obvious, as clouds are such a widely varying component, with the potential to make one day sunny and enjoyable, and the next rainy and miserable: unless, of course, you are a farmer. So that is the second major role of clouds; their pivotal role in the hydrological cycle[7].

We are now ready to embrace our central theme – the interactions of electromagnetic radiation with the constituents of the Earth's atmosphere and its surface. So we should develop the theory of the absorption, emission and scattering of radiation, as well as the key physics of thermal radiation[8]. This will be followed by two chapters devoted to the absorption and emission of radiation by atoms and molecules and the scattering (and absorption) of radiation by molecules, small particles (aerosols and ice crystals) and droplets[9].

The scattering of electromagnetic radiation is a central phenomenon in many areas of both pure and applied science and central to the propagation of radiation in the atmosphere of the Earth and other planets[10]. Scattering by molecules is responsible for the blue color of the sky and the red of sunset. Scattering by clouds reduces incoming solar radiation and makes cloudy days cooler[11]. Scattering by aerosol particles is responsible for the hazy days we often experience, especially in industrialized regions, as well as many other subtle effects which can impact on climate from the regional to the global level[12].

I. Optical levitation of spheres

The derivation of the force expressions obtained for a particle are dependent on the shape of the particle and the shape of the radiating beam. In our derivation we consider only spherical particles located in a Gaussian beam profile. This choice will enable us to compare numerical results with previously published experimental results of others. General expression for any particle shape and beam profile would be extremely complicated, if at all possible, to obtain in closed form. As will be observed in our force derivation, other particle shapes and beam profile geometries can easily be accommodated with little change in the overall derivation procedure.

A typical transparent sphere, shown in Fig. 1.1, has a stream of photons incident upon the lower surface at a position (r) with respect to the central axis. If the stream of photons is treated as a ray, part will be reflected at this surface and part will be refracted. The refracted photons will be deviated from the initial direction and be incident onto the upper surface of the sphere.

At this surface, part of the photons will be reflected and part will be refracted. We will not consider any additional reflections and refractions from the reflected contribution at the upper surface because they would be of minor intensity compared with the incident flux at the lower surface before refraction. This condition is especially true when the relative index of refraction between sphere and ambient medium is close to unity. The part refracted out of the sphere at the upper surface is assumed to propagate freely and not to encounter this or other spheres again.

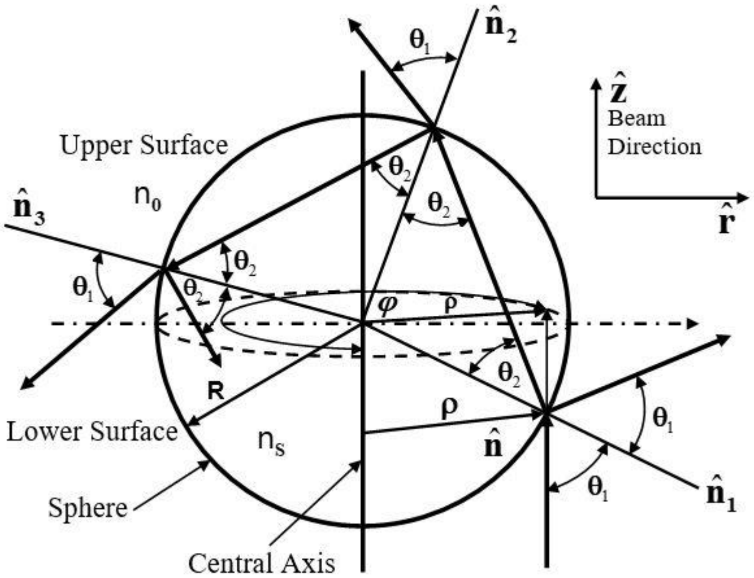


Figure 1.1 Stream of photons represented by a ray of light incident upon the lower side of a sphere. The sphere has index of refraction , and the surround has index of refraction . The ray is shown refracted and reflected at the lower and the upper surfaces.

In the derivation to follow, we decompose the effects caused by the stream of photons into one that acts axially and one that acts radially with respect to the sphere's central axis

1. Axial Force

The sphere is assumed to be located close to the central axis of the photon stream in such a way that the photons have nearly axial velocities before the interaction with the sphere. This is an acceptable approximation because levitation occurs when the sphere is centrally located with respect to a Gaussian-shaped, or other central maximum, photon flux beam. The fundamental equation governing the acceleration and levitation of particles in a stream of photons in Newton's second law:

$$\mathbf{F} = d\mathbf{p} \,/\, dt \,, \tag{1}$$

where $\mathbf{F}$ is the force on the particle, $d\mathbf{p}$ is the total momentum change on the particle produced by all photons, and dt is the interval of time over which $d\mathbf{p}$ is measured. Relating $d\mathbf{p}/\,dt$ to the physical properties of the light beam and particle shape is accomplished as follows:

A. The momentum of a single photon is given by the well-known de Broglie relation [1]

$$p = (h / \lambda_0) n_0 = (\hbar \omega_0 / c) n_0 , \qquad (2)$$

where h ($\hbar$) is Planck's constant, λ_0 is the wavelength of light in free space, ω_0 is the frequency of light in free space, c is the light speed at the vacuum, and n_0 is the is the refraction index of medium through which light propagates. As the photon is reflected or refracted at the surface of the sphere the axial component of the photon's momentum will, in general, change because of the change in direction of the photon. At the lower surface of the sphere in Fig. 1.1 the change in the axial component of the photon's momentum on reflection is given by

$$- (h / \lambda_0) n_0 (1 + \cos 2\theta_1) = - (\hbar \omega_0 / c) n_0 (1 + \cos 2\theta_1) , \qquad (3)$$

where   is the angle formed between the photon's incident direction and the normal to the surface of the sphere. We consider the collision between the photon and the sphere to be elastic; therefore the law of conservation of momentum is employed. The negative of expression (3) is the momentum given to the sphere from a photon reflected from the lower surface of the sphere:

$$\Delta p_{1rz} = (h / \lambda_0) n_0 (1 + \cos 2\theta_1) . \qquad (4)$$

The momentum transfer given to the sphere that results from a photon's being refracted at the lower surface, Δp_{1tz}, and reflected or refracted, Δp_{2rz} or Δp_{2tz} , at the top surface can be expressed as

$$\Delta p_{1rz} = (h / \lambda_0) n_0 (1 + \cos 2\theta_1), \qquad (5a)$$
$$\Delta p_{1tz} = (h / \lambda_0) [n_0 - n_S \cos(\theta_1 - \theta_2)], \qquad (5b)$$
$$\Delta p_{2rz} = (h / \lambda_0) n_S [\cos(\theta_1 - \theta_2) + \cos(3\theta_2 - \theta_1)], \qquad (5c)$$
$$\Delta p_{2tz} = (h / \lambda_0) \{ n_S \cos(\theta_1 - \theta_2) - n_0 \cos[2(\theta_1 - \theta_2)] \} \qquad (5d)$$
$$\Delta p_{3rz} = (h / \lambda_0) n_S [\cos(\theta_1 - 5\theta_2) - \cos(\theta_1 - 3\theta_2)], \qquad (5e)$$
$$\Delta p_{3tz} = (h / \lambda_0) \left[n_S \cos(\theta_1 - 3\theta_2) + n_0 \cos(2\theta_1 - 4\theta_2) \right]. \qquad (5f)$$

where θ_2 is obtained from θ_1 by Snell's law, $\sin\theta_2 = (n_0 / n_S) \sin\theta_1$, at the surface of the sphere, n_S is the refraction index of the particle sphere medium through which light propagates.

The first subscript indicates the lower surface, 1, or the upper surface, 2, of the sphere; the second subscript indicates the type of photon interaction, for reflection or for transmission; and the third subscript indicates the reference coordinate, for axial or for radial.

B. The relative number of photons that are reflected and refracted at each surface can be obtained from the power reflectance and transmittance coefficients. The power reflectance coefficient used is

$$| r_1 |^2 = \left| \tfrac{1}{2} \left(r_{TE} + r_{TM} \right) \right|^2 , \qquad (6)$$

where r_{TE} and r_{TM} are the complex amplitude reflectance for transverse-electric (TE) and transverse-magnetic (TM) polarization1/, respectively (the examples om some commonly used modes are shown in Fig. 1.2). The net result of using Eq. (6) is that the photon stream can be considered to be composed of an equal number of photons in both polarizations. The power reflectance and transmittance for the lower and upper surfaces obtained by use of the Fresnel coefficients2/ (Section 6.2 in [2], [3]) are

$$|r_1|^2=|r_2|^2=|r_3|^2=\frac{(n_0 n_S)^2(\cos^2\theta_1-\cos^2\theta_2)^2}{[n_0 n_S(\cos^2\theta_1-\cos^2\theta_2)+(n_0^2+n_S^2)\cos\theta_1\cos\theta_2]^2},$$
$$|t_1|^2=|t_2|^2=|t_3|^2=1-|r_1|^2 . \tag{7}$$

C. We must also relate the number of photons incident upon the sphere to the parameters that characterize the light beam. The intensity profile of the beam is chosen to be that of the lowest-order Gaussian mode [2]:

$$W(z)=\left(2P_0/\pi W(z)^2\right)\exp\left[2r^2/W(z)^2\right], \tag{8}$$

where r is the radial distance from the beam's axis, z is the distance measured along the beam's direction of propagation with $z=0$ located at the minimum waist, and $P_0=\frac{1}{2}\pi r_0^2 I_0$ is the total power in the beam. Here, I_0 is the intensity at the center of the beam at its waist, r_0 – the radius of laser beam at its waist. The value $W(z)$ is the beam width, given by

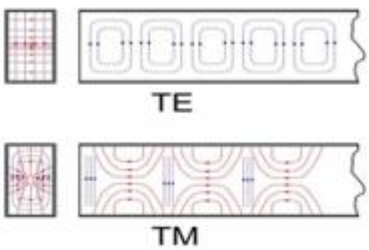
$$W(z)=W_0\left[1+(z/z_0)^2\right]^{1/2}, \tag{9}$$

and W_0, the waist, is

$$W_0=(\lambda_0 z_0/\pi)^{1/2}=(2cz_0/\omega_0)^{1/2}, \tag{10}$$

where z_0 is the position along the beam axis, λ_0, and ω_0 are the wave length and frequency of the beam light in free space, respectively. The intensity of the Gaussian laser beam $I(r,z)$ may be found from [4, 5],

$$I(r,z)=\frac{I_0}{(1-z/f)^2+(z/z_d)^2}\exp\left\{-\frac{r^2}{r_0^2[(1-z/f)^2+(z/z_d)^2]}\right\} \tag{11}$$



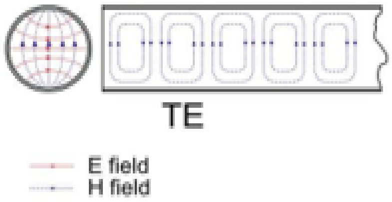


Figure 1.2: Examples of some commonly used TE and TM modes

where $z_d = kr_0$ – the diffraction length of the beam, $k = (\omega / c)n_S = (2\pi / \lambda)n_S$ – the wave number, ω – the frequency of radiation, λ – the wave length, f – the focal length of the lens used to focusing the laser beam. If the lens isn't used, the ratio z / f becomes zero and

$$I(r,z) = I_0 \left(r_0 / r_z\right)^2 \exp\left(- r^2 / r_z^2\right), \tag{11a}$$

with $r_z = r_0\sqrt{1 + (z / z_d)}$ being the beam radius at point z. The case when the particle's sphere is centered in the Gaussian beam is shown in Fig. 1.3.

D. The average photon flux, ϕ_{av}, is defined as the number of photons, N, per unit time and is given by

$$\phi_{\text{av}} = N / dt . \tag{12}$$

The power in the light beam, in terms of ϕ_{av} and the energy $E_{ph} = hc / \lambda = \hbar\omega$ of a single photon, is

$$P = \phi_{\text{av}} E_{ph} . \tag{13}$$

Using the results of steps (A) – (D) above, we can express the element of force produced on the sphere from the pencil-like stream of photons incident at point $\rho = r$ as

$$dF_{Az} = N \frac{dp_z}{dt}, \tag{14}$$

Where

$$dp_z = |r_1|^2 \Delta p_{1rz} + |t_1|^2 \Delta p_{1tz} + |r_1|^2 |t_1|^2 \Delta p_{2rz} + |t_1|^2 |t_1|^2 \Delta p_{2tz} + + |r_1|^2 |t_1|^2 \Delta p_{3rz} + |t_1|^2 |t_1|^2 \Delta p_{3tz}, \tag{15}$$

with Eq. (12), and recognizing that $\phi_{\text{av}}(r) = dP_{\text{tot}}(r) / E_{\text{ph}}$ and $P_{\text{tot}}(r) = I(r,z)dA$, we find the element of axial force, dF_{Az}:

where dA is the element of the particle's sphere surface.

The total axial force on the sphere that is due to the light beam is obtained by summing (integrating) all the force elements over the lower surface of the sphere:

$$F_z = F_{1rz} + F_{1tz} + F_{2rz} + F_{2tz}\,. \qquad (17)$$

Expressing the element of surface area in spherical coordinates, for the case when the laser beam radius is higher than the particle's sphere radius we obtain for the four axial force contributions

$$F_{1rz} = \int_0^{\pi/2} I(r,z)\frac{P}{c} n_0 (1+\cos 2\theta_1)\,|r_1|^2 R^2 \sin 2\theta_1 d\theta_1 \qquad (18a)$$

$$F_{1tz} = \int_0^{\pi/2} I(r,z)\frac{P}{c} [n_0 - n_S \cos(\theta_1-\theta_2)]\,|t_1|^2 R^2 \sin 2\theta_1 d\theta_1 \qquad (18b)$$

$$F_{2rz} = \int_0^{\pi/2} I(r,z)\frac{P}{c} n_S [\cos(\theta_1-\theta_2)+\cos(3\theta_2-\theta_1)]\,|t_1|^2|r_1|^2 R^2 \sin 2\theta_1 d\theta_1 \qquad (18c)$$

$$F_{2tz} = \int_0^{\pi/2} I(r,z)\frac{P}{c} \left[n_S \cos(\theta_1-\theta_2) - n_0 \cos 2(\theta_1-\theta_2)\right] |t_1|^2|t_1|^2 R^2 \sin 2\theta_1 d\theta_1 \qquad (18d)$$

$$F_{3rz} = \int_0^{\pi/2} I(r,z)\frac{P}{c} n_S [\cos(\theta_1-5\theta_2) - \cos(\theta_1-3\theta_2)]\,|t_1|^2|r_1|^2 R^2 \sin 2\theta_1 d\theta_1 \qquad (18e)$$

$$F_{3tz} = -\int_0^{\pi/2} I(r,z)\frac{P}{c} [n_S \cos(\theta_1-3\theta_2) - n_0 \cos(2\theta_1-4\theta_2)]\,|t_1|^2|t_1|^2 R^2 \sin 2\theta_1 d\theta_1 \qquad (18f)$$

with R the radius of the sphere, $\rho = R\sin\theta_1$, $z = R\left[\cos(\theta_1+2\theta_2)+\cos\theta_1\right]$.

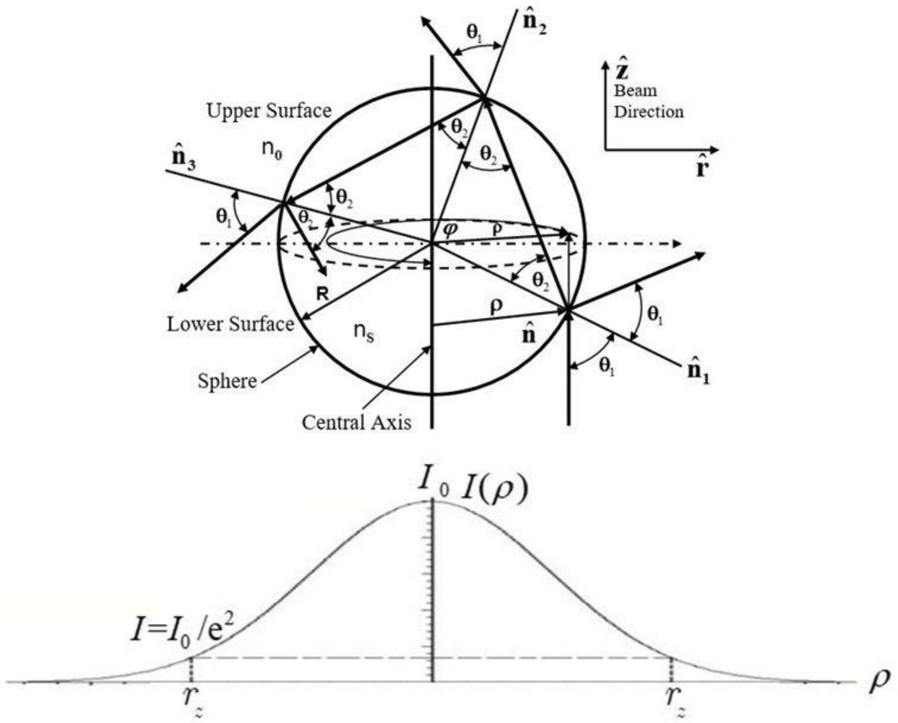


Figure 1.3: The particle sphere is centered in the Gaussian beam

When gravity acts in a direction opposite the flow direction of the photons, the net z-directed force is

$$F_{z\ \mathrm{net}} = F_z - \tfrac{4}{3}\pi R^3(\rho_S - \rho_0)g\,, \qquad (19)$$

where ρ_S and ρ_0 are the densities of the sphere and the ambient material, respectively, and g is the gravitational constant. Equation (19) permits the calculation of the net force on the sphere centered in a

Gaussian beam, based on the optical and physical properties of the light beam, the sphere, and the surrounding medium.

2. Radial Force

The sphere's radial component of the momentum change that is due to a single photon's being reflected or transmitted at the lower or upper surface of the sphere is given by

$$\Delta p_{1rr} = (h/\lambda_0) n_0 \sin 2\theta_1 \cos\varphi \qquad (20a)$$

$$\Delta p_{1tr} = (h/\lambda_0) n_S \sin(\theta_1 - \theta_2)\cos\varphi \qquad (20b)$$

$$\Delta p_{2rr} = (h/\lambda_0) n_S [\sin(3\theta_2 - \theta_1) - \sin(\theta_1 - \theta_2)]\cos\varphi \qquad (20c)$$

$$\Delta p_{2tr} = (h/\lambda_0)\{n_0 \sin[2(\theta_1 - \theta_2)] - n_S \sin(\theta_1 - \theta_2)\}\cos\varphi \qquad (20d)$$

$$\Delta p_{3rr} = (h/\lambda_0) n_S [\cos(\theta_1 - 3\theta_2) - \cos(\theta_1 - 5\theta_2)]\cos\varphi \qquad (20e)$$

$$\Delta p_{3tr} = (h/\lambda_0)[n_S \cos(\theta_1 - 3\theta_2) - n_0 \cos(2\theta_1 - 4\theta_2)] \qquad (20f)$$

The angle φ is the angle between the direction of the radial component of the momentum and the radial direction shown in Figs. 1.1 and 1.3. The lower subscript r indicates radial direction with respect to the central axis of the sphere. Following the steps leading to Eqs. (15) and (16), we can similarly write for the element of radial force, dF_{Ar}:

$$dp_r = |r_1|^2 \Delta p_{1rr} + |t_1|^2 \Delta p_{1tr} + |r_1|^2 |t_1|^2 \Delta p_{2rr} + |t_1|^2 |t_1|^2 \Delta p_{2tr}, \qquad (21)$$

$$dF_{Ar} = (2P / E_{\mathrm{ph}})\left[2 / \pi W(z)^2\right]\exp\left[2r^2 / W(z)^2\right] dp_r\, dA, \qquad (22)$$

where dA again is the element of the particle's sphere surface. The net radial force, which does not include a contribution from gravity, can be written as

$$F_{r\ \mathrm{net}} = F_{1rr} + F_{1tr} + F_{2rr} + F_{2tr}. \qquad (23)$$

$$F_{1rr} = -\int_0^{\pi/2}\int_0^{2\pi} I(r,z)\frac{n_0}{2c}\sin 2\theta_1 |r_1|^2 R^2 \cos\varphi\ \sin 2\theta_1 d\varphi\, d\theta_1, \qquad (24a)$$

$$F_{1tr} = \int_0^{\pi/2}\int_0^{2\pi} I(r,z)\frac{n_S}{2c}\sin(\theta_1 - \theta_2)|t_1|^2 R^2 \cos\varphi\ \sin 2\theta_1 d\varphi\, d\theta_1, \qquad (24b)$$

$$F_{2rr} = \int_0^{\pi/2}\int_0^{2\pi} I(\rho,z)\frac{n_S}{2c}\left[\sin(3\theta_2 - \theta_1) - \sin(\theta_1 - \theta_2)\right]|\tau_1|^2|\rho_1|^2 R^2\cos\varphi\,\sin 2\theta_1 d\varphi\, d\theta_1, \quad (24c)$$

$$F_{2rz} = \int_0^{\pi/2}\int_0^{2\pi} I(\rho,z)\left\{\frac{n_0}{2c}\sin[2(\theta_1 - \theta_2)] - \frac{n_S}{2c}\sin(\theta_1 - \theta_2)\right\}|\tau_1|^2|\tau_1|^2 R^2\cos\varphi\,\sin 2\theta_1 d\varphi\, d\theta_1. \quad (24d)$$

$$F_{3rr} = \int_0^{\pi/2}\int_0^{2\pi} I(\rho,z)\frac{n_S}{2c}[\cos(\theta_1 - 3\theta_2) - \cos(\theta_1 - 5\theta_2)]|\tau_1|^2|\rho_1|^2 R^2\cos\varphi\,\sin 2\theta_1 d\varphi\, d\theta_1, \quad (24c)$$

$$F_{3rz} = \int_0^{\pi/2}\int_0^{2\pi} I(\rho,z)\left[\frac{n_S}{2c}\cos(\theta_1 - 3\theta_2) - \frac{n_0}{2c}\cos(2\theta_1 - 4\theta_2)\right]|\tau_1|^2|\tau_1|^2 R^2\cos\varphi\,\sin 2\theta_1 d\varphi\, d\theta_1. \quad (24d)$$

When the sphere is not centered in the Gaussian beam, ρ is a function of φ as well as of θ_1 and is given by (Fig. 1.4)

$$\rho(\theta_1,\varphi) = (a^2 + R^2\sin^2\theta_1 + 2aR\sin\theta_1\cos\varphi)^{1/2}, \quad (25)$$

where is the relative offset between the sphere’s central axis and the Gaussian profile maximum.

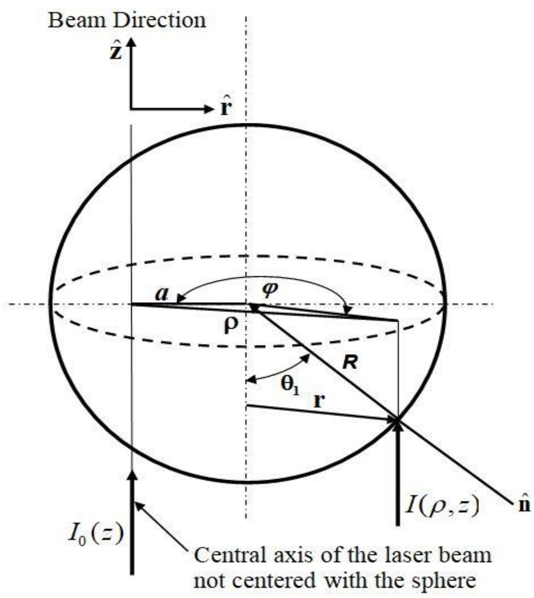


Figure 1.4: Sphere is not centered in the Gaussian beam.

In this case the Eqs, (18) has a view

$$F_{1rz} = \int_0^{\pi/2}\int_0^{2\pi} I(\rho,z)\frac{n_0}{2c}(1 + \cos 2\theta_1)|\rho_1|^2 R^2\sin 2\theta_1 d\varphi\, d\theta_1, \quad (26a)$$

$$F_{1\tau z} = \int_0^{\pi/2}\int_0^{2\pi} I(\rho,z)\left[\frac{n_0}{2c} - \frac{n_S}{2c}\cos(\theta_1 - \theta_2)\right]|\tau_1|^2 R^2\sin 2\theta_1 d\varphi\, d\theta_1, \quad (26b)$$

$$F_{2rz} = \int_0^{\pi/2}\int_0^{2\pi} I(\rho,z)\frac{n_S}{2c}[\cos(\theta_1 - \theta_2) + \cos(3\theta_2 - \theta_1)]|\tau_1|^2|\rho_1|^2 R^2\sin 2\theta_1 d\varphi\, d\theta_1, \quad (26c)$$

$$F_{2tz} = \int_0^{\pi/2}\int_0^{2\pi} I(r,z)\left[\frac{n_S}{2c}\cos(\theta_1-\theta_2)-\frac{n_0}{2c}\cos 2(\theta_1-\theta_2)\right]|t_1|^2|t_1|^2 R^2 \sin 2\theta_1 d\varphi\, d\theta_1, \qquad (26d)$$

$$F_{3rr} = \int_0^{\pi/2}\int_0^{2\pi} I(r,z)\frac{n_S}{2c}[\cos(\theta_1-3\theta_2)-\cos(\theta_1-5\theta_2)]|t_1|^2|r_1|^2 R^2\cos\varphi \sin 2\theta_1 d\varphi\, d\theta_1, \qquad (26e)$$

$$F_{3tz} = \int_0^{\pi/2}\int_0^{2\pi} I(r,z)\left[\frac{n_S}{2c}\cos(\theta_1-3\theta_2)-\frac{n_0}{2c}\cos(2\theta_1-4\theta_2)\right]|t_1|^2|t_1|^2 R^2\cos\varphi \sin 2\theta_1 d\varphi\, d\theta_1. \qquad (26f)$$

Equation (23) governs the centering of the sphere in the Gaussian profile. The use of Eqs. (19) and (23) permits suitable modeling of the levitation phenomena.

2.1: The particle sphere is centered in the Gaussian beam

1. For this case (shown in Fig. 1.2) the equations (18) and (24) have a view

$$F_{1rz} = \frac{\pi}{c} n_0 R^2 \int_0^{\pi/2} I(r,z)(1+\cos 2\theta_1)|r_1|^2 \sin 2\theta_1 d\theta_1, \qquad (27a)$$

$$F_{1tz} = \frac{\pi}{c} R^2 \int_0^{\pi/2} I(r,z)[n_0 - n_S\cos(\theta_1-\theta_2)]|t_1|^2 \sin 2\theta_1 d\theta_1, \qquad (27b)$$

$$F_{2rz} = \frac{\pi}{c} n_S R^2 \int_0^{\pi/2} I(r,z)[\cos(\theta_1-\theta_2)+\cos(3\theta_2-\theta_1)]|t_1|^2|r_1|^2 \sin 2\theta_1 d\theta_1, \qquad (27c)$$

$$F_{2tz} = \frac{\pi}{c} R^2 \int_0^{\pi/2} I(r,z)[n_S\cos(\theta_1-\theta_2) - n_0\cos 2(\theta_1-\theta_2)]|t_1|^2|t_1|^2 \sin 2\theta_1 d\theta_1, \qquad (27d)$$

$$F_{3rz} = \frac{\pi}{c} n_S R^2 \int_0^{\pi/2} I(r,z)[\cos(\theta_1-5\theta_2)-\cos(\theta_1-3\theta_2)]|t_1|^2|r_1|^2 \sin 2\theta_1 d\theta_1, \qquad (27e)$$

$$F_{3tz} = -\frac{\pi}{c} R^2 \int_0^{\pi/2} I(r,z)[n_S\cos(\theta_1-3\theta_2) - n_0\cos(2\theta_1-4\theta_2)]|t_1|^2|t_1|^2 \sin 2\theta_1 d\theta_1, \qquad (27f)$$

$$F_{1rr} = -\frac{2n_0R^2}{c}\int_0^{\pi/2} I(r,z)\sin 2\theta_1 |r_1|^2 \sin 2\theta_1 d\theta_1, \qquad (28a)$$

$$F_{1tr} = \frac{2n_SR^2}{c}\int_0^{\pi/2} I(r,z)\sin(\theta_1-\theta_2)|t_1|^2 \sin 2\theta_1 d\theta_1, \qquad (28b)$$

$$F_{2rr} = \frac{2n_SR^2}{c}\int_0^{\pi/2} I(r,z)[\sin(3\theta_2-\theta_1)-\sin(\theta_1-\theta_2)]|t_1|^2|r_1|^2 \sin 2\theta_1 d\theta_1, \qquad (28c)$$

$$F_{2tz} = \frac{2n_0}{c}R^2\int_0^{\pi/2} I(r,z)\left\{\sin[2(\theta_1-\theta_2)]-\frac{n_S}{n_0}\sin(\theta_1-\theta_2)\right\}|t_1|^2|t_1|^2 \sin 2\theta_1 d\theta_1. \qquad (28d)$$

$$F_{3rr} = \frac{2n_S}{c}R^2\int_0^{\pi/2} I(r,z)[\cos(\theta_1-3\theta_2)-\cos(\theta_1-5\theta_2)]|t_1|^2|r_1|^2 \sin 2\theta_1 d\theta_1, \qquad (28e)$$

$$F_{3tz} = \frac{2n_0}{c}R^2\int_0^{\pi/2} I(r,z)\left[\frac{n_S}{n_0}\cos(\theta_1-3\theta_2)-\cos(2\theta_1-4\theta_2)\right]|t_1|^2|t_1|^2 \sin 2\theta_1 d\theta_1. \qquad (28f)$$

The trigonometric functions $\sin\theta_2$ and $\cos\theta_2$ of the angle θ_2 in expressions (7) and sub-integral terms (27) – (28) may be found from

$$\sin\theta_2 = (n_0/n_S)\sin\theta_1, \quad \cos\theta_2 = \sqrt{1-(n_0/n_S)^2\sin^2\theta_1}, \qquad (29)$$

and the values ρ and z may be found as

$$\rho = R\sin\theta_1,\quad z = R\left[\cos(\theta_1+2\theta_2)+\cos\theta_1\right] = 2R\left\{\cos\theta_1-(n_0/n_S)^2\sin^2\theta_1\left[\cos\theta_1-\sqrt{1-(n_0/n_S)^2\sin^2\theta_1}\right]\right\}. \qquad (30)$$

2. For the simplest case when the Gaussian half-width W is sufficiently higher than the particle's diameter $2R$, i.e. $W_0 >> 2R$, we may limit the intensity function its spatial maximal value $I_{\max}$. Hence, the first term has a view,

$$F_{1rz} = \frac{\pi}{c}n_0R^2I_{\max}\int_0^{\pi/2}(1+\cos2\theta_1)|r_1|^2\sin2\theta_1 d\theta_1 =$$

$$= \frac{\pi}{c}n_0R^2I_{\max}\int_0^{\pi/2}(1+\cos2\theta_1)\frac{(\cos^2\theta_1-\cos^2\theta_2)^2\sin2\theta_1}{\left[(\cos^2\theta_1-\cos^2\theta_2)+\frac{n_0^2+n_S^2}{n_0n_S}\cos\theta_1\sqrt{1-(n_0/n_S)^2\sin^2\theta_1}\right]^2}d\theta_1 =$$

$$= \frac{4\pi}{c}n_0R^2I_{\max}\int_0^{\pi/2}\frac{(1-\sin^2\theta_1)\sin\theta_1}{\left[1+\frac{n_0^2+n_S^2}{n_0n_S\left[(n_0/n_S)^2-1\right]}\frac{1}{\sin^2\theta_1}\sqrt{\left[1-(n_0/n_S)^2\sin^2\theta_1\right](1-\sin^2\theta_1)}\right]^2}d(\sin\theta_1) =$$

$$= \frac{4\pi}{c}n_0R^2I_{\max}\int_0^{1}\frac{x(1-x^2)}{\left\{1+\frac{n_0^2+n_S^2}{n_0n_S\left[(n_0/n_S)^2-1\right]}x^{-2}\sqrt{\left[1-(n_0/n_S)^2x^2\right](1-x^2)}\right\}^2}dx =$$

$$= \frac{4\pi}{c}n_0R^2I_{\max}\int_0^{1}\frac{x(1-x^2)}{\left[1+Ax^{-2}\sqrt{(1-Bx^2)(1-x^2)}\right]^2}dx =$$

$$= \frac{2\pi}{c}n_0R^2I_{\max}\int_0^{1}\frac{(1-y)}{\left[1+(A/y)\sqrt{(1-By)(1-y)}\right]^2}dy = \frac{2\pi}{c}n_0R^2I_{\max}\int_1^{\infty}\frac{(u-1)u^{-2}}{\left[1+A\sqrt{(u-B)(u-1)}\right]^2}du\ .$$

where

$$A = \frac{n_0^2+n_S^2}{n_0n_S\left[(n_0/n_S)^2-1\right]},\quad B = (n_0/n_S)^2$$

Even for this simple case the last integral cannot be resolved under use elementary functions.

2.2: Derivation of the expressions (5) and (20) for the 3rd transmitted and reflected rays.

Explanation of the expressions (5) derivation is shown in detail. The used expressions for the angles (shown in Fig. 1.5) are:

$$\psi_+ = -(\tfrac{1}{2}\pi - \theta_1) + 2\pi - 4\theta_2 = \tfrac{3}{2}\pi + \theta_1 - 4\theta_2,$$
$$\psi_- = \tfrac{3}{2}\pi + \theta_1 - 4\theta_2 - \pi = \tfrac{1}{2}\pi + \theta_1 - 4\theta_2,$$
$$\alpha = \psi_+ + \theta_2 = \tfrac{3}{2}\pi + \theta_1 - 4\theta_2 + \theta_2 = \tfrac{3}{2}\pi + \theta_1 - 3\theta_2,$$
$$\alpha_\pi = \alpha - \pi = \tfrac{1}{2}\pi + \theta_1 - 3\theta_2,$$
$$\beta = \psi_+ + \theta_1 = \tfrac{3}{2}\pi + \theta_1 - 4\theta_2 + \theta_1 = \tfrac{3}{2}\pi + 2\theta_1 - 4\theta_2,$$
$$\beta_\pi = \beta - \pi = \tfrac{1}{2}\pi + 2\theta_1 - 4\theta_2,$$
$$\gamma = \psi_+ + \pi - \theta_2 = \tfrac{3}{2}\pi + \theta_1 - 4\theta_2 + \pi - \theta_2 = \tfrac{5}{2}\pi + \theta_1 - 5\theta_2,$$
$$\gamma_- = \psi_- - \theta_2 = \tfrac{1}{2}\pi + \theta_1 - 5\theta_2.$$

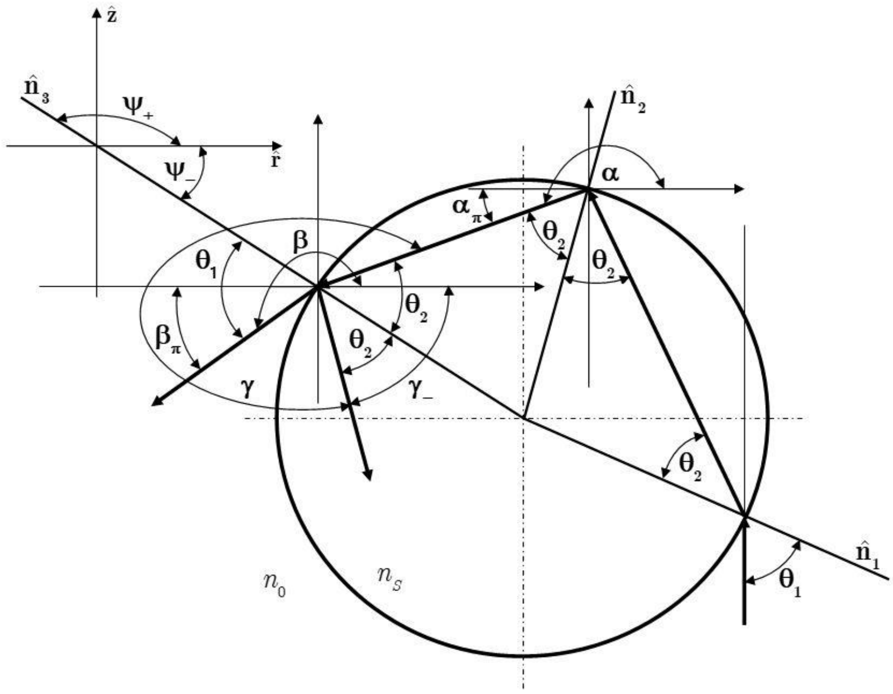


Figure 1.5: Stream of photons represented by the second transition ray of light; the sphere has index of refraction n_S, and the surround has index of refraction n_0.

Hence, the expressions (5) and (20) have views,

$$\Delta p_{1rz} = (h/\lambda_0) n_0 (1 + \cos 2\theta_1), \quad (5a)$$
$$\Delta p_{2rr} = (h/\lambda_0) n_S [\sin(3\theta_2 - \theta_1) - \sin(\theta_1 - \theta_2)]\cos\varphi \quad (20c)$$
$$\Delta p_{2tr} = (h/\lambda_0)\{n_0 \sin[2(\theta_1 - \theta_2)] - n_S \sin(\theta_1 - \theta_2)\}\cos\varphi \quad (20d)$$
$$\Delta p_{3rr} = (h/\lambda_0) n_S [\cos\alpha + \cos\gamma]\cos\varphi = (h/\lambda_0) n_S [\cos(\theta_1 - 3\theta_2) - \cos(\theta_1 - 5\theta_2)]\cos\varphi, \quad (20e)$$
$$\Delta p_{3tr} = (h/\lambda_0)[n_S \cos\alpha - n_0 \cos\beta] = (h/\lambda_0)[n_S \cos(\theta_1 - 3\theta_2) - n_0 \cos(2\theta_1 - 4\theta_2)] \quad (20f)$$
$$\Delta p_{3tz} = (h/\lambda_0)[n_S \sin\alpha - n_0 \sin\beta] = (h/\lambda_0)[n_S \cos(\theta_1 - 3\theta_2) + n_0 \cos(2\theta_1 - 4\theta_2)] \quad (5f)$$
$$\Delta p_{1rr} = (h/\lambda_0) n_0 \sin 2\theta_1 \cos\varphi \quad (20a)$$
$$\Delta p_{1tr} = (h/\lambda_0) n_S \sin(\theta_1 - \theta_2)\cos\varphi \quad (20b)$$

II. Force of a ray on a dielectric sphere

A ray of power P hits a sphere at an angle 0 where it partially reflects and partially refracts, giving rise to a series of scattered rays of power PR, PT2, PT2R, ..., PT2Rn ... , where the quantities R and T are the Fresnel reflection and transmission coefficients of the surface at θ_1 (Fig. 2.1). As seen in Fig. 2.1, these scattered rays make angles relative to the incident forward ray direction of $\pi + 2\theta_1$, $\alpha,\ \alpha + \beta,\ \ldots,\ \alpha + n\beta,\ \ldots$, respectively. The total force in the $\hat{\mathbf{z}}$ direction is the net change in momentum per second in the Z direction due to the scattered rays. Thus [6]:

$$F_z = \frac{n_0}{c}P - \frac{n_0}{c}P\left[R\cos(\pi + 2\theta_1) + \sum_{n=0}^{\infty} T^2R^n\cos(\alpha + n\beta)\right]$$

where n_0P/c is the incident momentum per second in the $\hat{\mathbf{z}}$ direction. Similarly for the $\hat{\mathbf{r}}$ direction, where the incident momentum per second is zero, one has:

$$F_r = 0 - \frac{n_0}{c}P\left[R\sin(\pi + 2\theta_1) - \sum_{n=0}^{\infty} T^2R^n\sin(\alpha + n\beta)\right]$$

As pointed out by van de Hulst in Chapter 12 of reference [7] and by Roosen [8], one can sum over the rays scattered by a sphere by considering the total force in the complex plane, $F_{tot} = F_z + iF_y$. Thus,

$$F_{tot} = \frac{n_0}{c}P\left\{(1 + R\cos 2\theta_1) + iR\sin 2\theta_1 - T^2\sum_{n=0}^{\infty} R^n e^{i(\alpha + n\beta)}\right\}$$

The sum over   is a simple geometric series which can be summed to give:

$$\begin{aligned} F_{tot} &= \frac{n_0}{c}P\left\{(1 + R\cos 2\theta_1) + iR\sin 2\theta_1 - T^2e^{i\alpha}\left[\frac{1}{1 - R\exp(i\beta)}\right]\right\} = \\ &= \frac{n_0}{c}P\left\{\left[1 + R\cos 2\theta_1 - T^2\frac{\cos\alpha - R\cos(\alpha - \beta)}{1 + R^2 - 2R\cos\beta}\right] + i\left[R\sin 2\theta_1 - T^2\frac{\sin\alpha - R\sin(\alpha - \beta)}{1 + R^2 - 2R\cos\beta}\right]\right\}. \end{aligned}$$

Hence,

$$\begin{aligned} F_z &= F_{tot}^{\mathrm{Re}} = \frac{n_0}{c}P\left[1 + R\cos 2\theta_1 - T^2\frac{\cos\alpha - R\cos(\alpha - \beta)}{1 + R^2 - 2R\cos\beta}\right], \\ F_r &= F_{tot}^{\mathrm{Im}} = \frac{n_0}{c}P\left[R\sin 2\theta_1 - T^2\frac{\sin\alpha - R\sin(\alpha - \beta)}{1 + R^2 - 2R\cos\beta}\right]. \end{aligned}$$

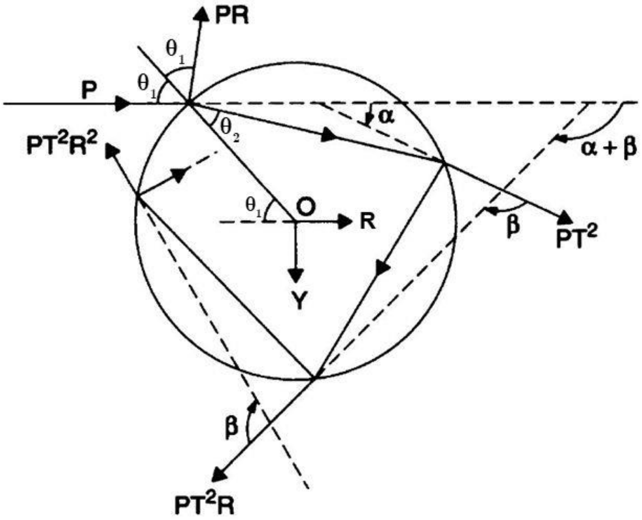


Figure 2.1: Geometry for calculating the force due to the scattering of a single incident ray of power P by a dielectric sphere, showing the reflected ray PR and infinite set of refracted rays PT2Rn.

If one rationalizes the complex denominator and takes the real and imaginary parts of F_{tot} , one gets the force expressions Al and A2 for F_z and F_y using the geometric relations $\alpha = 2\theta_1 - 2\theta_2$ and $\beta = \pi - 2\theta_2$, where and θ_2 are the angles of incidence and refraction of the ray. Substituting $\alpha = 2\theta_1 - 2\theta_2$ and $\beta = \pi - 2\theta_2$ one obtains,

$$F_z = F_{tot}^{\mathrm{Re}} = \frac{n_0}{c} P \left[1 + R\cos 2\theta_1 - T^2 \frac{\cos 2(\theta_1 - \theta_2) + R\cos 2\theta_1}{1 + R^2 + 2R\cos 2\theta_2} \right],$$
$$F_r = F_{tot}^{\mathrm{Im}} = \frac{n_0}{c} P \left[R\sin 2\theta_1 - T^2 \frac{\sin 2(\theta_1 - \theta_2) + R\sin 2\theta_1}{1 + R^2 + 2R\cos 2\theta_2} \right]. \qquad (31a)$$

For the case when the forces are of the Gaussian laser beam, it is necessary to integrate the expressions (A31a,b) over the surface of the particle's hemisphere onto which the laser beam is incident

$$F_z = F_{tot}^{\mathrm{Re}} = \frac{n_0}{c} R_{\mathrm{Sp}}^2 \int_0^{\pi/2}\int_0^{2\pi} I(\rho, z) \left[1 + R\cos 2\theta_1 - T^2 \frac{\cos 2(\theta_1 - \theta_2) + R\cos 2\theta_1}{1 + R^2 + 2R\cos 2\theta_2} \right] \sin 2\theta_1 d\varphi d\theta_1,$$
$$F_r = F_{tot}^{\mathrm{Im}} = \frac{n_0}{c} R_{\mathrm{Sp}}^2 \int_0^{\pi/2}\int_0^{2\pi} I(\rho, z) \left[R\sin 2\theta_1 - T^2 \frac{\sin 2(\theta_1 - \theta_2) + R\sin 2\theta_1}{1 + R^2 + 2R\cos 2\theta_2} \right] \cos\varphi \sin 2\theta_1 d\varphi d\theta_1, \qquad (31b)$$

where R_{Sp} is the particle's sphere radius. Here,

$$\rho(\theta_1, \varphi) = (a^2 + R_{\mathrm{Sp}}^2 \sin^2\theta_1 + 2aR_{\mathrm{Sp}} \sin\theta_1 \cos\varphi)^{1/2},$$
$$\sin\theta_2 = (n_0/n_S)\sin\theta_1, \quad \cos\theta_2 = \sqrt{1 - (n_0/n_S)^2 \sin^2\theta_1}, \quad z = R_{\mathrm{Sp}}\left[\cos(\theta_1 + 2\theta_2) + \cos\theta_1\right] = \qquad (32)$$
$$= 2R_{\mathrm{Sp}} \left\{ \cos\theta_1 - (n_0/n_S)^2 \sin^2\theta_1 \left[\cos\theta_1 - \sqrt{1 - (n_0/n_S)^2 \sin^2\theta_1}\right] \right\}.$$

For the case when the particle's sphere is centered in the Gaussian beam the value ρ may be found as $\rho = \rho(\theta_1) = R_{Sp}\sin\theta_1$, and the expressions (32) may be rewritten in more simple form,

$$F_z = F_{tot}^{\rm Re} = \frac{2n_0}{c}\pi R_{\rm Sp}^2 \int_0^{\pi/2} I(\rho,z)\left[1+R\cos 2\theta_1 - T^2\frac{\cos 2(\theta_1-\theta_2)+R\cos 2\theta_1}{1+R^2+2R\cos 2\theta_2}\right]\sin 2\theta_1 d\theta_1,$$

$$F_r = F_{tot}^{\rm Im} = \frac{4n_0}{c} R_{\rm Sp}^2 \int_0^{\pi/2} I(\rho,z)\left[R\sin 2\theta_1 - T^2\frac{\sin 2(\theta_1-\theta_2)+R\sin 2\theta_1}{1+R^2+2R\cos 2\theta_2}\right]\sin 2\theta_1 d\theta_1,$$

The Fresnel reflection and transmission coefficients of the surface at θ_1 may be found as [9]

$$R_{TE} = \left|r_{TE}\right|^2 = \left(\frac{n_0\cos\theta_1 - n_S\cos\theta_2}{n_0\cos\theta_1 + n_S\cos\theta_2}\right)^2,\quad T_{TE} = \left|1 - r_{TE}\right|^2 = \left(\frac{2n_S\cos\theta_2}{n_0\cos\theta_1 + n_S\cos\theta_2}\right)^2,$$

$$R_{TM} = \left|r_{TE}\right|^2 = \left(\frac{n_S\cos\theta_1 - n_0\cos\theta_2}{n_S\cos\theta_1 + n_0\cos\theta_2}\right)^2,\quad T_{TM} = \left|1 - r_{TM}\right|^2 = \left(\frac{2n_0\cos\theta_2}{n_S\cos\theta_1 + n_0\cos\theta_2}\right)^2.$$

For unpolarized beam the average of these coefficients is given [9]

$$R = \tfrac{1}{2}\left(R_{TE}+R_{TM}\right) = \frac{1}{2}\left[\left(\frac{n_0\cos\theta_1 - n_S\cos\theta_2}{n_0\cos\theta_1 + n_S\cos\theta_2}\right)^2 + \left(\frac{n_S\cos\theta_1 - n_0\cos\theta_2}{n_S\cos\theta_1 + n_0\cos\theta_2}\right)^2\right] =$$

$$= \frac{(n_0 n_S)^2(\cos^2\theta_1 - \cos^2\theta_2)^2 + \cos^2\theta_1\cos^2\theta_2(n_0^2 - n_S^2)^2}{(n_0\cos\theta_1 + n_S\cos\theta_2)^2(n_S\cos\theta_1 + n_0\cos\theta_2)^2} = \frac{(n_0 n_S)^2(\cos^4\theta_1 + \cos^4\theta_2) + \cos^2\theta_1\cos^2\theta_2(n_0^4 + n_S^4)}{[n_0 n_S(\cos^2\theta_1 + \cos^2\theta_2) + \cos\theta_1\cos\theta_2(n_0^2 + n_S^2)]^2},$$

$$T = \tfrac{1}{2}\left(T_{TE}+T_{TM}\right) = \frac{1}{2}\left[\left(\frac{2n_S\cos\theta_2}{n_0\cos\theta_1 + n_S\cos\theta_2}\right)^2 + \left(\frac{2n_0\cos\theta_2}{n_S\cos\theta_1 + n_0\cos\theta_2}\right)^2\right] =$$

$$= \frac{2\left(n_S\cos\theta_2\right)^2\left[(n_S\cos\theta_1 + n_0\cos\theta_2)^2 + (n_0\cos\theta_1 + n_S\cos\theta_2)^2\right]}{(n_0\cos\theta_1 + n_S\cos\theta_2)^2(n_S\cos\theta_1 + n_0\cos\theta_2)^2} =$$

$$= \frac{2\left(n_S\cos\theta_2\right)^2\left[(n_S\cos\theta_1 + n_0\cos\theta_2)^2 + (n_0\cos\theta_1 + n_S\cos\theta_2)^2\right]}{[n_0 n_S(\cos^2\theta_1 + \cos^2\theta_2) + \cos\theta_1\cos\theta_2(n_0^2 + n_S^2)]^2}.$$

Note that some authors use another expression for Fresnel transmission coefficients (see for example [10])

$$T_{TE} = \left|t_{TE}\right|^2 = 1 - \left|r_{TE}\right|^2,\quad T_{TM} = \left|t_{TM}\right|^2 = 1 - \left|r_{TE}\right|^2,$$

$$T = \tfrac{1}{2}\left(T_{TE} + T_{TM}\right) = 1 - R = 1 - \frac{1}{2}\left[\left(\frac{n_0\cos\theta_1 - n_S\cos\theta_2}{n_0\cos\theta_1 + n_S\cos\theta_2}\right)^2 + \left(\frac{n_S\cos\theta_1 - n_0\cos\theta_2}{n_S\cos\theta_1 + n_0\cos\theta_2}\right)^2\right] =$$

$$= \frac{2(n_0 n_S)^2\cos\theta_1\cos\theta_2\left[2\cos\theta_1\cos\theta_2 + (\cos^2\theta_1 + \cos^2\theta_2)\dfrac{n_0^2 + n_S^2}{n_0 n_S}\right]}{[n_0 n_S(\cos^2\theta_1 + \cos^2\theta_2) + \cos\theta_1\cos\theta_2(n_0^2 + n_S^2)]^2} =$$

$$= \frac{2(n_0 n_S)^2\cos\theta_1\cos\theta_2\left[(\cos\theta_1 + \cos\theta_2)^2 + (\cos^2\theta_1 + \cos^2\theta_2)\left(\dfrac{n_0}{n_S} + \dfrac{n_S}{n_0} - 1\right)\right]}{[n_0 n_S(\cos^2\theta_1 + \cos^2\theta_2) + \cos\theta_1\cos\theta_2(n_0^2 + n_S^2)]^2}.$$

The last expression directly corresponds to the formulas given on Wikipedia. Note that there can be singularity point if

$$n_0 n_S(\cos^2\theta_1 + \cos^2\theta_2) + \cos\theta_1\cos\theta_2(n_0^2 + n_S^2) = 0,\ \text{i.e.}\ \cos\theta_1 = \cos\theta_2 = 0$$

The last equality is equivalent to

$$\cos q_1 = \cos q_2 = \sqrt{[1- (n_0 / n_S)^2] + (n_0 / n_S)^2 \cos^2 q_1} .$$

To realize last equality, it is necessary to have

$$[1- (n_0 / n_S)^2]\cos^2 q_1 = [1- (n_0 / n_S)^2], \text{ or}$$

$\cos q_1 = \cos q_2$, only if $\cos q_1 = \cos q_2 = 1.0$, not zero. Hence, there is no singular points at the interval $[0, p / 2]$.

As a final result we should find the solution from integral,

$$\begin{aligned}
F_z = F_{tot}^{\mathrm{Re}} &= \frac{2n_0}{c} I_{\max} \pi R_{\mathrm{Sp}}^2 \int_{-\pi/2}^{\pi/2} \left[1 + R\cos 2\theta_1 - T^2 \frac{\cos 2(\theta_1 - \theta_2) + R\cos 2\theta_1}{1 + R^2 + 2R\cos 2\theta_2} \right] \sin 2\theta_1 d\theta_1 = \\
&= \frac{2n_0}{c} I_{\max} \pi R_{\mathrm{Sp}}^2 \int_{-\pi/2}^{0} \left[1 + R\cos 2\theta_1 - T^2 \frac{\cos 2(\theta_1 - \theta_2) + R\cos 2\theta_1}{1 + R^2 + 2R\cos 2\theta_2} \right] \sin 2\theta_1 d\theta_1 + \\
&+ \frac{2n_0}{c} I_{\max} \pi R_{\mathrm{Sp}}^2 \int_{0}^{\pi/2} \left[1 + R\cos 2\theta_1 - T^2 \frac{\cos 2(\theta_1 - \theta_2) + R\cos 2\theta_1}{1 + R^2 + 2R\cos 2\theta_2} \right] \sin 2\theta_1 d\theta_1 = \\
&= \frac{4n_0}{c} I_{\max} \pi R_{\mathrm{Sp}}^2 \int_{0}^{\pi/2} \left[1 + R\cos 2\theta_1 - T^2 \frac{\cos 2(\theta_1 - \theta_2) + R\cos 2\theta_1}{1 + R^2 + 2R\cos 2\theta_2} \right] \sin 2\theta_1 d\theta_1 = \\
&= -\frac{8n_0}{c} I_{\max} \pi R_{\mathrm{Sp}}^2 \int_{-1}^{1} \left[1 + R\cos 2\theta_1 - T^2 \frac{\cos 2(\theta_1 - \theta_2) + R\cos 2\theta_1}{1 + R^2 + 2R\cos 2\theta_2} \right] \cos\theta_1 d(\cos\theta_1).
\end{aligned}$$

due to its symmetry relative to z-axis (the function under the integral sign is even, see Appendix 2.1).

The integral action of radial forces in our case is equal zero due to their anti-symmetry relative to z-axis (the function under the integral sign is odd, see Appendix 2.1).

$$\begin{aligned}
F_r = F_{tot}^{\mathrm{Im}} &= \frac{4n_0}{c} I_{\max} R_{\mathrm{Sp}}^2 \int_{-\pi/2}^{\pi/2} \left[R\sin 2\theta_1 - T^2 \frac{\sin 2(\theta_1 - \theta_2) + R\sin 2\theta_1}{1 + R^2 + 2R\cos 2\theta_2} \right] \sin 2\theta_1 d\theta_1 = \\
&= \frac{4n_0}{c} I_{\max} R_{\mathrm{Sp}}^2 \int_{-\pi/2}^{0} \left[R\sin 2\theta_1 - T^2 \frac{\sin 2(\theta_1 - \theta_2) + R\sin 2\theta_1}{1 + R^2 + 2R\cos 2\theta_2} \right] \sin 2\theta_1 d\theta_1 + \\
&+ \frac{4n_0}{c} I_{\max} R_{\mathrm{Sp}}^2 \int_{0}^{\pi/2} \left[R\sin 2\theta_1 - T^2 \frac{\sin 2(\theta_1 - \theta_2) + R\sin 2\theta_1}{1 + R^2 + 2R\cos 2\theta_2} \right] \sin 2\theta_1 d\theta_1 = \\
&= \frac{4n_0}{c} I_{\max} R_{\mathrm{Sp}}^2 \int_{0}^{1} \left[R\sin 2\theta_1 - T^2 \frac{\sin 2(\theta_1 - \theta_2) + R\sin 2\theta_1}{1 + R^2 + 2R\cos 2\theta_2} \right] \cos\theta_1 d(\cos\theta_1) - \\
&- \frac{4n_0}{c} I_{\max} R_{\mathrm{Sp}}^2 \int_{0}^{1} \left[R\sin 2\theta_1 - T^2 \frac{\sin 2(\theta_1 - \theta_2) + R\sin 2\theta_1}{1 + R^2 + 2R\cos 2\theta_2} \right] \cos\theta_1 d(\cos\theta_1) = 0.
\end{aligned}$$

Here,

$$R = \tfrac{1}{2}\left(R_{TE} + R_{TM}\right),\ T = \tfrac{1}{2}\left(T_{TE} + T_{TM}\right) = 1 - R,$$

$$R_{TE} = \left|r_{TE}\right|^2 = \left(\frac{n_0 \cos q_1 - n_S \cos q_2}{n_0 \cos q_1 + n_S \cos q_2}\right)^2,\ T_{TE} = 1 - \left|r_{TE}\right|^2 = 1 - \left(\frac{n_0 \cos q_1 - n_S \cos q_2}{n_0 \cos q_1 + n_S \cos q_2}\right)^2,$$

$$R_{TM} = \left|r_{TE}\right|^2 = \left(\frac{n_S \cos q_1 - n_0 \cos q_2}{n_S \cos q_1 + n_0 \cos q_2}\right)^2,\ T_{TM} = 1 - \left|r_{TM}\right|^2 = 1 - \left(\frac{n_S \cos q_1 - n_0 \cos q_2}{n_S \cos q_1 + n_0 \cos q_2}\right)^2,$$

$$\cos q_2 = \sqrt{1 - (n_0 / n_S)^2 \sin^2 q_1} = \sqrt{[1 - (n_0 / n_S)^2] + (n_0 / n_S)^2 \cos^2 q_1}.$$

For comparison our results with the similar in literature it better to take it in [11]. The bifurcation diameter of the TE01 mode is $10.4m$m

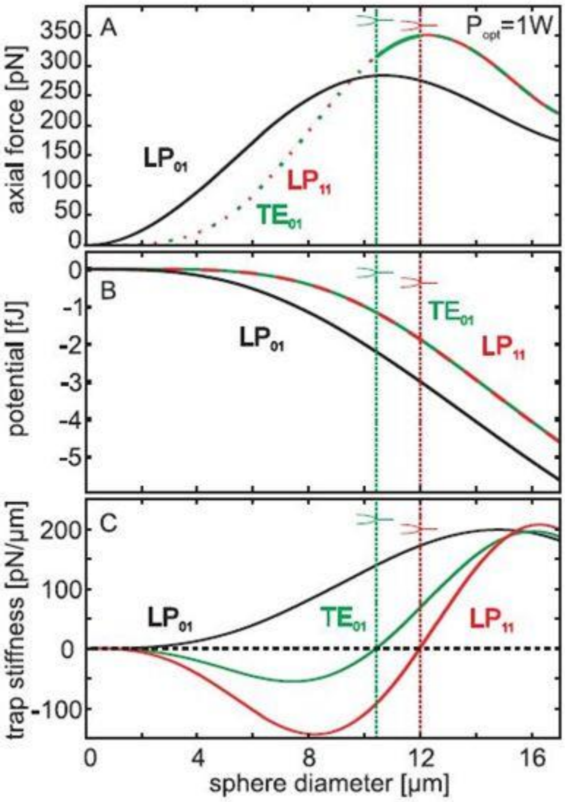


Figure 2.2: Axial optical force (A), radial trapping potencial (B) and trap stiffness (C) for a spherical borosilicate particle of variable radius, located on the fibber axis. The bifurcation diameters of TE01 and LP11 mode are indicated by vertical dotted lines.Comparison with experimental data (see Fig. 2.2) gives 472pN (calculation) and 350pN (experiment).

$$\left.\frac{8n_0}{c} I_{\max} \pi R_{\mathrm{Sp}}^2\right|_{R_{\mathrm{Sp}}=12.0\,\mu m} = \frac{2n_0}{c} I_{\max}[\mathrm{W/\,m^2}]\pi D_{\mathrm{Sp}}^2(\mu m) \times 10^{-12} \approx$$

$$\approx \tfrac{2}{3} n_0 I_{\max}[\mathrm{W/\,m^2}]\pi D_{\mathrm{Sp}}^2(\mu m) \times 10^{-20}[\mathrm{N}] \approx$$

$$\approx \tfrac{2}{3} 1.33 \frac{1.0}{\tfrac{1}{4}\pi(1.04 \times 10^{-5})^2} \pi 1.2^2 \times 10^{-20}[\mathrm{N}] \approx$$

$$\approx \tfrac{8}{3} 1.33 \cdot (1.2 / 1.04)^2 \times 10^{-10}[\mathrm{N}] \approx 4.72 \times 10^{10} = 472\mathrm{pN},$$

$$c \approx 3 \times 10^8[\mathrm{m/\,s}],\quad D_{\mathrm{Sp}} = 2R_{\mathrm{Sp}} - \text{the droplet diameter.}$$

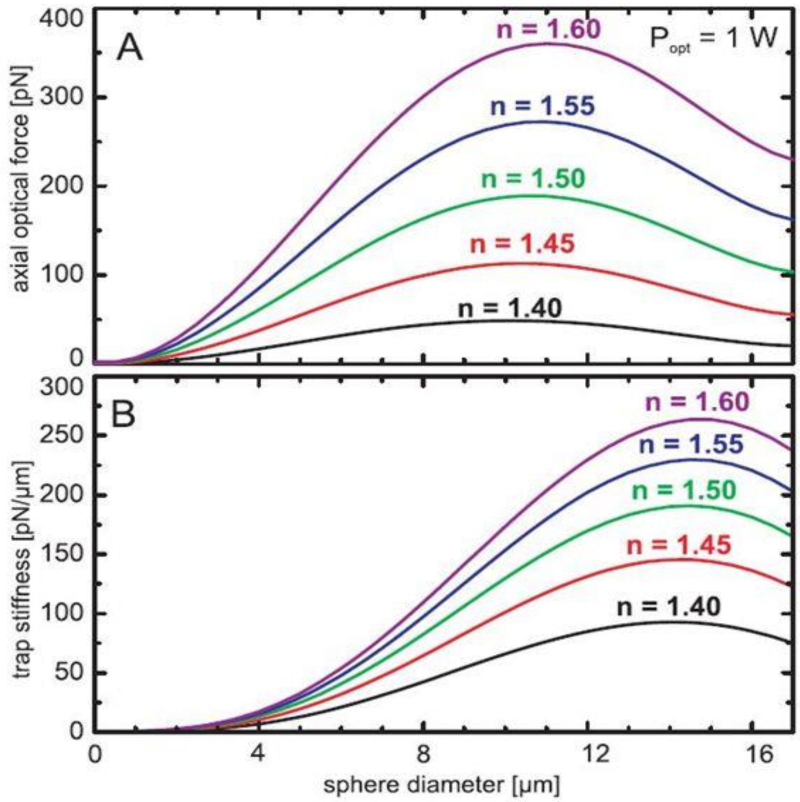


Figure 2.3: A: an axial optical force and B: trap stiffness versus radius for spherical particles of different refractive index. The calculations were performed for LP11 mode and 1 W optical power and a waveguide medium of refractive index 1.33.

III. Liquid Droplet Velocity

The problem is to find liquid droplet velocity inside air gas flow with the velocity under laser beam force The motion equation may be written as

$$m_d \left(dv_d / dt \right) = F_D = -\tfrac{1}{2} \rho_g C_D \left| v_g - v_d \right|^2 A_d + F_z , \tag{33}$$

where $m_d = \frac{4}{3} p r_d R_{Sp}^3$ is the liquid droplet mass, $A_d = pR_{Sp}^2$ is the contact surface, with the droplet's diameter $D_p = 2R_{Sp}$, v_d and v_g are the liquid droplet and gaseous media velocities. We'll assume the Stokes coefficient C_D to be constant

Due to laser beam force acting at the droplet with the diameter $D_p = 2R_{Sp}$ is equal

$$F_z = \frac{2n_0}{c} I_{\max} \pi R_{\mathrm{Sp}}^2 \int_0^{\pi/2} \left[1 + R\cos 2\theta_1 - T^2 \frac{\cos 2(\theta_1 - \theta_2) + R\cos 2\theta_1}{1 + R^2 + 2R\cos 2\theta_2} \right] \sin 2\theta_1 d\theta_1 = \pi R_{\mathrm{Sp}}^2 \mathrm{F}_z \tilde{I}(t),$$
$$\mathrm{F}_z = \frac{2n_0}{c} \int_0^{\pi/2} \left[1 + R\cos 2\theta_1 - T^2 \frac{\cos 2(\theta_1 - \theta_2) + R\cos 2\theta_1}{1 + R^2 + 2R\cos 2\theta_2} \right] \sin 2\theta_1 d\theta_1, \tag{34a}$$

$$\tilde{I}(t) = I_{\max} \times \begin{cases} 1, & t \in \left[k(\Delta\tau_{\mathrm{Imp}} + \Delta\tau_0), (k+1)\Delta\tau_{\mathrm{Imp}} + k\Delta\tau_0\right], \quad k = 0,1,2,\dots, \\ 0, & t \in \left[(k+1)\Delta\tau_{\mathrm{Imp}} + k\Delta\tau_0, (k+1)(\Delta\tau_{\mathrm{Imp}} + \Delta\tau_0)\right], \quad k = 0,1,2,\dots . \end{cases} \tag{34b}$$

Here $\Delta\tau_{\mathrm{Imp}}$, and $\Delta\tau_0$ are the widths of laser impulse and interrupt time, correspondingly. Eq. (33) has a view

$$\begin{aligned} & d(v_d - v_g) / dt = -\tfrac{3}{8}(\rho_g / \rho_d R_{Sp}) C_D \left|v_g - v_d\right|^2 + \tfrac{3}{4}(1/ \rho_d R_{Sp}) \mathrm{F}_z \tilde{I}(t), \\ & (v_d - v_g)\big|_{t=0} = v_{d,0} - v_{g,0}. \end{aligned}$$

Denoting $x = v_d - v_g$ we obtain the problem

$$\begin{aligned} & d\bar{v} / dt = -\tfrac{3}{8}(\rho_g / \rho_d R_{Sp}) C_D \bar{v}^2 + \tfrac{3}{4}(1/ \rho_d R_{Sp}) \mathrm{F}_z \tilde{I}(t), \\ & \bar{v}\big|_{t=0} = \bar{v}_0. \end{aligned} \tag{35a}$$

Denoting $A = \frac{3}{8}(\rho_g / \rho_d R_{Sp}) C_D$, $B = \frac{3}{4}(1/ \rho_d R_{Sp}) \mathrm{F}_z I_{\max}$ we get

$$\begin{aligned} & d\bar{v} / dt = -A\bar{v}^2 + B\left[\tilde{I}(t) / I_{\max}\right], \\ & \bar{v}\big|_{t=0} = \bar{v}_0. \end{aligned} \tag{35b}$$

The equation above is the Riccati equation, which does not accept analytical solution for the case when the power source is δ-function type. Nevertheless, we can start the iteration process due to the power source function has a form of step function (34b).

1. Step 1, $\boldsymbol{k = 0}$: Its solution for the time intervals $t \in \left[0, \Delta\tau_{\mathrm{Imp}}\right]$ is

$$\frac{d\bar{v}}{\bar{v}^2 - B/A} = -A\,dt \quad \Rightarrow \quad \int_{\bar{v}_0}^{\bar{v}} \frac{d\bar{v}}{\bar{v}^2 - \left(\sqrt{B/A}\right)^2} = -At$$

Hence,

$$\ln\left(\frac{\bar{v} - \sqrt{B/A}}{\bar{v} + \sqrt{B/A}}\right)\left(\frac{\bar{v}_0 + \sqrt{B/A}}{\bar{v}_0 - \sqrt{B/A}}\right) = -2\sqrt{AB}t \quad \Rightarrow \quad \frac{\bar{v} - \sqrt{B/A}}{\bar{v} + \sqrt{B/A}} = \frac{\bar{v}_0 - \sqrt{B/A}}{\bar{v}_0 + \sqrt{B/A}} \exp\left(-2\sqrt{AB}t\right),$$

$$\bar{v} = \frac{2\sqrt{B/A}\left(\bar{v}_0 + \sqrt{B/A}\right)}{\left(\bar{v}_0 + \sqrt{B/A}\right) - \left(\bar{v}_0 - \sqrt{B/A}\right)\exp\left(-2\sqrt{AB}t\right)} - \sqrt{B/A}.$$

Finally, we obtain

$$\bar{v} = \sqrt{B/A}\, \frac{(\bar{v}_0 + \sqrt{B/A}) + (\bar{v}_0 - \sqrt{B/A})\exp\left(-2\sqrt{AB}t\right)}{(\bar{v}_0 + \sqrt{B/A}) - (\bar{v}_0 - \sqrt{B/A})\exp\left(-2\sqrt{AB}t\right)}. \tag{36}$$

Assuming that $v_{d,0} = v_g$ at $t = 0$ we obtain for relative velocity $\bar{v}$ the trivial initial condition $\bar{v}_0 = 0$, and so the solution for the 1st time interval (the width of laser impulse) $t \in [0, \Delta\tau_{\text{imp}}]$, is

$$\bar{v}\Big|_{t\in[0,\Delta\tau_{\text{imp}}]} = \sqrt{B/A}\,\frac{1-\exp\left(-2\sqrt{AB}t\right)}{1+\exp\left(-2\sqrt{AB}t\right)} \tag{37a}$$

For the 1st interrupt time interval $t \in [\Delta\tau_{\text{imp}}, \Delta\tau_{\text{imp}} + \Delta\tau_0]$ we should to resolve uniform equation (35b) with the boundary condition for v_d obtained from (4a) at $t = \Delta\tau_{\text{imp}}$, i.e.

$$d\bar{v}/dt = -A\bar{v}^2, \qquad \bar{v}\Big|_{t=\Delta\tau_{\text{Imp}}} = \bar{v}\Big|_{t=\Delta\tau_{\text{Imp}}},$$

Where

$$\bar{v}\Big|_{t=\Delta\tau_{\text{imp}}} = \sqrt{B/A}\,\frac{1-\exp\left(-2\sqrt{AB}\Delta\tau_{\text{imp}}\right)}{1+\exp\left(-2\sqrt{AB}\Delta\tau_{\text{imp}}\right)}$$

is the value v_d at the end, $t = \Delta\tau_{\text{imp}}$, of the 1st laser impulse interval $t \in [0, \Delta\tau_{\text{imp}}]$. The problem (36) has the solution

$$\bar{v}\Big|_{t\in[\Delta\tau_{\text{Imp}},\Delta\tau_{\text{Imp}}+\Delta\tau_0]} = \frac{1}{\left(\bar{v}\Big|_{t=\Delta\tau_{\text{Imp}}}\right)^{-1} + A(t-\Delta\tau_{\text{imp}})}. \tag{38a}$$

To start the next step we should find the boundary condition for $\bar{v}$ at $t = \Delta\tau_{\text{imp}} + \Delta\tau_0$,

$$\bar{v}\Big|_{t=\Delta\tau_{\text{Imp}}+\Delta\tau_0} = 1\Big/\left[\left(\bar{v}\Big|_{t=\Delta\tau_{\text{Imp}}}\right)^{-1} + A\Delta\tau_0\right]. \tag{38b}$$

2. Step 2, $\boldsymbol{k = 1}$: Simultaneously, using (38b) as a boundary condition we obtain

$$\bar{v}\Big|_{t\in[\Delta\tau_{\text{imp}}+\Delta\tau_0,\,2\Delta\tau_{\text{imp}}+\Delta\tau_0]} = \sqrt{B/A}\,\frac{\left(\bar{v}\Big|_{t=\Delta\tau_{\text{Imp}}+\Delta\tau_0} + \sqrt{B/A}\right) + \left(\bar{v}\Big|_{t=\Delta\tau_{\text{Imp}}+\Delta\tau_0} - \sqrt{B/A}\right)\exp\left\{-2\sqrt{AB}\left[t-(\Delta\tau_{\text{imp}}+\Delta\tau_0)\right]\right\}}{\left(\bar{v}\Big|_{t=\Delta\tau_{\text{Imp}}+\Delta\tau_0} + \sqrt{B/A}\right) - \left(\bar{v}\Big|_{t=\Delta\tau_{\text{Imp}}+\Delta\tau_0} - \sqrt{B/A}\right)\exp\left\{-2\sqrt{AB}\left[t-(\Delta\tau_{\text{imp}}+\Delta\tau_0)\right]\right\}}, \tag{39a}$$

and the boundary condition for the next step is $\bar{v}$ at $t = 2\Delta\tau_{\text{imp}} + \Delta\tau_0$,

$$\bar{v}\Big|_{t=2\Delta\tau_{\text{imp}}+\Delta\tau_0} = \sqrt{B/A}\,\frac{\left(\bar{v}\Big|_{t=\Delta\tau_{\text{Imp}}+\Delta\tau_0} + \sqrt{B/A}\right) + \left(\bar{v}\Big|_{t=\Delta\tau_{\text{Imp}}+\Delta\tau_0} - \sqrt{B/A}\right)\exp\left(-2\sqrt{AB}\Delta\tau_{\text{imp}}\right)}{\left(\bar{v}\Big|_{t=\Delta\tau_{\text{Imp}}+\Delta\tau_0} + \sqrt{B/A}\right) - \left(\bar{v}\Big|_{t=\Delta\tau_{\text{Imp}}+\Delta\tau_0} - \sqrt{B/A}\right)\exp\left(-2\sqrt{AB}\Delta\tau_{\text{imp}}\right)}$$

and, using (39a) we get

$$\bar{v}\Big|_{t\in\left[2\Delta t_{\mathrm{imp}}+\Delta t_0,\,2\Delta t_{\mathrm{imp}}+2\Delta t_0\right]} = \frac{1}{\left(\bar{v}\Big|_{t=2\Delta t_{\mathrm{Imp}}+\Delta t_0}\right)^{-1} + A\left[t-\left(2\Delta t_{\mathrm{imp}}+\Delta t_0\right)\right]}. \qquad (39b)$$

Similarly, the boundary condition for $\bar{v}$ at $t = 2(\Delta t_{\mathrm{imp}} + \Delta t_0)$ is

$$\bar{v}\Big|_{t=2(\Delta t_{\mathrm{Imp}}+\Delta t_0)} = \frac{1}{\left(\bar{v}\Big|_{t=2\Delta t_{\mathrm{Imp}}+\Delta t_0}\right)^{-1} + A\Delta t_0}$$

3. Step $\boldsymbol{k}$: Generally, for every k - and $(k+1)$ - intervals in (2),

$$t \in \left[k(\Delta\tau_{\mathrm{Imp}}+\Delta\tau_0),(k+1)\Delta\tau_{\mathrm{Imp}}+k\Delta\tau_0\right],$$

And

$$t \in \left[(k+1)\Delta\tau_{\mathrm{Imp}}+k\Delta\tau_0,(k+1)(\Delta\tau_{\mathrm{Imp}}+\Delta\tau_0)\right]$$

we obtain the following two expressions:

$$\bar{v}\Big|_{t\in\left[k(\Delta t_{\mathrm{Imp}}+\Delta t_0),(k+1)\Delta t_{\mathrm{Imp}}+k\Delta t_0\right]} =$$

$$= \sqrt{B/A}\,\frac{\left(\bar{v}\Big|_{t=k(\Delta t_{\mathrm{Imp}}+\Delta t_0)}+\sqrt{B/A}\right)+\left(\bar{v}\Big|_{t=k(\Delta t_{\mathrm{Imp}}+\Delta t_0)}-\sqrt{B/A}\right)\exp\left\{-2\sqrt{AB}\left[t-k(\Delta t_{\mathrm{imp}}+\Delta t_0)\right]\right\}}{\left(\bar{v}\Big|_{t=k(\Delta t_{\mathrm{Imp}}+\Delta t_0)}+\sqrt{B/A}\right)-\left(\bar{v}\Big|_{t=k(\Delta t_{\mathrm{Imp}}+\Delta t_0)}-\sqrt{B/A}\right)\exp\left\{-2\sqrt{AB}\left[t-k(\Delta t_{\mathrm{imp}}+\Delta t_0)\right]\right\}},$$

$$\bar{v}\Big|_{t\in\left[(k+1)\Delta t_{\mathrm{Imp}}+k\Delta t_0,(k+1)(\Delta t_{\mathrm{Imp}}+\Delta t_0)\right]} = \frac{1}{\left(\bar{v}\Big|_{t=(k+1)\Delta t_{\mathrm{Imp}}+k\Delta t_0}\right)^{-1} + A\left\{t-\left[(k+1)\Delta t_{\mathrm{imp}}+k\Delta t_0\right]\right\}}$$

where

$$\bar{v}\Big|_{t=k(\Delta t_{\mathrm{Imp}}+\Delta t_0)} = \frac{1}{\left(\bar{v}\Big|_{t=k\Delta t_{\mathrm{Imp}}+(k-1)\Delta t_0}\right)^{-1} + A\Delta t_0},$$

$$\bar{v}\Big|_{t=(k+1)\Delta t_{\mathrm{Imp}}+k\Delta t_0} = \sqrt{B/A}\,\frac{\left(\bar{v}\Big|_{t=k(\Delta t_{\mathrm{Imp}}+\Delta t_0)}+\sqrt{B/A}\right)+\left(\bar{v}\Big|_{t=k(\Delta t_{\mathrm{Imp}}+\Delta t_0)}-\sqrt{B/A}\right)\exp\left(-2\sqrt{AB}\Delta t_{\mathrm{imp}}\right)}{\left(\bar{v}\Big|_{t=k(\Delta t_{\mathrm{Imp}}+\Delta t_0)}+\sqrt{B/A}\right)-\left(\bar{v}\Big|_{t=k(\Delta t_{\mathrm{Imp}}+\Delta t_0)}-\sqrt{B/A}\right)\exp\left(-2\sqrt{AB}\Delta t_{\mathrm{imp}}\right)}.$$

Here

$$A = \tfrac{3}{8}\left(\rho_g/\rho_d R_{Sp}\right)C_D, \quad B = \tfrac{3}{4}\left(1/\rho_d R_{Sp}\right)F_z I_{\max},$$

$$\sqrt{B/A} = \sqrt{2F_z I_{\max}/\left(\rho_g C_d\right)},$$

$$\sqrt{AB} = \tfrac{3}{4}\left(1/\rho_d R_{Sp}\right)\sqrt{\tfrac{1}{2}\left(\rho_g C_d\right)F_z I_{\max}} = \tfrac{3}{2}\left(1/\rho_d R_{Sp}\right)\sqrt{\tfrac{1}{2}\left(\rho_g C_d\right)F_z I_{\max}}.$$

CONCLUSION

In this paper we have shown that the axial and radial forces applied to micrometer-sized spheres can be obtained from a ray-optics model. The theory can easily be adapted to other particle shapes and beam profiles without changing the procedure for deriving the forces. Firstly, the velocity of a drop of liquid in a flow of air and gas under the influence of a laser beam was found; an analytical expression for laser impulse and interrupt time periods is presented.

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