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RESEARCH ARTICLE

A DOUBLE - INTERSECTION PRODUCTION STRATEGY OF AN INVENTORY POLICY OF DETERIORATING ITEM UNDER PARTIAL BACKLOGGING REGIME

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ABSTRACT

This study develops an inventory optimization model for perishable item (blood) using deteriorating inventory theory. The framework integrates time-dependent demand D(u) = Ae—αdu with Weibull-based hazard h(u) = αβuβ—1e—α(u)β under partial back-logging regime. Numerical illustration was used to validate that the deterioration hazard function rises, and intersects the declining demand function at u = 1.7 time units, cli-maxed at u = 3 time units and then falls to intersect the declining demand function ones again at u = 4.3 time units thereby creating a window for a double - intersection production strategy

Keywords: Optimal production, Negative Exponential Demand, Time - Dependent Weibull Deterioration, Partial Backlogging, Optimization.

INTRODUCTION

Imagine hastily pacing the floor of a health facility to the blood bank section to get blood for a ’dying’ patient and the blood bank attendant says, ” sorry we have run out of blood”.

The implication of such a statement is that either the management of that facility is not using a suitable blood bank inventory system or they do not have one at all [11]. This is a devastating scenario and life could be lost and by extension national/global health security could be threatened.

If it is true that ”blood is life”, the reverse may not also be false. Keeping more units of blood than is required per time may make no business sense since it is bound to be wasted due to spoilage. Therefore, maintaining optimal stock levels relative to patients’ demand is a primary concern of those in charge of blood bank unit of Hospitals. If there should be much blood units at a time the demand for it is low, then holding and deterioration costs increase leading to low profit margin and if there are no enough units when demand is high, shortage cost ensues with massive loss of goodwill and confidence on the part of both patients and even their loved ones. Creating a balance, proper inventory management becomes inevitable

Inventory management of perishable goods remains a complex challenge, particularly in systems where demand declines over time and items deteriorate rapidly. Traditional models often fall short by assuming either constant demand or ramp type demand, ignoring production costs, or oversimplifying deterioration patterns. For instance:
Kumar in [14] put forward

$$\frac{dI}{dt} = -ae^{b-ct}\quad 0 \le t \le \mu;\quad I(\mu) = s$$
$$\frac{dI}{dt} = -\frac{ae^{b-ct}}{1+\delta(T-t)};\quad T_1 \le t \le T \tag{1}$$

as an Economic Order Quantity (EOQ) model for deteriorating items under time - dependent exponential demand rate, incorporating penal cost for shortages. The model assumes that deterioration occur at constant rate with demand rate R(t) as ae^{b-ct}.. The question is why assume deterioration at constant rate even when that is not what it is in real life and why is Weibull deterioration function not used which according to [17] is apt for a deteriorating item?

Even though the model provides useful insights for inventory systems with growing demand patterns ae^{b-ct}., there arise this question - to what extent will demand grow since we are considering deteriorating item? Will there not be a time when demand declines due to spoilage of the item? Again, the model assumes zero lead time, so what happens to its applicability to situations where replenishment delays are inevitable?

Khan, et al. in [13] focus on a single item inventory system with non -instantaneous deterioration, nonlinear stock - dependent demand and employs partial backlogging. The model is:

$$\frac{dI_1}{dt} = -\eta[I_1(t)]^{\eta}\quad 0 < t \le t_s$$
$$\frac{dI_2}{dt} + \theta I_2(t) = -\eta[I_1(t)]^{\eta}\quad t_s < t \le t_1$$
$$\frac{dI_3}{dt} = -\frac{\eta}{1+\delta(t_1+t_2-t)}\quad t_1 \le t \le t_1+t_2 \tag{2}$$

Again, does D(t) = η[I1(t)]η model the demand logistics of perishables better than D(t) = Ae—αdt? How will this model maximize profit when no mention is made of production logistics?

Again, Abdeldain, et al. in [1] proposes a production model

$$\frac{dq_{1r}(t)}{dt} + \theta_r(t)q_{1r}(t) = -\frac{X_r}{N_r}\quad 0 \le t \le N_r \tag{3}$$

for deteriorating items with stochastic demand, single period lead time and varying deterioration cost. The goal is to determine the optimal scheduling period and order quality, minimize both the wastage cost due to deterioration and the expected average total cost subject to a constraint on expected deterioration cost. Questions!

i. Where is the production cost function that could enrich the cost structure, in this model?

ii. Why is there no specified trend form for the demand function?

iii. Why full backlogging when partial backlogging reflects realistic handling of shortages?

In reality, almost every perishable product experiences a negative exponential demand due to market saturation, changing preferences, or limited shelf life of the product. Moreover, deterioration in such systems is rarely uniform. A two - parameter Weibull distribution offers greater flexibility to represent real life deterioration patterns more accurately than the classical exponential model. However, models combining Weibull deterioration with non - constant demand are still limited in literature. Compounding the issue is the inadequate treatment of shortages. In practice, not all unmet demand is lost - some portion of it may be partially backlogged depending on customers' willingness to wait [24]. Many existing models either assume complete backlogging or complete lost sales, both of which fail to reflect practical realities [25]. Additionally, production cost is frequently omitted in inventory cost structures [5, 7, 8, 11, 22], despite being a significant component in manufacturing and healthcare logistics. This oversight can lead to sub-optimal policies in real world applications where production costs are dynamic and substantial. Consequently, there is a critical need for a comprehensive inventory model that incorporates:

i. Negative exponential demand;
ii. A Weibull deterioration function;
iii. Partial backlogging; and
iv. Production cost considerations.

The work of [20], Pius, et al. outlined a mathematical inventory framework, that excludes production cost which consequently reduces its ability to fully capture the cost structure faced by manufacturing firms. This omission leaves a gap in the theory that this work seeks to address. We have incorporated production cost and related production decisions thereby enhancing the horizon of application of our model and its usefulness for managerial decision making especially in production - inventory systems. Our proposed model offers a more realistic approach by treating both the deterioration rate and customer demand as time - sensitive variables, which is especially relevant for seasonal products. We also model production as being dependent on this fluctuating demand and account for partial backlogging. This work builds upon the foundational inventory model established by [3, 4, 9, 11].

While many models primarily treated demand as a function of price, [18] synthesized these concepts into a more comprehensive framework because they introduced a deteriorating inventory model that integrates preservation technology with both price and stock sensitive demand thereby capturing a more complex and realistic market response and as such ad-dressed the gap created in the work of [23]. In many models where production is logistics incorporated, the common assumption of production cost being a linear function of time lacks realism. Addressing this, [27], broke new ground by developing an overtime production inventory model for deteriorating items with nonlinear price and stock - dependent demand though partial backlogging was not considered. In this work, partial backlogging is included in a unified sense quite unlike [10] who introduced back - ordering into an EOQ model set within an inflationary, cloudy-fuzzy environment.

In [19], an inventory model with price-dependent demand is developed and preservation technology under a partial backlogging strategy is implemented by Pervin, et al. but with no recourse to Weibull deterioration hazard function. Similarly, in [7] a generalized inventory model featuring selling - price dependent demand and time dependent deterioration is formulated but excluded a demand dependent production logistics as we have in this work. Researchers have explored alternative demand patterns; [22] introduced an integrated vendor buyer model with a novel quadratic demand function, incorporating an inspection policy and preservation technology instead of a negative exponential demand function that appear to be a better demand tool, while in [1] a deteriorating inventory model that integrates preservation technology with both price and stock sensitive demand is introduced, capturing a more complex and realistic market response. It is worthy of note that the deterioration of items is more of time dependent than price and/or stock.

In all these articles,as well as those of [5, 7, 8, 11, 22] production scheduling is a factor that is often-overlooked. Moreover, in [25] the concept of multivariable demand is used at different stages of the model to improve profit and to control time dependent deterioration. They recommended in that paper that a Weibull distribution function should be used in future studies instead of an ordinary time dependent deterioration of their kind. This forms the motivation of this work.

A critical limitation persists across most inventory literature: the reliance on static demand assumptions - whether constant [3], time-dependent [24], stock-dependent or price-dependent [7], [12], applied uniformly over the entire inventory cycle. This approach is fundamentally inadequate for managing items with very low shelf-lives and high deterioration rates, where demand is inherently more volatile and dynamic.
Despite extensive research, a significant gap remains in the modeling of high-deterioration, low-shelf-life items. Previous studies have largely overlooked the critical aspect of a lead - time dependent partial backlogging, applying unrealistic full backlogging instead. Furthermore, the common practice of modeling demand based on a single, static parameter for an entire cycle fails to capture the dynamic market behavior for such perishable products. Addressing these gaps, the current study introduces a novel, multi-stage inventory model. Its key innovations include:

1. A two-parameter demand function that varies across different stages of the inventory cycle, reflecting realistic shifts in market demand;
2. The integration of a more accurate, time-dependent deterioration rate;
3. A demand - dependent production strategy that directly links manufacturing to market requirements.

This unified approach provides manufacturers with a more practical and robust framework for inventory control, ultimately enabling better stock management and enhanced profit maximization for highly perishable goods.
This model incorporates several key realistic assumptions to enhance its applicability:

(i) Time-Dependent Deterioration

The rate of product deterioration is modeled as a function of time, providing a more accurate representation for low-shelf-life items compared to constant-rate assumptions;

(ii) Partial Backlogging with first in first out (FIFO) strategy:

Shortages are permitted and are partially backlogged. During the stock-out period, it is assumed that deterioration ceases as there is no physical inventory. Once production resumes, back - ordered demand is fulfilled on a first-come, first-served (FIFO) basis.

The manuscript consists of five sections. Section 2 outlines the assumptions and notations, develops the mathematical formulation of the proposed model. Analytical solutions of the governing differential equations are derived section 3. Section 4 illustrates the model through a numerical example, followed by a sensitivity analysis. Finally, conclusions are drawn in Section 5.

Preliminaries

The development of the inventory model is based on the following assumptions:

1. **Blood Procurement and Demand**

The blood Procurement rate is greater than the maximum demand rate to ensure that inventory can be built up initially . The rate of demand for blood at time, t is given by a negative exponential function:

$$D(u) = Ae^{-\alpha du}, \quad \text{where} \quad A > 0 \text{ and } \alpha_d > 0 \qquad (4)$$

2. Deterioration

Blood units in inventory are subjected to deterioration over time. The deterioration rate follows a two - parameter Weibull distribution characterized by shape parameter $\beta > 0$, scale parameter α and location parameter.

Deterioration applies only when physical blood is present. Finally, during periods of shortage and backlogging, deterioration is assumed to be zero.

3. Inventory Phases

The inventory cycle is divided into three distinct phases, namely:

i. phase I ($0 \le u < u1$);

ii. phase II ($u1 \le u < u2$);

iii. phase III ($u2 \le u < U$).

These three phases may respectively be seen as interval of time where there is blood accumulation either by procurement from blood stockpile or from blood donors and inventory depletion due to demand and spoilage, interval of time where there is shortage - when the blood bank attendant tells the patient ”sorry we have run out of stock” and partial backlogging sets in since some patients will be rushed out to another health facility and only a fraction agrees to wait for the next blood arrival, as well as interval of time where there is replenishment (restocking).

4. Shortages and Backlogging

Shortages are allowed and partially backlogged. The backlogging rate is dependent on the waiting time until the next replenishment. Patients with less critical cases might wait but those with critical cases may find an alternative.

5. Replenishment and Lead Time

There is replenishment just after u2. According to [8], the lead time for production and delivery is minimized so that the waiting time is not unnecessarily prolonged. The reorder point depends on the lead time.

6. Costs

Contrary to most classical models where blood procurement cost is deliberately excluded, the proposed model considers various cost components, including procurement cost, holding cost, deterioration cost, shortage cost and backlogging cost. These costs functions are continuous and differentiable.

7. Initial Conditions

The inventory level at the beginning of the cycle is I0. it is a known positive quantity. At u1, the inventory level depletes to zero and at time u2, shortages begin, and backlogged demands are accounted for.

(i) Notations and Parameters

In this section, the notations and parameters used throughout the modeling and analysis of the blood inventory system are defined for clarity and ease of reference.

Symbols **Description**

t Time variable within a cycle ($0 \leq u \leq U$)

U Length of the entire inventory

u1 Time at which blood depletes to zero (end of positive inventory)

u2 Time at which shortage and backlogged blood are replenished I(u) Inventory level at time u

I1(u) Inventory level during procurement and accumulation phase ($0 \leq u < u1$)

I2(u) Inventory level during depletion phase (u1 ≤ u ≤ u2 I3(u)

Inventory level during replenishment phase ($u2 \leq u < U$)

A Initial demand rate (units per unit time)

α Deterioration rate

β Shape parameter

γ Location parameter

αd Demand decay rate (exponential parameter)

σ Production rate factor relative to demand rate

β Shape parameter of the Weibull deterioration distribution

ch Unit holding cost per unit time

cp Unit production cost

ζ(u) Time - dependent deterioration rate at time u given by αβuβ—1

cs Shortage cost per unit short per unit time

cb Backlogging cost per unit backlogged demand per unit time

D(u) Demand rate at time u: D(u) = Ae—αdu

s(u) Rate of satisfied demand or service rate at time, u

δ(u) Fraction of customers willing to back-order at time u

(depends on waiting time)

cd Deterioration cost per unit per time

Figure (1) is the schematic diagram of the model. It shows that at I_0 there is production (may be stockpile) a function of demand. The multiplier σ ¿ 1 ensures that procurement exceeds demand, hence, there is physical stock in the first phase $I_1$of the model. The inventory reduces with time due to the joint action of demand and deterioration. At the second phase I_2, inventory is exhausted, there is no physical stock, and as a result there is no deterioration but unmet demand. The unmet demand is partially backlogged and inventory is replenished at the third phase I_3 to offset the unmet demand. This phase experiences deterioration of unused blood unites, inventory depletes once again due to the effect of demand spoilage and the cycle repeats itself for $I_3(U) = I_0$

(ii) Mathematical Formulation of the Model

The inventory cycle length is partitioned into three intervals, namely:

1. the accumulation/procurement phase during which inventory builds up due to production exceeding demand;
2. the depletion phase where inventory is reduced to zero by satisfying customer demand without procurement or replenishment; unmet demand is backlogged and

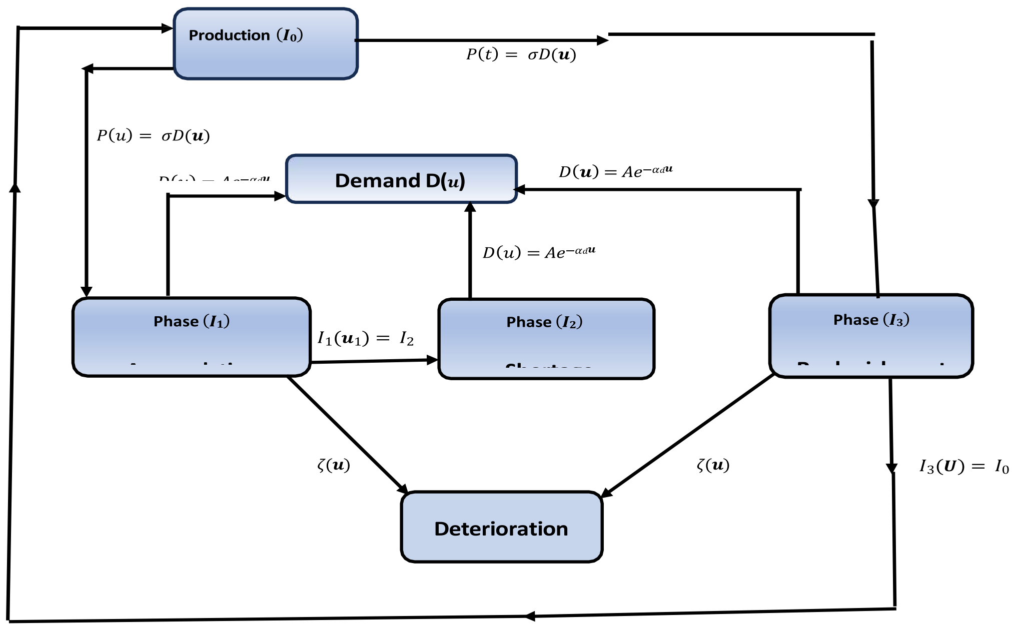

Figure 1: Schematic Diagram of model

3. the replenishment/ restocking phase during which the hitherto unmet but partially backlogged demand is met by new arrivals. Spoilage sets in on unused inventory.

Each phase is governed by a differential equation that captures the dynamics of the proposed inventory system, incorporating both the deterioration blood units over time and the varying demand rate. The deterioration of blood is modeled using time - dependent three -parameter Weibull distribution while the demand follows a negative exponential function of time u. The following subsections present the formulation of the differential equations for each phase of the cycle, along with the appropriate initial conditions to ensure a consistent and realistic representation of the inventory system.

(iii) Inventory Accumulation Phase

In this phase, there is initial procurement of blood either from stockpiles or from blood donors to hospitals. There is high demand met from fresh inventory, yet products begin to expire based on their temperature and chemical stability. Cold - chain logistics apply. The proposed model is:

$$\frac{dI_1}{du} + \alpha\beta u^{\beta-1} I_1(u) = \sigma D(u) - D(u); \quad 0 \le u < u_1 \tag{5}$$

with I_0 is the initial inventory level at the beginning of the cycle. $Ae^{-\alpha du}$ reflects decaying demand of post - outbreak.

(iv) Unfulfilled Demand

Here stock is exhausted, Patients may need to wait (Partial backlogging) even though each delay increases health risk and institutional penalties. The model is

$$\frac{dI_2}{du} = -Ae^{-\alpha du} \qquad \text{with} \quad I_2(u_1) = 0 \tag{6}$$

(v) Restocking Phase

In this phase , new blood units arrive. There is a fulfillment of backlogged demand and unused blood units from the new batch will again face expiry risk. The scenario in this phase is modeled by:

$$\frac{dI_3}{du} + \alpha\beta u^{\beta-1} I_3(u) = \sigma D(u) - D(u); \quad 0 \le u < u_1 \tag{7}$$

Equations (5), (6)and (7) forms the proposed system as:

$$\begin{cases} \frac{dI_1(u)}{du} + \alpha\beta u^{\beta-1} I_1(u) = (\sigma - 1)A^r e^{-\alpha_d u} \; ; I_1(0) = I_0 \\ \frac{dI_2(u)}{du} = -A^r e^{-\alpha_d u} \quad I_2(u_1) = I_1(u_1) = 0 \\ \frac{dI_3(u)}{du} + \alpha\beta u^{\beta-1} I_3(u) = (\sigma - 1)A^r e^{-\alpha_d u} \; ; I_3(u_2) = I_2(u_2) = \eta_0 \end{cases}$$

Main Result

In this section, the Differential Equations governing each phase of the inventory cycle are systematically solved. The method of integrating factor is employed where necessary to obtain explicit analytic expressions for each inventory level as a function of time.

Solution for the Phases

The solution to equation (5) is:

$$I_1(u) = e^{-\alpha u^\beta}\left[I_0^r + (\sigma - 1)A^r \int_0^u e^{\alpha s^\beta - \alpha s} ds\right]. \tag{8}$$

The solution to equation (6) is:

$$I_2(u) = \frac{A^r}{\alpha_d}\left(e^{-\alpha_d u} - e^{-\alpha_d u}\right) \tag{9}$$

and (10)

and that of equation (7) is:

$$I_3(u) = e^{-\alpha u^\beta}\left[\eta_0 e^{\alpha u_2^\beta} + (\sigma - 1)A \int_{u_2}^{u} e^{\alpha s^\beta - \alpha s} ds\right]. \tag{11}$$

Production Cost function C_p is:

$$C_p = c_p(\sigma - 1)A^r\left[\int_0^{u_1} e^{-\alpha_d u}\,du + \int_{u_1}^{u_2} e^{-\alpha_d u}\,du\right] \tag{12}$$

whereas the Holding cost function is:

$$C_h = C_h = c_h\left[\int_0^{u_1} I_1(u)du + \int_{u_2}^{U} I_3(u)du\right] \tag{13}$$

If C_d denotes the deterioration cost, then:

$$C_d = C_d = c_d\left[\int_0^{u_1} \alpha\beta u^{\beta-1} I_1(u)du + \int_{u_2}^{U} \alpha\beta u^{\beta-1} I_3(u)du\right] \tag{14}$$

and Shortage Cost C_s function is given by:

$$C_s = C_s = c_s A \int_{u_1}^{u_2} (u_2 - u)e^{-\alpha_d u}\,du \tag{15}$$

Thus, the total Cost Function $TCF = C_p + C_h + C_d + C_s$ and The Total Average Cost TAC is $\frac{TCF}{T}$. Therefore,

$$TAC = \frac{1}{U}\left[c_p(\sigma - 1)A^r\left[\int_0^{u_1} e^{-\alpha_d u}\,du + \int_{u_1}^{u_2} e^{-\alpha_d u}\,du\right] + c_h\left[\int_0^{u_1} I_1(u)du + \int_{u_2}^{U} I_3(u)du\right] + c_d\left[\int_0^{u_1} \alpha\beta u^{\beta-1} I_1(u)du\right.\right.$$

$$+ \int_{u_2}^{U} \alpha\beta u^{\beta-1} I_3(u)du\Big] + c_s A \int_{u_1}^{u_2} (u_2-u)e^{-\alpha_d u}\,du$$

$$= \frac{1}{U}\Big[c_p(\sigma-1)A\Big(\frac{1-e^{-\alpha_d u_2}}{\alpha_d}\Big) + c_s A\Big(\frac{u_2-u_1}{\alpha_d}e^{-\alpha_d u_1} + \frac{e^{-\alpha_d u_2}-e^{-\alpha_d u_1}}{\alpha_d^2}\Big) + c_d\frac{(\sigma-1)}{\alpha_d}\Big(1-e^{-\alpha_d u_1}+e^{-\alpha_d u_2}-e^{-\alpha_d U}\Big)$$
$$+ I_1^r + \eta_0 - I_3(U) + c_h\Big(\int_0^{u_1} I_1(u)du + \int_{u_2}^{U} I_3(u)du\Big)\Big] \qquad (16)$$

If the partial derivatives of TAC wrt u_1, u_2 and U are equated to zero then:

$$\frac{\partial TAC^r}{\partial u_1} = \frac{1}{U}\Big[-c_s A^r(u_2-u_1)e^{-\alpha_d u_1} + c_d(\sigma-1)e^{-\alpha_d u_1}\Big]$$
$$= \frac{1}{U}A^r e^{-\alpha_d u_1}\Big[c_d(\sigma-1) - c_s(u_2-u_1)\Big] = 0 \qquad (17)$$

$$\frac{\partial TAC^r}{\partial u_2} = \frac{1}{U}\Big[c_p(\sigma-1)A^r e^{-\alpha_d u_2} + \frac{c_s A^r}{\alpha_d}\Big(e^{-\alpha_d u_1} - e^{-\alpha_d u_2}\Big)$$
$$+ c_d(\sigma-1)e^{-\alpha_d u_2} - c_h I_3(u_2)\Big] = 0 \qquad (18)$$

$$\frac{\partial TAC^r}{\partial U} = -\frac{1}{U^2}(C_p + C_s + C_d + C_h) + \frac{1}{U}\Big[c_d(\sigma-1)e^{-\alpha_d U}$$
$$- c_d I_3^r(U) + c_h I_3(U)\Big] = 0 \qquad (19)$$

Equations (17), (18) and (19) are simultaneously solved to find the decision variables u_1, u_2 and U.

Numerical Example

To illustrate the model, the following numerical values in appropriate unit are assign to the parameters: $A = 100.00$, $\alpha = 0.1$, $\beta = 1.5$, $\gamma = 0.80$, $\sigma = 1.50$, $c_p = 1.00$, $c_h = 0.5$, $c_d = 0.80$, $c_s = 0.30$, $\alpha_d = 0.3$, $I_0 = 50.00$, and $\eta_0 = 0.5$. and solve with the aid of a Python solver to obtain the optimal point as $u_1 = 4.313399$, $u_2 = 4.363399$, and $U = 29.929455$.

Sensitivity Analysis

The sensitivity analysis is carried out by varying each parameter by ±20% and ±50% whereas the other parameters are kept unchanged. Tables 2 − 4 summarize it all.

Table 1: α Sensitivity Analysis Table

Variation	α value	$\Delta\alpha$	u_1	Δu_1	u_2	Δu_2	U	ΔU	U Elasticity
−50%	0.05	-50.00	4.36	0.00%	4.41	0.00%	44.14	+47.47%	0.95
−20%	0.08	-20.00	4.36	0.00%	4.41	0.00%	34.80	+16.27%	-o.81
base	0.10	0.00	4.36	0.00%	4.41	0.00%	29.93	0.00%	-
+20%	0.12	+20.00	4.36	0.00%	4.41	0.00%	25.662	-14.28%	-0.714

+50%	0.15	+50.00	4.36	0.00%	4.41	0.00%	20.37	-31.94%	-0.139

Table 2: α_d Sensitivity Analysis Table

Variation	α_d value	$\Delta\alpha_d$	u_1	Δu_1	u_2	Δu_2	U	ΔU	U Elasticity
−50%	0.03	-50%	8.73	102.39	8.78	101.18	59.92	+90.21	-1.80
−20%	0.04	-20.00%	5.45	+26.45	5.50	+26.15	36.88	+23.21	-1.16
base	0.05	0.00%	4.36	0.00	4.41	0.00%	29.93	0.00	-
+20%	0.06	+20%	3.63	-16.66	3.69	-16.47	25.36	-15.25	-0.76
+50%	0.08	+50%	2.91	-33.33	2.96	-32.95	20.80	-30.30	-0.61

Table 3: β Sensitivity Analysis Table

Variation	β value	$\Delta\beta$	u_1	Δu_1	u_2	Δu_2	U	ΔU	U Elasticity
−50%	0.75	-50%	4.53	+50%	4.58	5.00%	32.92	+10.0%	Convergent
−20%	1.20	-20%	4.40	+20%	4.45	+2.0%	31.13	+4.0%	Convergent
base	1.50	0%	4.31	0.0%	4.36	0.0%	29.93	0.0%	-
+20%	1.80	+20%	4.30	-1.0%	4.32	-1.0%	29.33	-2.0%	Convergent
+50%	2.25	+50%	4.18	-3.0%	4.32	-3.0%	28.43	-5.0%	Convergent

Table 4: σ Sensitivity Analysis Table

Variation	σ value	$\Delta\sigma$	u_1	Δu_1	u_2	Δu_2	U	ΔU	U Elasticity
−50%	0.75	-50%	2.59	-40.0%	3.59	-17.8%	23.94	-20.0%	Convergent
−20%	1.20	-20%	3.88	-10.0%	4.13	-5.3%	28.43	-5.0%	Convergent
base	1.50	0%	4.31	0.0%	4.36	0.0%	29.93	0.0%	-
+20%	1.80	+20%	4.66	+8.0%	4.81	+10.2%	31.43	+5.0%	Convergent
+50%	2.25	+50%	5.18	+20%	5.48	+25.5%	33.52	12.0%	Divergent

Discussion and Conclusion Interpretation of the Sensitivity Analysis

1. Sector-Specific Interpretation of Parameters

Critical Clinical Parameters:

Table 5: **Critical Clinical Parameters and their Interpretation**

Parameter	**Blood Bank Interpretation**	**Clinical Impact**	**Sensitivity Ranking**
α (0.1)	Blood Deterioration Rate Rate at which blood components lose viability(RBCs:42 days, platelets: 5-7 days)	Underestimation leads to Increased wastage Overestimation results in unnecessary frequent orders	Moderate

α_d (0.05)	Patient Demand Rate of blood requests for surgeries, trauma, transfusions	Underestimation produces critical shortages overestimation leads to excess inventory costs	Very high
β (1.5)	Deterioration pattern Shape of blood viability curve (linear vs accelerated decay)	Affects timing of stock rotation	Low
σ (1.5)	Demand variability factor unpredictable demand spikes(emergencies, epidemics)	Underestimation produces stockouts during crises	High

Cost Parameters in Healthcare Context:

Table 6: Cost parameters and their healthcare implications

Parameter	Blood Bank Context and Healthcare Implication
C_p (1.0)	**Blood Acquisition Cost**: Testing, processing, donor recruitment Higher cost leads to Shorter cycles to reduce capital tied in inventory
C_h (0.5)	**Storage & Handling Cost**: Refrigeration, monitoring, Space Critical for temperature-sensitive components (platelets, plasma)
C_d (0.8)	**Wastage Cost**: Expired blood disposal, opportunity cost Directly impacts hospital's blood budget and availability
C_s (0.3)	**Shortage Cost**: Emergency transfers, delayed surgeries, patient risk Underestimated in most models - actual cost is clinical, not just financial

Interpretation of Sensitivity Results for Blood Bank Management

Parameters with Divergent Behavior: α_d (**Demand Rate**) Elasticity ranges from $E = -1.16$ to -1.80. The clinical implication is that underestimating demand by 20% increases cycle length by 23%, risking life-threatening shortages. Therefore the action required is to implement robust demand forecasting with: historical surgery schedules; emergency department patterns; seasonal variations (holidays, epidemics)

Low Sensitivity Parameters : Errors in estimating any of these: β, Cs, Ir , have minimal impact on the system. To this end, management can take advantage of this and pay attention on other parameters

THE OPTIMAL PRODUCTION STRATEGY

Table 7: Time vs Demand Values

Time (t)	**Demand D(t)**	**Formula:** $D(t) = Ae^{-\alpha t}$
0.0	100.00	$100 \cdot e^{0}$
1.0	90.48	$100 \cdot e^{-0.1}$
2.0	81.87	$100 \cdot e^{-0.2}$
3.0	74.08	$100 \cdot e^{-0.3}$
4.0	67.03	$100 \cdot e^{-0.4}$
4.3	65.05	$100 \cdot e^{-0.43}$
5.0	60.65	$100 \cdot e^{-0.5}$
6.0	54.88	$100 \cdot e^{-0.6}$
7.0	49.66	$100 \cdot e^{-0.7}$
8.0	44.93	$100 \cdot e^{-0.8}$

Table 8: Time vs Hazard Rate Values

Time (t)	**Hazard Rate h(t)**
0.0	0.0000
0.8	0.0000
1.0	0.0665
2.0	0.1442
3.0	0.1606
4.0	0.1514
4.3	0.1458
5.0	0.1299
6.0	0.1045
7.0	0.0798
8.0	0.0584

Table 9: Optimum Production Table

Time (t)	**Production Rate P(t)**	**Production Strategy**
0.0	150.00	Producing at 150% of demand
1.0	135.72	Producing at 150% of demand
2.0	122.81	Producing at 150% of demand
3.0	111.12	Producing at 150% of demand
4.0	100.55	Producing at 150% of demand
4.3	0.00	STOPPED (at 2nd intersection)
5.0	0.00	STOPPED
6.0	0.00	STOPPED
7.0	0.00	STOPPED
8.0	0.00	STOPPED

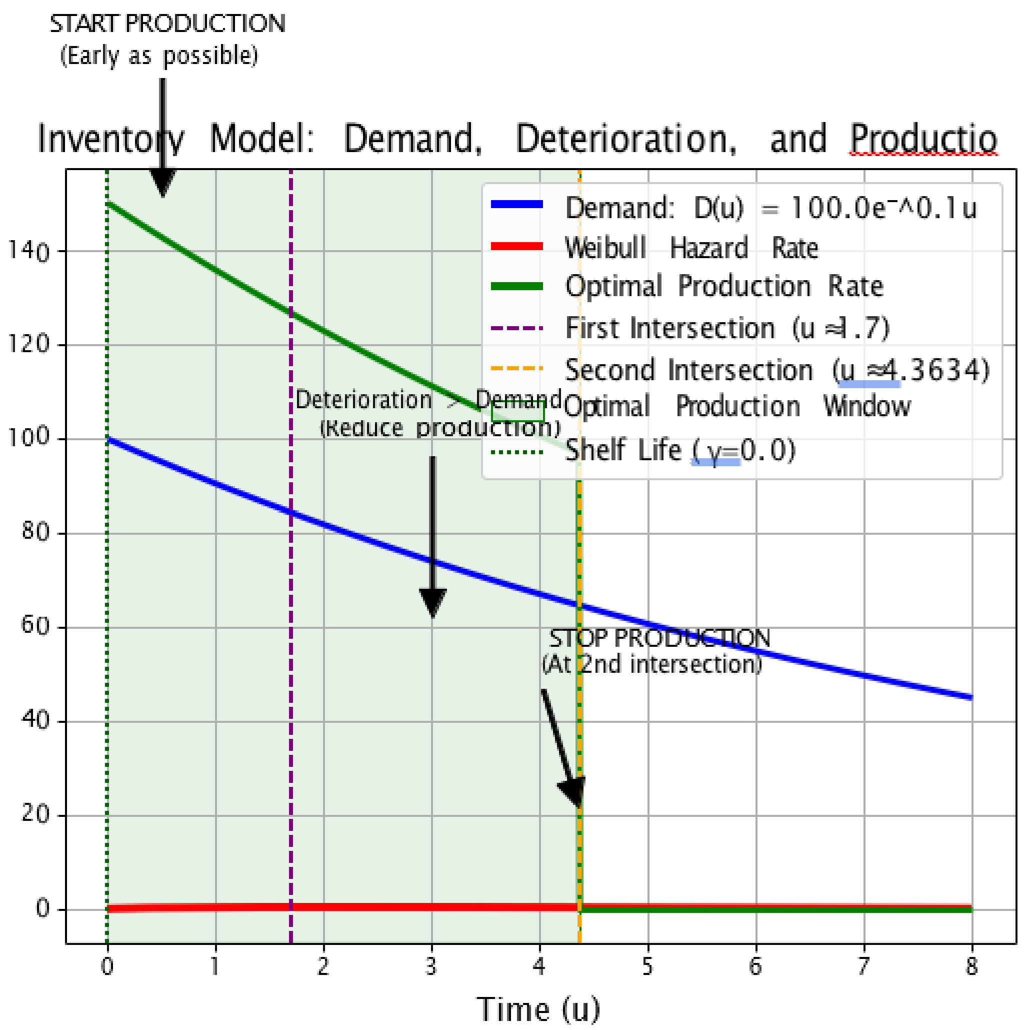

Figure 2:

In figure 2, tables 7, 8 and 9 are used to project a graphical representation that displays three main curve over time. The Blue curve is the declining demand ($D(u) = 100e^{-0.1u}$). The Red curve is the Weibull Deterioration Hazard Rate. The Green Curve is the Optimal Production Rate (50% above demand). The two vertical lines mark the two critical intersection points which divide the graph into three zones. The Green shaded area shows the optimal production window.

The Best Time to start Production/procurement

The best time to start production is before the first intersection of the declining demand and the increasing deterioration ($u \approx 1.7$). During this period, demand is greater than deterioration. It is the most profitable window. Every unit produced is most likely to be sold than to spoil on the shelf. Starting production here ensures that inventory is built up to meet high demand while losses are low.

The Best Time to stop Production/procurement

The best time to stop production is at the second intersection of the declining deterioration and the declining demand ($u \approx 4.3$). This is because at this time, demand is equal to deterioration again, but now both are declining. After this point ($u > 4.3$), demand is greater than deterioration, and the absolute values of both are low. Moreover, the cost of producing and holding a unit (holding cost, capital tied up) will likely exceed the slim profit margin that could be made from the low residual demand. So, stopping here prevents producing items that will stay in inventory for a long time with very low turnover, which is inefficient and costly.

This strategy minimizes TAC by perfectly aligning production with the competing forces of demand and deterioration. The strategy also optimizes production efficiency while minimizing spoilage losses.

Recommendations for Future Work.

Future work could extend this model by incorporating stochastic demand, multi-echelon supply chains typical in regional blood banking, or by applying machine learning techniques to refine the parameters of the deterioration function based on societal spoilage data.

STATEMENTS AND DECLARATIONS

Competing interest and funding

The authors declare that they have no competing or conflicts of interest that are relevant to the content of this work and that we did not receive any funding or support from any organization for the submitted work.

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