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TRANSITIVITY AND IMPRIMITIVITY ACTION OF THE PRODUCT OF THREE ALTERNATING GROUPS ACTING ON THE CARTESIAN PRODUCT OF THREE SETS OF ORDERED QUADRUPLES

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ABSTRACT

In this paper, we determine the transitivity and primitivity actions of the product of three alternating groups acting on the cartesian product of three sets of ordered quadruples through the determination of the number of orbits and using the definition primitivity and blocks respectively[1]. When $n\geq 6$, the action of the product of three alternating groups, $A_n\times A_n\times A_n$, acts both transitively and imprimitively on the cartesian product of three sets of ordered quadruples, T^[4] ×U^[4] ×W^[4] [2].

Keywords: Transitivity Action, Primitivity Action, Ordered Sets of Quadruples, Cartesian Product and Alternating Group

INTRODUCTION

Cameron (1972) worked on the suborbits of multiply transitive permutations and later in 1973 studied the suborbits of primitive groups.[3] Ndarinyo et al., (2015) showed that the alternating group $A_n=5,6,7$ acts transitively on unordered and ordered triples from the set $P=1,2,\ldots,n$ when $n\leq 7$ through determination of the number of orbits[4]. Muriuki et al., (2017) showed that for the action of direct product of three symmetric groups on Cartesian product of three sets, the action is both transitive and imprimitive for all $n\geq 2$ and the associated rank is 2^3. Mutua et al., (2018) showed that the direct product of $S_n\times A_n$, of the symmetric group S_n by the alternating group A_n on the cartesian product $X\times Y$ has its action both transitive and imprimitive when $n\geq 3$. Nyaga (2018) proved that the direct product action of the alternating group on the Cartesian product of three sets is transitive[5]. Maraka et al., (2021) showed that the action of the cartesian product of the alternating group, $A_n\times A_n\times A_n$, on the cartesian product of P^[3] ×S^[3] ×V^[3] , the cartesian product of ordered sets of triples is transitive when $n\geq 5$[6]. Maraka et al., (2022) showed that the action of the cartesian product of the alternating group, $A_n\times A_n\times A_n$, on the cartesian product of P^[3] ×S^[3] ×V^[3] , the cartesian product of ordered sets of triples is imprimitive when $n\geq 5$.[7] Based on these results we investigate some properties of $A_n\times A_n\times A_n$, the cartesian product action of the alternating group acting on T^[4] ×U^[4] ×W^[4] , the cartesian product of ordered sets of triples[8].

The product of three alternating groups, A_n×A_n×A_n, acts on T^[4] ×U^[4] ×W^[4] , by the rule;

$g_1\{([1,2,3,4],[1,2,3,5],\ldots,[n,n-1,n-2,n-4],[n,n-1,n-2,n-3])\}\times g_2\{([n+1,n+2,n+3,n+4],[n+1,n+2,n+3,n+5],\ldots,[2n,2n-1,2n-2,2n-4],[2n,2n-1,2n-2,2n-3])\}\times g_3\{([2n+1,2n+2,2n+3,2n+4],[2n+1,2n+2,2n+4,2n+5],\ldots,[3n,3n-1,3n-2,3n-4],[3n,3n-1,3n-2,3n-3])\}=\{([1,2,3,4],[1,2,3,5],\ldots,[n,n-1,n-2,n-4],[n,n-1,n-2,n-3]),g_2([n+1,n+2,n+3,n+4],[n+1,n+2,n+3,n+5],\ldots,[2n,2n-1,2n-2,2n-4],[2n,2n-1,2n-2,2n-3]),g_3([2n+1,2n+2,2n+3,2n+4],[2n+1,2n+2,2n+4,2n+5],\ldots,[3n,3n-1,3n-2,3n-4],[3n,3n-1,3n-2,3n-3])\};$

$\forall\, g_1,g_2,g_3\in A_n,\ \{([1,2,3,4],[1,2,3,5],\ldots,[n,n-1,n-2,n-4],[n,n-1,n-2,n-3])\}\in T^{[4]}$, set of ordered quadruples from the set $P=\{1,2,3,\ldots,n\}$; $\{([n+1,n+2,n+3,n+4],[n+1,n+2,n+3,n+5],\ldots,[2n,2n-1,2n-2,,2n-4],[2n,2n-1,2n-2,2n-3])\}\in U^{[4]}$, set of ordered quadruples from the set

$U=\{n+1,n+2,\ldots,2n\}$; and $\{([2n+1,2n+2,2n+3,2n+4],[2n+1,2n+2,2n+3,2n+5],\ldots,[3n,3n-1,3n-2,3n-4],[3n,3n-1,3n-2,3n-3])\}\in W^{[4]}$, set of ordered quadruples from the set $W=\{2n+1,2n+2,\ldots,3n\}$.

PRELIMINARIES

1.1.Notation and Terminology

In this paper, we shall represent the following notations as: ∑- sum over i; A_n -an alternating group of degree n and order n!/2 ; |G| – the order of a group G; |G:H| -Index of H in G; T^[4] – the set of an ordered triple from set T={1,2,3,...,n}; U^[4] – the set of an ordered triple from set U={n+1,n+2,...,2n}; W^[4] – the set of an ordered triple from set W={2n+1,2n+2,...,3n}; [a,b,c,d] -Ordered quadruple; A_n×A_n×A_n -Cartesian product of alternating group A_n; T^[4] ×U^[4] ×W^[4] -Cartesian product of ordered sets of triples T^[4] ,U^[4] and W^[3] [9].

Definition 1.1.1: Group action (Kinyanjui et al., 2013): Let T be a non-empty set. A group G is said to act on the left of T if for each $g\in G$ and each $t\in T$ there corresponds a unique element $gt\in G$ such that:

(i) $(g_1g_2)=g_1(g_2t),\ g_1,g_2\in G$ and $t\in T$.

(ii) For any $t\in T$, $et=T$, where e is the identity in G.

The action of G from the right on T can be defined in the same manner.[10]

Definition 1.1.2: Orbit (Njagi, 2016): Let G act on a set T . Then T is partitioned into disjoint equivalent classes called orbits or transitivity classes of the action. For every $t\in T$ the orbit containing t is called the orbit of t and is denoted by $Orb_G(t)$.

Definition 1.1.3: Fixed point (Kinyanjui et al., 2013): Let G act on a set T. The set of elements of T fixed by $g\in G$ is called the fixed-point set of G and is denoted by $Fix(g)$. Thus $Fix(g)=\{t\in T|gt=t\}$.

Definition 1.1.4: Transitive group (Cameron, 1970): If the action of a group G on set T has only one orbit, then we say that G acts transitively on T. In other words, G acts transitively on T if for every pair of points $t, u \in T$, there exists $g \in G$ such that $gt = u$[11].

Theorem 1.1.4: (Orbit – Stabilizer Theorem, Rose, 1978, p.72): Let G act on a set T. Then $|Orb_G(t)| = |G: Stab_G(t)|$.

Definition 1.1.5: Blocks and primitivity (Nyaga et al., 2011):

Assume that the action of G on T is transitive. For every subset A of T such that each $g \in G$, let $gA = \{ga: a \in A\} \subseteq T$. A subset A of T is referred to as a block for the action if, for every $g \in G$, either $gA = A$ or $gA \cap A = \emptyset$. $\emptyset, T$ and all singleton subsets of T are definitely blocks known as trivial blocks. If these are the only blocks, then G acts primitively on T otherwise, G acts imprimitively[12].

Definition 1.1.6: Direct product action (Cameron et al, 2008): Let (G_1, P_1) and (G_2, P_2) be permutation groups. The direct product $G_1 \times G_2$ acts on the disjoint union $P_1 \cup P_2$ by the rule $p(g_1, g_2) = \begin{cases} pg_1, if\ p \in P_1, \\ pg_2, if\ p \in P_2 \end{cases}$ and on the Cartesian product $P_1 \times P_2$ by the rule $(p_1, p_2)(g_1, g_2) = (p_1 g_1, p_2 g_2)$.

Theorem 1.1.7: (Armstrong, 2013): The $G_1 \times G_2 \times G_3$-orbit containing $(t, u, w) \in T \times U \times W$ is given by $Orb_{G_1}(t) \times Orb_{G_2}(u) \times Orb_{G_3}(w)$ and the stabilizer of (t, u, w) is given by $Stab_{G_1}(t) \times Stab_{G_2}(u) \times Stab_{G_3}(w)$.

RESULTS

Lemma 2.1: The action of the product of three alternating groups $A_6 \times A_6 \times A_6$, acting on the cartesian product of three sets of ordered quadruples, $T^{[4]} \times U^{[4]} \times W^{[4]}$ is transitive.

Proof: Let $G = A_6 \times A_6 \times A_6$ act on $T^{[4]} \times U^{[4]} \times V^{[4]}$ where;

gap> Arrangements([1,2,3,4,5,6],4);
$T^{[4]}$ = { [[1, 2, 3, 4], [1, 2, 3, 5], [1, 2, 3, 6], ... ,[6, 5, 4, 1], [6, 5, 4, 2], [6, 5, 4, 3]]} ;
gap> Arrangements([7,8,9,10,11,12],4);
$U^{[4]}$ = { [[7, 8, 9, 10], [7, 8, 9, 11], [7, 8, 9, 12],. . . . [12, 11, 10, 7], [12, 11, 10, 8], [12, 11, 10, 9]]} and;
gap> Arrangements([13,14,15,16,17,18],4);
$W^{[4]}$ = { [[13, 14, 15, 16], [13, 14, 15, 17], [13, 14, 15, 18],. . . . [18, 17, 16, 13], [18, 17, 16, 14], [18, 17, 16, 15]] }.
The cartesian product of $T^{[4]} \times U^{[4]} \times W^{[4]}$ is generated using the GAP software with;
$\left| T^{[4]} \times U^{[4]} \times W^{[4]} \right| = 46\ 656\ 000$. G is generated by
$< \{(123456), (1234)\}, \{(7\ 8\ 9\ 10\ 11\ 12), (7\ 8\ 9\ 10)\}, \{(13\ 14\ 15\ 16\ 17\ 18), (13\ 14\ 15\ 16)\} >$ using the GAP software. ([1,2,3,4], [7,8,9,10], [13,14,15,16]) is fixed by an element $(g_t, g_u, g_w) \in G$ if and only if 1,2,3 and 4 comes from a single cycle in g_t ; 7,8,9 and 10 comes from a single cycle in g_u and 13,14,15 and 16 comes from a single cycle of g_w .

The $|Stab_G([1,2,3,4],[7,8,9,10],[13,14,15,16])| = 1.$

Using the Orbit-Stabilizer Theorem,

$$|Orb_G([1,2,3,4],[7,8,9,10],[13,14,15,16])| = |G: Stab_G([1,2,3,4],[7,8,9,10],[13,14,15,16])|$$

$$= \frac{|G|}{|Stab_G([1,2,3,4],[7,8,9,10],[13,14,15,16])|}$$

$$= \frac{46\,656\,000}{1} = 46\,656\,000 = \left|T^{[4]} \times U^{[4]} \times W^{[4]}\right|$$

Hence, the action of $A_6 \times A_6 \times A_6$ on $T^{[4]} \times U^{[4]} \times W^{[4]}$ is transitive.

Lemma 2.2: The action of the product of three alternating groups $A_7 \times A_7 \times A_7$, acting on the cartesian product of three sets of ordered quadruples, $T^{[4]} \times U^{[4]} \times W^{[4]}$ is transitive.

Proof: Let $G = A_7 \times A_7 \times A_7$ act on $T^{[4]} \times U^{[4]} \times W^{[4]}$ where;

gap> Arrangements([1,2,3,4,5,6,7],4);
$T^{[4]}$ = {[[1, 2, 3, 4], [1, 2, 3, 5], [1, 2, 3, 6], [1, 2, 3, 7 , . .. [7, 6, 5, 1], [7, 6, 5, 2], [7, 6, 5, 3], [7, 6, 5, 4]]};
gap> Arrangements([8,9,10,11,12,13,14],4);
$U^{[4]}$ = { [[8, 9, 10, 11], [8, 9, 10, 12], [8, 9, 10, 13], [8, 9, 10, 14], . . . , . , .,[14, 13, 12, 8], [14, 13, 12, 9], [14, 13, 12, 10], [14, 13, 12, 11]} and;
gap> Arrangements([15,16,17,18,19,20,21],4); $W^{[4]}$ =
{[[15, 16, 17, 18], [15, 16, 17, 19], [15, 16, 17, 20],
... , [21, 20, 19, 16], [21, 20, 19, 17], [21, 20, 19, 18]]

The cartesian product of $T^{[4]} \times U^{[4]} \times W^{[4]}$ is generated using the GAP software with;
$\left|T^{[4]} \times U^{[4]} \times W^{[4]}\right| = 592\,704\,000.$ G is generated by
$<$
$\{(1234567),(1234)\},\{(8\ 9\ 10\ 11\ 12\ 13\ 14),(\ 8\ 9\ 10\ 11)\},\{(15\ 16\ 17\ 18\ 19\ 20\ 21),(15\ 16\ 17\ 18)\} >$
using the GAP software. ([1,2,3,4],[8,9,10,11],[15,16,17,18]) is fixed by an element $(g_t, g_u, g_w) \in G$ if and only if 1,2,3 and 4 comes from a single cycle in g_t ; 8,9,10 and 11 comes from a single cycle in g_u and 15,16,17 and 18 comes from a single cycle of g_w .

The $|Stab_G([1,2,3,4],[8,9,10,11],[15,16,17,18])| = 27.$

Using the Orbit-Stabilizer Theorem,

$$|Orb_G([1,2,3,4],[8,9,10,11],[15,16,17,18])| = |G: Stab_G([1,2,3,4],[8,9,10,11],[15,16,17,18])|$$

$$= \frac{|G|}{|Stab_G([1,2,3,4],[8,9,10,11],[15,16,17,18])|}$$

$$= \frac{16\,003\,008\,000}{27} = 592\,704\,000 = \left|T^{[4]} \times U^{[4]} \times W^{[4]}\right|$$

Hence, the action of $A_7 \times A_7 \times A_7$ on $T^{[4]} \times U^{[4]} \times W^{[4]}$ is transitive.

Theorem 2.3: The action of the product of three alternating groups A_n , $A_n \times A_n \times A_n$, acting on the cartesian product of three sets of ordered quadruples, $T^{[4]} \times U^{[4]} \times W^{[4]}$ is transitive if and only if $n \geq 6$.

Proof: Let $G = G_p \times G_s \times G_v = A_n \times A_n \times A_n$ act on $T^{[4]} \times U^{[4]} \times W^{[4]}$. It suffices to verify that $\left|T^{[4]} \times U^{[4]} \times W^{[4]}\right|$ is equal to $|Orb_G([1,2,3,4],[n+1,n+2,n+3,n+4],[2n+1,2n+2,2n+3,2n+4])|$.

Let $|M| = |Stab_G([1,2,3,4],[n+1,n+2,n+3,n+4],[2n+1,2n+2,2n+3,2n+4])|$.

So, $(g_t, g_u, g_w) \in G = A_n \times A_n \times A_n$ fixes $([1,2,3,4],[n+1,n+2,n+3,n+4],[2n+1,2n+2,2n+3,2n+4]) \in T^{[4]} \times U^{[4]} \times W^{[4]}$ if and only if $1,2,3$ and 4 comes from 1-cycle of g_t ; $n+1, n+2, n+3$ and $n+4$ comes from 1-cycle of g_u and $2n+1, 2n+2, 2n+3$ and $2n+4$ comes from 1-cycle of g_w .

The cardinality of $T^{[4]} \times U^{[4]} \times W^{[4]}$ is given as;

$$\left| T^{[4]} \times U^{[4]} \times W^{[4]}\right| = \left(\frac{n!}{(n-4)!}\right)^3$$

The $Stab_G([1,2,3,4],[n+1,n+2,n+3,n+4],[2n+1,2n+2,2n+3,2n+4])$ is isomorphic to: $A_{n-4} \times A_{n-4} \times A_{n-4}$.

Therefore, $|M| = |Stab_G([1,2,3,4],[n+1,n+2,n+3,n+4],[2n+1,2n+2,2n+3,2n+4])| = \left|Stab_{G_t}([1,2,3,4]) \times Stab_{G_u}([n+1,n+2,n+3,n+4]) \times Stab_{G_w}([2n+1,2n+2,2n+3,2n+4])\right|$

$$|M| = \frac{(n-4)! \times (n-4)! \times (n-4)!}{2 \times 2 \times 2} = \left(\frac{(n-4)!}{2}\right)^3$$

Applying the Orbit-Stabilizer Theorem we get;

$$|Orb_G([1,2,3,4],[n+1,n+2,n+3,n+4],[2n+1,2n+2,2n+3,2n+4])|$$
$$= |G : Stab_G([1,2,3,4],[n+1,n+2,n+3,n+4],[2n+1,2n+2,2n+3],2n+4)|$$

$$|G| = \frac{n! \times n! \times n!}{2 \times 2 \times 2} = \left(\frac{n!}{2}\right)^3$$

$$\frac{|G|}{|R|} = \frac{\left(\frac{n!}{2}\right)^3}{\left(\frac{(n-4)!}{2}\right)^3} = \left(\frac{n!}{(n-4)!}\right)^3 .$$

Therefore;

$$\frac{|G|}{|M|} = \left(\frac{n!}{(n-4)!}\right)^3 = \left|T^{[4]} \times U^{[4]} \times W^{[4]}\right|$$

Hence, $A_n \times A_n \times A_n$ acts transitively on $T^{[4]} \times U^{[4]} \times W^{[4]}$ if $n \geq 6$.

Corollary 2.4: For $n < 6$, the $|Stab_G([1,2,3,4],[n+1,n+2,n+3,n+4],[2n+1,2n+2,2n+3,2n+4])| = |A_{n-4} \times A_{n-4} \times A_{n-4}| < 1$. The order of a permutation group must be finite (countable) and not a negative or fractional number. The order of the stabilizer is less than 1 for all $n < 6$. Hence, $n \geq 6$.

Theorem 2.5: The action of the cartesian product of the alternating group A_n, $A_n \times A_n \times A_n$, acting on the cartesian product of ordered sets of quadruples, $T^{[4]} \times U^{[4]} \times W^{[4]}$, is imprimitive for $n \geq 6$.

Proof: The action of $A_n \times A_n \times A_n$ on $T^{[4]} \times U^{[4]} \times W^{[4]}$ is transitive by Theorem 2.4 for $n \geq 6$.

Consider, $\forall\, g_1, g_2, g_3 \in A_n, \{([1,2,3,4],[1,2,3,5],\ldots,[n,n-1,n-2,n-4],[n,n-1,n-2,n-3])\} \in T^{[4]}$, set of ordered quadruples from the set $P = \{1,2,3,\ldots,n\}$; $\{([n+1,n+2,n+3,n+4],[n+1,n+2,n+3,n+5],\ldots,[2n,2n-1,2n-2,,2n-4],[2n,2n-1,2n-2,2n-3\,])\} \in U^{[4]}$, set of ordered quadruples from the set

$U = \{n+1,n+2,\ldots,2n\}$; and $\{([2n+1,2n+2,2n+3,2n+4],[2n+1,2n+2,2n+3,2n+5],\ldots,[3n,3n-1,3n-2,3n-4],[3n,3n-1,3n-2,3n-3])\} \in W^{[4]}$, sets of ordered quadruples from the set $W = \{2n+1,2n+2,\ldots,3n\}$.

We have;

$L = \{([1,2,3,4],[1,2,3,5],\ldots,[n,n-1,n-2,n-4],[n,n-1,n-2,n-3]) \times ([n+1,n+2,n+3,n+4],[n+1,n+2,n+3,n+5],\ldots,[2n,2n-1,2n-2,,2n-4],[2n,2n-1,2n-2,2n-3\,]) \times ([2n+1,2n+2,2n+3,2n+4],[2n+1,2n+2,2n+3,2n+5],\ldots,[3n,3n-1,3n-2,3n-4],[3n,3n-1,3n-2,3n-3])\}$.

Let L' be the non-trivial subset of L; $L' = T^{[4]'} \times U^{[4]'} \times W^{[4]'}$ such that $|L'|$ divides $|L|$.

Therefore; $\dfrac{\frac{n!}{(n-4)!}\times\frac{n!}{(n-4)!}\times\frac{n!}{(n-4)!}}{\frac{n!}{(n-4)!}} = \dfrac{|L|}{|L'|}$

$$\text{Now, } L' = \begin{Bmatrix} ([1,2,3,4],[n+1,n+2,n+3,n+4],[2n+1,2n+2,2n+4,2n+5]),\ldots, \\ ([1,2,3,4],[n+1,n+2,n+3,n+4],[3n,3n-1,3n-2,3n-4]), \\ ([1,2,3,4],[n+1,n+2,n+4,n+5],[2n+1,2n+2,2n+3,2n+4]) \end{Bmatrix}.$$

So, $|L'| = \frac{n!}{(n-4)!}$

For every element of L' there exist $(g_t, g_u, g_w) \epsilon G$ with $\frac{n!}{(n-4)!}$ cycles permutation;

$\{([1,2,3,4],[1,2,3,5],\ldots,[n,n-1,n-2,n-4],[n,n-1,n-2,n-3]),([n+1,n+2,n+3,n+4],[n+1,n+2,n+4,n+5],\ldots,[2n,2n-1,2n-2,2n-4],[2n,2n-1,2n-2,2n-3]),([2n+1,2n+2,2n+3,2n+4],[2n+1,2n+2,2n+4,2n+5],\ldots,[3n,3n-1,3n-2,3n-4],[3n,3n-1,3n-3,3n-3])\}$ such that for every $(g_p, g_s, g_v) \epsilon G$; $[1,2,3,4]$ is fixed in g_t, $[n+1,n+2,n+3,n+4]$ is fixed in g_u and that $[2n+1,2n+2,2n+3,n+4]$ belongs to a single cycle of g_w, then, (g_p, g_s, g_v) either fixes an element of $L' = T^{[4]'} \times U^{[4]'} \times W^{[4]'}$ or takes one element of L' to another so that;

$(g_t, g_u, g_w) T^{[4]'} \times U^{[4]'} \times W^{[4]'} = T^{[4]'} \times U^{[4]'} \times W^{[4]'}$. Any other $(g_t, g_u, g_w) \epsilon G$ moves an element of L' to an element not in L' so that; $(g_t, g_u, g_w)\, T^{[4]'} \times U^{[4]'} \times W^{[4]'} \cap T^{[4]'} \times U^{[4]'} \times W^{[4]'} = \emptyset$. This argument shows that L' is a non-trivial block for the action and the conclusion follows from definition 1.1.5 hence the action is imprimitive for $n \geq 6$.

CONCLUSION

The action of the product of three alternating groups, $A_n \times A_n \times A_n$, acts both transitively and imprimitively on the cartesian product of three sets of ordered quadruples, $T^{[4]} \times U^{[4]} \times W^{[4]}$, if and only if $n \geq 6$.

CONFLICT OF INTEREST

The author hereby declares that NO conflict of interest is present in this work.

DISCLAIMER (ARTIFICIAL INTELLIGENCE)

The author hereby declares that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

REFERENCES

1. Armstrong. (2013). Groups and symmetry. Springer-Verlag, New York Inc.
2. Cameron, P.J., Gewurz, D.A., Merola, F. (2008). Product action. Discrete Math. 386–394.
3. Higman, D.G. (1970). Characterization of families of rank 3 permutation groups by sub degrees. I. Arch.Math., 21: 151-156.
4. Kinyanjui, J.N., Musundi, S.W., Rimberia, J., Sitati, N.I. and Makila P. (2013). Transitivity Action of A_n (n=5, 6, 7, 8) on ordered and unordered pairs. IJMA-4(9), 77-88.
5. Maraka K. Moses, Musundi W. Sammy, Lewis N. Nyaga, "Primitivity Action of the Cartesian Product of an Alternating Group Acting on a Cartesian Product of Ordered Sets of Triples". London Journals Press, Vol. 22(7): 25-32, 2022.
6. Maraka, K. M., Musundi, S. W. and Nyaga, L.N. (2021). Transitivity Action of the Cartesian Product of the Alternating Group Acting on a Cartesian Product of Ordered Sets of Triples. Asian Research Journal of Mathematics, 17(12), 53-62.
7. Muriuki, G. D., Namu, N. L., & Kagwiria, R. J. (2017). Ranks, sub degrees and suborbital graphs of direct product of the symmetric group acting on the cartesian product of three sets. Pure and Applied Mathematics Journal, 6(1), 1.
8. Mutua, A.K., Nyaga, L.N. and Gachimu, R.K. (2018). Combinatorial properties, invariants and structures of the action of S_n×A_n on X×Y. International Journal of Sciences: Basic and Applied Research (IJSBAR), Volume 40, No. 2, pp 109-115.
9. Ndarinyo, C.W. and Rimberia, J.K. (2015). Transitivity Action of A_n (n=5,6,7) on Unordered and Ordered Triples. IJMSE, 6(7), 21-27.
10. Njagi, L.K. (2016). Ranks, subdegrees and suborbital graphs of symmetric group S_n acting on ordered pairs. Journal of advance research in applied science (issn: 2208-2352), 3(2), 51-70.

11. Nyaga, L. N. (2018). Transitivity of the direct product of the Alternating group acting on the cartesian product of three sets. International journal of mathematics and its applications, (ISSN:2347-1557), rn, 55, 7.

12. Rose, J. S. (1978). A Course in Group Theory. Cambridge University Press, Cambridge.Z.