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PRIMES MADE SIMPLE

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ABSTRACT

In this article, the author presents yet another alternative approach to proving the binary Goldbach-Euler conjecture [2]. Drawing upon the ternary and binary problems, the author highlights the inherent systematic nature of prime numbers by proving a theorem regarding the arithmetic mean of a prime number. The article concludes with calculations that substantiate the aforementioned findings.

Keywords: Solutions to current problems in number theory,

INTRODUCTION

1 Theorem

If the sum of three prime numbers, starting from 7, constitutes any odd number, then the sum of two prime numbers constitutes any even number, starting from 4. The sum of three prime numbers, equaling any odd number, implies the following:

$$p_1 + p_2 + p_3 + 2 = p_4 + p_5 + p_6 \tag{1}$$

From which it trivially follows:

$$p_1 + p_2 - p_6 + 2 = p_4 + p_5 - p_3 \tag{2}$$

From this, it follows that the difference between the sum of two primes and a single prime is any odd number, starting from 1 (2 + 2 – 3 = 1). Thus, we have:

$$p_1 + p_2 + p_3 = p_4 + p_5 - p_6 \tag{3}$$

$$p_1 + p_2 + p_3 + p_6 = p_4 + p_5 \tag{4}$$

The proof is as follows: the sum of four prime numbers is identical to the sum of two prime numbers. The sum of three prime numbers plus any odd prime number yields any even number starting from 10. We will prove (4) using the method of mathematical induction.

The sum of three prime numbers:

$$p_1 + p_2 + p_3 = 2(K+2) + 1 \tag{5}$$

For K=1 we have $p_1 = 2,\ p_2 = 2,\ p_3 = 3,\ p_6 = 3$ .Then in (4) $p_4 + p_5 = 10$ and $p_4 = p_5 = 5$ or $p_4 = 3$, $p_5 = 7$.

We assume that for some K=k, condition (4) is satisfied; in this case:

$$2(k+2) + 1 + p_6 = p_4 + p_5 \tag{6}$$

It is necessary to prove that, for K = k + 1, equality (4) also holds. We will prove this by contradiction:

$$2(k+2+1) + 1 + p_{12} \neq p_{10} + p_{11} \tag{7}$$

Subtract (6) from (7):

$$2 + p_{12} - p_6 \neq p_{10} + p_{11} - p_4 - p_5 \tag{8}$$

$$2 + p_{12} + p_4 + p_5 \neq p_{10} + p_{11} + p_6 \tag{9}$$

This implies that, within the set of prime numbers, there do not exist six primes that would satisfy the equality expressed in (9), a result that clearly contradicts (1) and (9).

Furthermore, the sum of two primes yields any even number starting from 10.For numbers up to 10, this is arithmetically confirmed.

2. Theorem

Any odd prime number, starting from 7, can be expressed as the difference of a doubled prime number and a prime number; or, alternatively, any prime number, starting from 5, is the arithmetic mean of two other primes.

Proof.

The author asserts that the binary Goldbach–Euler problem was proven above as an alternative [2]. From which it follows:

$$p_1 + p_2 + 2 = p_3 + p_4 \tag{10}$$

hence:

$$p_1 - p_3 + 2 = p_3 - p_2 \tag{11}$$

This means that, just like the sum of two primes (starting from 4), the difference also equals any even number, starting from 2.

Thus, the ternary Goldbach problem has an alternative form:

$$p_1 + p_2 - p_3 + 2 = p_4 + p_5 - p_6 \qquad (12)$$

And it can be formulated as follows:

Any odd number, starting from 3, can be expressed as the difference between the sum of two odd primes and a single odd prime.

Therefore, starting from three, any odd prime:

$$p_1 + p_2 - p_3 = p_4 \qquad (13)$$

where the numbers in (13) are odd primes.

Given $p_1 + p_2$ that this applies to any even number starting from 6, it is valid to proceed as follows- we set them equal to each other $p_3 = p_4$ and continue:

$$p_1 + p_2 = 2\,p_3\,, \quad p_3 = \frac{p_1 + p_2}{2} \qquad (14)$$

By contradiction we show that in (14) $p_1 = p_2 = p_3$ not the only solution, but rather we have a second solution, where $p_1 \neq p_2$.

Because p_1 and p_2 two odd numbers, and p_3 arithmetic mean from which it follows:

$$p_1 - p_3 = p_3 - p_2 = a \qquad (15)$$

Thus, the difference:

$$p_1 - p_2 = 2\,a \qquad (16)$$

And since *a* is an even number, the difference is divisible without a remainder by at least four:

$$p_1 - p_2 = 4\,b \qquad (17)$$

where *b* is any integer. The impossibility of the difference between two prime numbers being equal to (17) implies that, within the set of odd primes, there exists a prime number that is not an arithmetic mean namely

$$2\,p_2 + 4\,b \neq 2\,p_3\,, \quad p_3 - p_2 \neq 2\,b \qquad (18)$$

A Clear Contradiction (11)!

Which was to be proved.

3. The difference between the simple arithmetic mean and the lower and upper simple values.

The difference is a = 2 only in the single case when $p_3 = 5$, what is related to the uniqueness of the three prime twins 3, 5, 7.

In the event of $p_2 = 3$ and $p_3 > 5$ *a* -even without a prime factor 3.

If $p_2 \neq 3$ then the difference is a = 6, or is a multiple a = 6k, where k is a rational number. Suppose that *a* is not a multiple of three; in this case, there exists an odd prime:

$p = 3 * ... + d$, a= 3*...+c, where p is an odd prime, d and c are the remainder equal to 1 or 2. Then, according to (14):

$$p_2 + a = p_3 \tag{19}$$

$$p_3 + a = p_1 \tag{20}$$

for different d and c (14) p_3 a composite number divisible by 3.

Given equal values of *d* and *c*, the difference *a* is inevitably divisible by three.Thus, every prime number starting from 5 is the arithmetic mean of a larger and a smaller number; furthermore—beginning with 11—the difference between the larger and the mean, as well as between the mean and the smaller number, is equal to 6 or is a multiple of six.

$$p_1 - p_2 = 12k \tag{21}$$

where:

k is an integer.

4. The Interrelationship of Prime Numbers

According to the theorem above, from every prime number of smaller magnitude follow in greater order through an interval of 6 or multiples, at intervals of 12 units and its multiples -a pattern that can be utilized as an additional tool for identifying prime numbers of the highest magnitude.To represent the entire table up to 10,000, in accordance with the above, it is necessary to shift to an interval of 6 units and its multiples, and to present two sequences of prime numbers based on the following formulas:

$$p_1 = 7 + 6k \tag{22}$$

and the sequence itself (with *k* in curly brackets):

7{1}13{2}19{4}31...5011{834}...9949{1657}9967{1660}9973{1661}

The second sequence:

$$p_1 = 11 + 6k \tag{23}$$

11{1}17{1}23 {1}29...5021{835}...9923{1652}9929{1653}9941{1655}

Next, we will perform a calculation for a value exceeding 10000. As the numerical value for a prime number, we will adopt the arithmetic mean of the primes within the range up to 10000; then, using a step-by-step selection method, we will determine larger values:

$p_3 = 9973, \quad p_2 = 9887, \quad p_1 = 10159, \quad a = 6 * 31 = 186$

$p_3 = 9887, \quad p_2 = 9623, \quad p_1 = 10151, \quad a = 6 * 44 = 264$

$p_3 = 9931, \quad p_2 = 9721, \quad p_1 = 10141, \quad a = 6 * 35 = 210$

$p_3 = 9949, \quad p_2 = 9859, \quad p_1 = 10139. \quad a = 6 * 15 = 90$

$p_3 = 9941, \quad p_2 = 9749, \quad p_1 = 10133, \quad a = 6 * 32 = 192$

$p_3 = 9871, \quad p_2 = 9631, \quad p_1 = 10111, \quad a = 6 * 40 = 240$

$p_3 = 9791, \quad p_2 = 9479, \quad p_1 = 10103, \quad a = 6 * 52 = 312$

$p_3 = 9781, \quad p_2 = 9463, \quad p_1 = 10099, \quad a = 6 * 53 = 318$

$p_3 = 9721, \quad p_2 = 9349, \quad p_1 = 10093, \quad a = 6 * 62 = 372$

$p_3 = 9791, \quad p_2 = 9491, \quad p_1 = 10091, \quad a = 6 * 50 = 300$

$p_3 = 9941, \quad p_2 = 9803, \quad p_1 = 10079, \quad a = 6 * 23 = 138$

$p_3 = 9949, \quad p_2 = 9829, \quad p_1 = 10069, \quad a = 6, * 20 = 120$

$p_3 = 9929, \quad p_2 = 9791, \quad p_1 = 10067, \quad a = 6, * 23 = 138$

$p_3 = 9791, \quad p_2 = 9521, \quad p_1 = 10061, \quad a = 6, * 45 = 270$

$p_3 = 9859, \quad p_2 = 9679, \quad p_1 = 10039, \quad a = 6 * 30 = 180$

$p_3 = 9857, \quad p_2 = 9677, \quad p_1 = 10037, \quad a = 6 * 30 = 180$

$p_3 = 9829, \quad p_2 = 9649, \quad p_1 = 10009, \quad a = 6 * 30 = 180$

$p_3 = 9749, \quad p_2 = 9491, \quad p_1 = 10007, \quad a = 6 * 43 = 258$

Similarly, we identify other prime numbers greater than 10000; we determine prime numbers within new, larger intervals-a task made feasible by powerful computers capable of identifying prime numbers with on the order of 10 to the 24th power digits.

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